Review of Literature and Survey of the Developed Inventory Models

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Abstract: The review of the literature is an important feature of the research work that connects the past work with the current work. The literature on the inventory modelling is growing quickly. The goal of this article is to present a complete and updated information about the developed inventory models under different situations.

Keywords: Inventory models, deteriorating items, and trade credit.

1. Introduction

Inventory modelling is a mathematical modelling that helps commercial enterprise in determining the optimum level of inventories that should be maintained in a production process, managing the frequency of ordering, deciding on quantity of goods or raw materials to be stored, tracking the flow of supply of raw materials and goods to provide continuous service to customers without any delay in delivery. This article deals with a review of the available literature on inventory modelling with and without deterioration under trade credits. This paper also gives a different classification of work done on various inventory models under different situations and business environments. The survey of various inventory models in this paper is given in the following sections:

(1). Survey of inventory models for decaying items; (2). Survey of production inventory models; (3). Survey of inventory models with trade credits; (4). Survey of inventory models with shortages; (5). Survey of inventory models with inflation.

2. Survey of Inventory Models for Decaying Items

Decay or deterioration of a product is a realistic situation associated with an inventory system. Approximately, all products deteriorate and lose their values partially or completely over time, when they are kept in stock as an inventory for fulfilling the future demand, which may occur due to one or many factors i.e. storage conditions, weather conditions or due to humidity. Therefore, the effect of deterioration is very important natural phenomenon.

In the literature of inventory theory, the deteriorating inventory models have been continually modified so as to accumulate more practical features of the real inventory systems. The economic order quantity (EOQ) model developed by Harris

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was the first mathematical inventory model to assist corporations in minimizing total inventory costs. In this classical model deterioration rate was considered as constant. Regardless of the simplicity of the EOQ model, it is still applied in the inventory modelling. The inventory problem of deteriorating items was first studied by Whitin [106]. Author studied fashion items deteriorating at the end of the storage period. The fundamental EOQ model was extended by Ghare and Schrader [83] considering the deterioration rate as a negative exponential function of time. They also proposed the following inventory model for deteriorating items:

\[
\frac{dI(t)}{dt} + \theta I(t) = -f(t),
\]

Where \(\theta\) stands for the deteriorating rate of item, \(I(t)\) stands for inventory level at any time \(t\), and \(f(t)\) is the demand rate level at any time \(t\). This inventory model laid foundations for the follow-up study in the inventory control for deteriorating products.

Aggarwal and Jaggi [95] have developed ordering policies of deteriorating items under permissible delay in payments. The demand and deterioration rate were assumed as constant in their study. Retailers pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments was discussed by Hwang and Shinn [31]. Chu et al. [78] demonstrated that the total cost function presented in Aggarwal and Jaggi [95] is not convex in general and improved their solution procedure. Jamal et al. [4] differentiated the purchasing price from the selling price in a cost minimization EOQ model for deteriorating items and obtained the retailers optimal cycle time as well as payment time when paying the remaining balance at a time between the credit period.

Chang et al. [32] have developed an inventory model for deteriorating items with partial backlogging and permissible delay in payments. Having assumed the same selling price and purchasing costs, Chang et al. [33] obtained the optimal replenishment time in an EOQ inventory model with two types of deterioration rates, linear as well as Weibull. Chang et al. [16] established an EOQ model for deteriorating items. In this model, the supplier provided conditional trade credit to the purchaser. Ouyang et al. [62] studied an economic order quantity model for deteriorating items when the supplier offers cash discount and permissible delay in payment to the retailer. Considering different selling price and purchasing costs, Teng et al. [49] established an EOQ model for deteriorating items with price sensitive demand. They found the optimum selling price and replenishment policies and challenged the results obtained in Goyal [89], Aggarwal and Jaggi [95], and Jamal et al. [4]. Ouyang et al. [59] established an EOQ model for non-instantaneous deteriorating items under permissible delay in payment and generalized some previous studies. Chang [53] challenged the process of proofs in Ouyang et al. [59] and presented the complete proofs for their proposed theorems to overcome their shortcoming.

Under trade credit policy, Khanra et al. [92] determined the optimal order quantity in an EOQ model for a deteriorating item with quadratic time dependent demand. With quite similar assumptions to Ouyang et al. [59] and Chung [53], Musa and Sani [5] generalized Goyals model [89] by incorporating non-instantaneous deterioration. Das et al. [8] studied an integrated supply chain system consisting of a suppliers production inventory model and a retailers EOQ model where the supplier provides the retailer with a delay period for deteriorating item. Liao et al. [43] assumed greater demand rate before deterioration sets in, improved and completed Musa and Sani [5] mathematically, and simplified their proposed solution procedure. Wang et al. [109] have proposed an EOQ model of a seller and determined his optimal cycle time and credit period for the deteriorating products with credit dependent demand and maximum lifetime. Chen and Teng [88] established an EOQ model under suppliers trade credit financing for continuously deteriorating items with maximum lifetime.

Shah and Chaudhari [75] formulated an integrated inventory model for three players dealing with deteriorating item with fixed lifetime and demand rate quadratically decreasing and credit period dependent. Khanna et al. [1] developed a model for deteriorating imperfect quality items with allowable shortages and permissible delay in payments. Singh et al. [98] introduced an economic order quantity model for deteriorating products having stock dependent demand with trade credit period and preservation technology. Kumar and Kumar [77] developed an inventory model with inventory-dependent demand
rate for deterioration items. Singh and Singh [99] developed an optimal inventory policy for deteriorating items with stock level and selling price dependent demand under the permissible delay in payments.

3. Survey of Production Inventory Models

The economic production quantity (EPQ) model is an extension of EOQ model. The EOQ model assumes that the quantity ordered will arrive complete and immediately after ordering. EPQ model on the other hand takes a more realistic approach, assuming the orders are available or received in an incremental manner. The classical EPQ/EOQ model has been widely used in practice because of its simplicity. But over the last few decades, numerous researches have been done to extend the EPQ model making it closer to real life situations.

Porteus [24] first presented an EPQ model with an imperfect production process that could be improved through capital investment. Balkhi and Benkherouf [114] developed a general EPQ model for deteriorating items, where demand and production rates are time varying, but the rate of deterioration is constant. Wee and Law [36] studied economic production lot size model for deteriorating items taking account of the time value of money. Abad [79] extended the optimal pricing and lot-sizing EOQ model to an economic production quantity (EPQ) model. Salameh and Jaber [66] provided an economic production quantity model for items with imperfect quality. Balkhi [113] further generalized the EPQ model to allow for time varying deterioration rate.

Abad [81] studied the pricing and lot sizing problem for a perishable good under finite production, exponential decay and partial backordering and lost sales. In fact, Abads production inventory model is similar to that in Balkhi and Benkherouf [114]. Lately, Goyal and Giri [90] investigated a similar production inventory problem in which the demand, production and deterioration rates of a product were assumed to vary with time. However, pricing was not under consideration and the backlogging rate was assumed to be a constant fraction. Finally, Goyal and Giri [90] provided a numerical example to show that their model outperforms Balkhi and Benkherous model [114] in terms of the least expensive total cost per unit time.

Chen and Chen [46] formulated a pricing and production lot size/scheduling with finite capacity for a deteriorating item over a finite time horizon. Jolai et al. [25] developed an economic production lot size model with deteriorating items, stock dependent demand, and partial backlogging. Ouyang et al. [63] proposed an integrated production inventory model with quality improvement and lead time reduction. Parveen and Rao [70] further investigated an integrated production inventory model with quality improvement, lead time, and setup cost reduction. Hu and Liu [26] analyzed an optimal replenishment policy for the EPQ model with permissible delay in payments and allowable shortages. Roy and Samanta [6] presented an extension of Goyal model [89] in which two different rates of production for deteriorating items, and different selling and purchasing prices were assumed.

Soni and Patel [76] discussed an optimal strategy for an integrated inventory system involving variable production and defective items under retailer partial trade credit policy. Singh et al. [97] studied an economic production lot size model with volume flexibility and rework under shortages. Dem and Singh [30] derived a production model with quality consideration. Yedes et al. [112] presented an integrated inventory model with an imperfect production process. To reflect the actual production and inventory situation, quality issues should be considered in the model. Palanivel and Uthayakumar [68] developed and EPQ model for deteriorating items with variable production cost, time dependent holding cost and partial backlogging under inflation. Sicilia et al. [47] established a production inventory model with power demand pattern. In this model, shortages are allowed which are partially backordered over an infinite time horizon. Tayal et al. [104] developed an EPQ model for non-instantaneous deteriorating item with time dependent holding cost and exponential demand rate. Singh et al. [101] developed a production inventory model for perishable products with trade credit period and investment in
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preservation technology. Singh and Singh [23] developed a production inventory model for deteriorating products considering both stock and demand dependent production rate under variable holding cost.

4. Survey of Inventory Models with Trade Credits

In the classical inventory modelling, the authors assumed that retailer pays purchasing cost of the items as soon as the items are received. However, such an assumption is not necessarily what happens in the real world. In practice, the supplier offers the retailer a delay period, known as the trade credit period, in paying for purchasing cost. During this trade credit period, the retailer can accumulate revenues by selling items and by earning interest. Usually, there is no interest charge if the outstanding amount is paid within this credit period. In the competitive business environment, trade credit is now an invaluable promotional tool for the suppliers to increase profitability through stimulating more sales and a unique opportunity for the retailers to reduce demand uncertainty.

Trade credit was first presented by Haley and Higgins [20] who examined the effect of trade credit on the optimal inventory policy. Although this study provides useful insights into the importance of the trade credit period in inventory control decisions, there are some limitations on their analyses and conclusions. Later, Chapman et al. [12] have developed the optimal ordering policies under different considerations such as a classical EOQ model, paying when the products are sold during a trade credit period, and paying over a fixed time period.

Goyal [89] pioneered in developing the inventory model when a supplier offers credit period in settling the account, so that no interest will be charged on the outstanding amount if the account is settled within the allowable delay period. Author ignored the difference between the selling and purchasing prices. Dave [107] redeveloped Goyals model [89] by taking different selling and purchasing prices. Mandal and Phaujdar [10] have studied Goyals model [89] by including interest earned from the sales revenue on the stock remaining beyond the credit period. Chung [52] simplified the solution procedure proposed in Goyals model [89] by demonstrating a simple procedure to determine the optimal time interval between consecutive orders.

Teng [51] has generalized Goyals model [89] by differentiating between the unit purchasing cost and selling price and has concluded that some retailers may order less quantity and benefit from the permissible delay more frequently. Ouyang et al. [60] modified Goyals model [89] by assuming two level credit policy. Besides assuming not necessarily equal selling and purchasing prices, Chung and Huang [55] extended Goyals model [89] by assuming that the retailer pays a partial amount of total purchasing at the end of the trade credit period. Abad and Jaggi [82] considered the seller, buyer channel in which the end demand was price sensitive and the seller may offer trade credit to the buyer. A general framework of Goyals model [89] is developed by Huang [110]. In the Huangs model [110], not only selling and purchasing prices have been differentiated, but also the retailer offered partial trade credit to his/her customers. Ouyang et al. [62] generalized Goyals model [89] to obtain an optimal order policy for the retailer when the supplier offers not only a cash discount but also a permissible delay in payment.

Under two credit periods with progressive interest charges offered by a bank, Goyal et al. [91] formulated the retailers EOQ model and obtained the closed form optimal solution. They assumed that paying before the first defined period would cause no interest charge, while paying after the second predetermined period would result in larger interest on the unpaid balance. Soni and Shah [37] dealt with the problem of determining a retailers optimal ordering policies when demand is stock dependent and progressive credit periods are offered by the supplier.

Sana and Chaudhuri [103] established an EOQ model under different types of deterministic demand, including constant, time dependent, stock dependent, price dependent, and both price and stock dependent demands. They assumed that the supplier offers the retailer some credit periods with different rates of price discount. Chung and Huang [56] analyzed an ordering
policy with allowable shortage and permissible delay in payments. When the demand is a linearly increasing function of time, Teng et al. [50] determined the optimal order quantity and cycle time in an EOQ framework with permissible delay in payment. Cheng et al. [64] formulated EOQ model under trade credit approach and different financial policies. Shah et al. [74] developed an integrated inventory model with trapezoidal price sensitive demand with two part trade credit. Li et al. [45] studied both joint ordering problem and inventory games of multiple retailers who buy the same commodities from a supplier and are offered a permissible delay in payment. Ouyang et al. [61] discussed the integrated inventory model with the order size dependent trade credit and a constant demand. Lashgari et al. [67] proposed a lot sizing model with partial upstream advanced payment and partial downstream delayed payment. Tyagi [7] investigated an inventory system as a cost minimization problem to find the retailers optimal inventory cycle time and optimal order quantity. Shah [73] developed a retailers optimal policy for deteriorating items with a fixed lifetime under order linked conditional trade credit. Khanna et al. [2] developed an inventory control model for deteriorating imperfect quality items with allowable shortages under the condition of permissible delay in payments with price dependent demand.

5. Survey of Inventory Models with Shortages

When the shortages occur, it is assumed that entire demand is either completely backlogged or completely lost. But in realistic situations, it is not basically true. Practically, during the stock out period, some customers are willing to wait for next refill and receive their orders at the end of the shortage period, while others would turn to buy from other places. This situation is called the partial backlogging or lost sales in the inventory modelling. Customers who experience stockout may not purchase the goods again from the respective suppliers, and they may turn to another store to purchase the goods. Consequently, a larger proportion of sales are lost, leading to a smaller profit. As a result, taking into account the partial backlogging factor is necessary for inventory modelling.

In many of the inventory models discussed in this chapter, either shortages are not allowed, or if occur, they are considered to be completely backlogged. However, in todays highly competitive market, providing varieties of products to the consumers due to globalization, partial backorder is a more realistic one. For fashionable items and high tech products with short product life cycle, the willingness of a customer to wait for backlogging during the shortage period decreases with the waiting time. During the stock out period, the backorder rate is generally considered as a non increasing linear function of backorder replenishment lead time through the amount of shortages. The larger the expected shortage quantity is, the smaller the backorder rate would be. The remaining fraction of the shortage is lost. This type of backlogging is called time dependent partial backlogging. Montogomery et al. [21] was the first to introduce partial backlogging of unsatisfied demand for non perishable items. Wee [38] proposed the production model for decaying products with a fixed rate of production and demand function with partial backordered.

Padmanabhan and Vrat [28] developed an EOQ model for deteriorating items with stock dependant demand. they proposed three models: completely backlogging, partial backlogging and without backlogging, for the purpose of maximizing profit. Wee [39] studied the model of perishable items with partial backlogging of unsatisfied demand. Wee [35] developed an inventory model with deteriorating items, quantity discount, pricing, and partial backlogging. Abad discussed many pioneering and inspiring backlogging rates as functions of waiting time. Abad [80] developed an optimal pricing and lot sizing inventory model for a reseller considering selling price dependent demand. Abad [81] formulated optimal lot sizing policies for perishable goods in a finite production inventory model with partial backlogging and lost sales. Dye et al.[19] developed an inventory model for deteriorating items with price dependent demand. In this model, shortages are allowed. Kumari et al. [85] presented an inventory model for deteriorating items with partial backlogging under the conditions of
permissible delay in payments. Chang et al. [15] investigated a partial backlogging inventory model for non instantaneous deteriorating items. They assumed that the demand of the items are stock dependent, and proposed a mathematical model to find the minimum total relevant cost and an optimal order quantity of the model. Liao et al. [111] investigated a distribution free newsvendor model with balking and lost sales penalty. Mishra and Singh [108] investigated an inventory model for decomposing products which is dependent on holding costs as well as demand rate. They also considered a fixed rate of deterioration with shortages which are partially backordered. Agrawal et al. [86] developed an inventory model with two storage facilities with different rate of decaying products with the ramp type demand function and partially backlogged. Kumar and Rajput [93] presented a partial backlogging inventory model for deteriorating items with constant deterioration rate and ramp type demand rate. The backlogging rate of unsatisfied demand assumed as a function of waiting time. Khurana and Chaudhary [22] presented an optimal pricing and ordering policy for deteriorating items with price and stock dependent demand and partial backlogging. Rastogi et al. [72] developed an EOQ model with variable holding cost and partial backlogging under the credit limit policy.

6. Survey of Inventory Models with Inflation

In the classical inventory modelling, most decision makers think that the inflation does not have significant influence on the demand of the products and services. In reality, inflation has a major effect on the demand of the certain goods and services. Since inflation is an economic concept, then it plays an important role in the inventory control models because of its impotency. In economics, inflation is a rise in the general level of prices of goods and services over a period of time. As inflation increases, the value of money goes down, which decreases the future worth of saving and forces one for more current spending. Thus, the inflation plays a vital role in the inventory system and production management though the decision makers may face difficulties in arriving at answers related to decision making. Therefore, the effect of inflation cannot be ignored in developing the optimal inventory policy.

Buzacott [42] developed the first EOQ model taking inflationary effects into account. In this model, a uniform inflation was assumed for all the associated costs and an expression for the EOQ was derived by minimizing the average annual cost. Bierman and Thomas [9] investigated the inventory decision policy under an inflationary condition in a standard EOQ model. Misra [84] gave a note on optimal inventory management under inflation and developed a discount cost model in which the effects of both inflation and the time value of money are considered. Chandra and Bahner [65] developed models to investigate the effects of inflation and time value of money on a finite horizon policy. Datta and Pal [105] developed a model with linear demand to investigate the effects of inflation on a finite horizon policy. An EOQ model for deteriorating items was developed by Bose et al. [87]. They developed inventory model with linear trend in demand, allowing inventory shortages and backlogging. The effects of inflation and time value of money were incorporated into the model.

Su et al. [18] developed an inventory model under inflation for initial stock dependent consumption rate and exponentially decay. Ray and Chaudhuri [48] developed a finite time horizon deterministic economic order quantity inventory model with shortages under inflation. Liao et al. [29] presented a model under inflation and permissible delay in payment when the consumption rate is initial stock dependent and the deterioration rate is dependent on time. The effects of inflation and time value of money on an economic order quantity model have been discussed by Moon and Lee [41]. They have considered the normal distribution as a product life cycle in addition to the exponential distribution. Chang [17] considered an EOQ model with deteriorating items under inflation and a situation in which the supplier has provided a permissible delay in payments to the purchaser.

Moon et al. [40] developed models for ameliorating/deteriorating items with time varying demand patterns over a finite
planning horizon, taking into account the effects of inflation and time value of money. A partial backlogging inventory models for deteriorating items were discussed by Yang [34]. In this model, the inflationary effect was considered and deterioration rate was taken as constant. Hou and Lin [57] proposed an EOQ model for deteriorating items with price and stock-dependent selling rates under inflation.

Jaggi et al.[13] presented the optimal inventory replenishment policy for deteriorating items under inflationary conditions. The demand rate was assumed to be a function of inflation and optimal solution for the proposed model was derived. Dey et al. [44] discussed a finite time horizon inventory problem for a deteriorating item having two separate warehouses is developed with interval valued lead time under inflation. Singh and Jain [100] extended a model designed for decomposing matter in the inflationary surroundings by considerations that the retailer had the preserve cash to sell out the suppliers starting but obtain advantages of his credit period of time. Chung and Liao [54] developed an EOQ model under order quantity dependent trade credit and DCF approach. Effects of variable inflationary conditions on an inventory model with inflation proportional demand rate were discussed by Mirzazadeh[3]. Sarkar et al.[11] formulated an economic manufacturing quantity (EMQ) model with a time dependent demand under the inflationary environment. Singh and Singh [14] developed a production process with exponential demand rate, Weibull deterioration under the effect of inflation.

Hou and Lin [58] presented an EOQ inventory model for deteriorating items under trade credit. They obtained the optimal ordering and pricing policies and discussed the effects of inflation, deterioration, and permissible delay in payment. Singh et al. [102] developed a probabilistic inventory model for Weibull deteriorating items with flexibility and reliability under the effect of inflation. Singh et al. [96] presented a mathematical production inventory model for deteriorating items with time dependent demand rate under the effect of the inflation and shortages. Kumar and Rajput [94] developed a general inventory model for deteriorating items with constant deterioration rate and ramp type demand under stock dependent consumption rate. The effect of inflation and the time value of money is introduced into the model. Palanivel and Uthayakumar [69] presented an economic order quantity (EOQ) model for non instantaneous deteriorating items with permissible delay in payments under the effect of inflation.

7. Conclusion

In this article most of the factors such as deterioration, shortage, inflation, and trade credit, etc., which affect inventory modelling has been considered and relevant literature survey has been provided. In future, this article will be very profitable for all researchers in the field of inventory modelling.

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