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Optimal Fuzzy Approach: Transportation Modelling

Sohan Tyagi[1,*]

1 Department of Mathematics, SRM University, NCR Campus, Modinagar, Ghaziabad, Uttar Pradesh, India.

Abstract: The paper presents a classification and analysis of the results achieved using fuzzy logic to the model of transportation and traffic problems. In the recent years, research has been focused to estimating and techniques to help solve different engineering, management, control and computational problems. Natural systems teach us that very simple individual organisms can form system capable of performing highly tasks by dynamically interacting with each other. The main goal of the paper is to show how we can use fuzzy optimization approach when solving traffic and transportation. The fuzzy Ant system described in the paper represents an attempt to handle the uncertainty that sometimes exists in transportation problems. The potential application of the fuzzy Ant system in the field of traffic and transportation are discussed. The basic premises of fuzzy logic systems are presented as well as detailed analysis of fuzzy logic system developed to solve various traffic and transportation problems.

Keywords: Fuzzy logic, transportation modeling, Bee system, vagueness, traveling salesman problems, fuzzy Ant system.

1. Introduction

Transportation researchers in many scientific disciplines have been trying to deal with fuzzy optimization of transportation problems. In the past several stochastic models have been developed to solve complex traffic and transportation problems. The uncertainties can be divided into two different types ; one is randomness due to the non-deterministic nature of choice behaviour problems, which is measured, based on probability distribution, and another is fuzziness due to information and poor familiarity with network attributes and others which is measured, based on possibility distribution. In the field of choice behaviour modeling, discrete choice behaviour models have been widely employed to deal with the uncertainty of randomness, while fuzzy optimal are mainly used to deal with uncertainty of fuzziness in transportation modeling. The main goal of this paper is to show how we can use fuzziness or vagueness when solving complex problems in traffic and transportation. The organization of the rest of this theme of the paper which is described below, To solve traffic and transportation problems by fuzzy optimal techniques the fuzzy ant system that has been proposed to some transportation problems characterized by uncertainty. In this paper the fuzzy reasoning were carried out to evaluate diversion motivations of drivers, in the case of their current routes were dissatisfied.

2. Fuzzy Logic System for Transportation Modelling

Basic results linked to the development of fuzzy logic from [15], Introducing a concept of Appropriate Reasoning Zadeh successfully showed that vague logical statements enable to the formation of algorithms that can use vague data to derive

* *E-mail: sohanridhi@gmail.com*

vague inferences. Zadeh assumed his approach would be beneficial above all in the study of complex humanistic systems. Realizing that Zadeh's approach could be successfully applied to industrial plant controllers. What is a fuzzy logic system? [11] explains the concept of a fuzzy logic system which is as follows: In general a fuzzy logic system in a non linear mapping of an input data (feature) vector into a scalar output and we would also note that a fuzzy logic systems maps (most often crisp) inputs into crisp outputs. The basic elements of every fuzzy logic system maps (most often crisp) inputs into crisp outputs. Input data are most often crisp values. The task of the fuzziness is to map crisp numbers into fuzzy sets and models based on fuzzy logic consist fuzzy rule base. The final value chosen is most often either the value corresponding to the highest grade of membership function. Drivers, passengers or dispatchers make decisions about route choice, mode of transportation, most suitable departure time or dispatching trucks. The fuzzy reasoning support the application of the current input against each rule and the derivation of the appropriate outcome

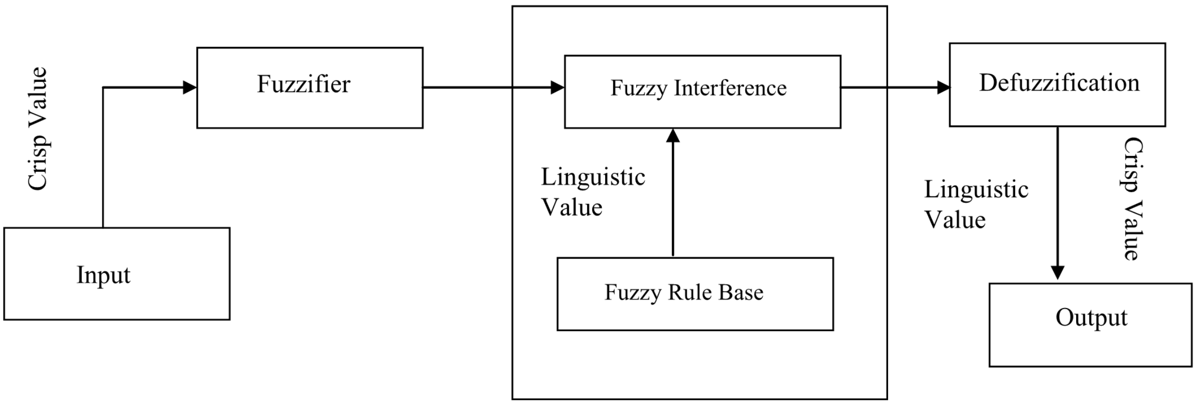

Figure 1. **Fuzzy Inference Process**

3. Solving Complex Transportation Modelling by Fuzziness

Several practical real world problems were formulated and solved using mathematical programming techniques during the lack of information and uncertainty. It is important to note, however, that the majority of real-world problems solved by some of the optimization techniques were of small dimensionality. Many traffic and transportation problems and modeling are combinational by nature. Typical representatives of this type of problems are the vehicle fleet planning, static, dynamic routing and scheduling of vehicles and crews for airlines, railroads, truck operations and public transportation services, designing transportation networks and optimizing alignments. For highways and public transportation routes through fuzziness and different locations problems. There are some models in the literature based on fuzziness that try to solve certain transportation problems. The classical transportation problem refers to a special class of linear programming problems. In a typical problem a product is to be transported from m sources to n destinations and their capacities are $a_1, a_2, a_3, \ldots, a_i$ and $b_1, b_2, b_3, \ldots, b_j$ respectively. In addition, there is a penalty C_{mn} associated with transporting a unit of product from source i to destination j. This penalty may be cost or delivery time or optimal value etc. A variable x_{ij} represents the unknown quantity to be shipped from source i to destination j. Generally, the real life problems are modelized with multi-objective which are measuring in different scales and same time in conflict. Moreover, it is difficult for the decision making to combine the objective functions in one or over all utility function. The mathematical model of the multi-objective transportation problem as under follows:

$$\begin{aligned} \text{minimize} \quad & F^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij} \\ \text{subject to} \quad & \sum_{i=1}^{m} x_{ij} = a_i, \qquad i = 1, ..., m \\ & \sum_{j=1}^{n} x_{ij} = b_j, \qquad j = 1, ..., n \\ & \forall\ x_{ij} \geq 0 \end{aligned} \tag{1}$$

where, $F^k(x) = \{F^1(x), F^2(x), F^3(x), \ldots, F^k(x)\}$ is a vector of k objective functions and the superscript on both $F^k(x)$ and C^k_{ij} are used to identify the number of objective function without loss of generality it will be assumed that $\sum a_{ij} = \sum b_{ij}$.

4. Types of Solution

The existing solutions procedures of the problem can be divided into two category represents the procedures that are seeking non-dominated solution and best compromise solution among the set of efficient solutions.

4.1. Non-Dominated Solutions

A feasible vector $X^0 \in S$ (where S is the feasible region) gives a non-dominated solution of the problems, if there is no other feasible vector $X \in S$ such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{ij}X_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{ij}X^0_{ij}\ \forall\ k$$

and

$$\sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{ij}X_{ij} \neq \sum_{i=1}^{m}\sum_{j=1}^{n} C^k_{ij}X^0_{ij} \tag{2}$$

4.2. Compromise Solution

A feasible vector $X^* \in S$ is a compromise solution of transportation problems if and only if $X^* \in S$ and $F(X^*) \leq \wedge_{x \in E} F(x)$, where $\wedge$ means 'minimum' and E is the set of efficient solutions. From a practical point of view the knowledge of the set of efficient solutions E is not always necessary. In such a case a procedure is needed to determine a compromise solution. As a consideration different approaches have been developed in the context of multi-objective transportation problem to find the compromise solution such as goal programming, interactive and utility approaches etc.

5. Computing with Artificial Bees: Case Study of Travelling Salesman Problems

Self-organisation of bees is based on a few relatively simple rules of individual insect's behavior [1, 8, 9, 12, 15]. In spite of the existence of a large number of different social insect species, and variation in their behavioural patterns, it is possible to describe individual insects.

Lucic and Teodorovic [3, 6] and Lucic [4] recently developed and initial version of the artificial life model inspired by bees behaviour in nature. They use bees behaviour in nature as a source of ideas for development of an artificial system called the bee system. The bee system is composed of agents or virtual creatures called artificial bees. The basic assumption is that artificial bees are capable of discovering 'good' solution for difficult to optimization problem.

The primary goal of this paper is to show the possible applications of the artificial systems inspired by collective social insects intelligence in solving traffic and transportation problems. The development of the new heuristic algorithm for the travelling salesman problem using the bee-system will serve as an illustration. For such applications and will show the characteristics of the proposed concepts. It is important to say that the travelling salesman problem closely related to the broad class of transportation problems whose typical representatives are the vehicle fleet planning and dynamic routing and scheduling of vehicle and cross for airlines, truck operations railroads and public transportation services.

6. Solution of Travelling Salesman Problem by the Bee System

Let us consider $H = (N, A)$ the network in which the bees are collecting the graph in which the travelling salesman route should be discovered. Where $N = \{u_1, u_2, u_3, \ldots, u_n\}$ be the set of nodes to be visited, and $A = \{(u_i, u_j) : i \neq j\}$ be the set of links connecting these nodes. Let us also randomly locate the hive in one of the nodes when the artificial bees are trying to collect as much nectar as possible. Let us also assume that the nectar quantity that is possible to collect flying along a certain link is inversely proportional to the link length.

In other words, the shorter the link, higher and the nectar quantity collected along that link. This means that the greatest possible nectar quantity could be collected when flying along the shortest travelling salesman route. Our artificial bees will collect the nectar during the certain prescribed time interval. After that, we will randomly change the hive position and the artificial bees will be started to collect the nectar from the new location one of the most successful and widely accepted discrete choice models. We have assumed that the probability of choosing node j by k$^{\text{th}}$ bee, located in node i during stage $(v + 1)$ and iteration (ω) which is equals to

$$q_{ij}^{k}(v+1,\omega) = \begin{cases} \dfrac{e^{-ad_{ij}\frac{\omega}{\sum_{s=\max}^{\omega=1} n_{ij}(s)}}}{\sum_{l\in N_k(v,w)} e^{\frac{\omega}{\sum_{s=\max(\omega-b,1)}^{\omega-1} n_{il}(s)}}} \\ 0, & \text{otherwise} \end{cases}$$

$$i = g_k(v,\omega),\ j \in N_k(v,\omega)\ \forall\, k, v, \omega \tag{3}$$

where,

i, j	$=$ nodes indexes $(i, j = 1, 2, 3, \ldots, \lvert N\rvert)$
ad_{ij}	$=$ length of link (i, j)
k	$=$ bee index $(k = 1, 2, 3, \ldots, B)$
B	$=$ The total number of bees in the hive
ω	$=$ Iteration index $(\omega = 1, 2, 3, \ldots, M)$
M	$=$ Maximum number of iteration
v	$=$ Stage index $\left(v = 1, 2, 3, \ldots, \frac{\lvert N\rvert-1}{r}\right)$
$n_{il}(s)$	$=$ Total number of bees that visited link $(i, 1)$ in s^{th} iteration.
$g_{ik}(v, \omega)$	$=$ Last node that bee k visits at the end of stage v in iteration ω.
$N_k(v, \omega)$	$=$ Set of unvisited nodes for bee k at stage v in iteration ω.

Since we have to discuss equation (3) in more details. The greater distance between node i node j-the lower the probability that the k-th bee located in the node i will choose node j during stage v and iteration. The greater number of iterations, the higher influence of the distance. In other words, at the beginning of the search process, artificial bees have more freedom of flight. We assume that every bee can obtain the information about the nectar quantity collected by every other bee. The probability that at the beginning of stage $v+1$, bee k will use the same partial tour that is defined in stage v in iteration ω equals to

$$P_k\left(v+1,\omega\right)=e^{-\frac{L_k(v,\omega)-\min\limits_{S\in Z(v,\omega)}[L_S(v,\omega)]}{v\omega}} \tag{4}$$

where, $L_k(v,\omega)$ is the length of partial route that is discovered by bee K in stage v in iteration while foraging in stage v, every artificial bees has the ability to notice the total number of bees in every link. The maximum number of stages the bees can recall represents memory length. From equation (4) we can see that if a bee has discovered the shortest partial travelling salesman tour in stage v in iteration ω . The bee will fly along the same partial tour with the probability equal to one. It is assumed that the probability P^* of the event that the artificial bee will continue foraging at the food source without recurring nestmates is very low

$$P^*\ll 1 \tag{5}$$

In the case when at the beginning of stave $v+1$, the bees does not use the same partial travelling salesman tour. Let every partial travelling salesman tour G that is being advertised in the area and has two main attributes (a) the total length (b) the number of bees that are advertising the partial route. We introduce the normalized value of the total length of the partial travelling salesman tour and the normalized values are defined in the following way:
We have considered in this paper that the probability that the partial route G will be chosen by any bee that decided to choose the new route equals.

$$P_\zeta\left(v,\omega\right)=\frac{e^{\rho\beta_\zeta(v,\omega)-\theta\alpha_\zeta(v,\omega)}}{\sum\limits_{S\in y(v,\omega)}e^{\rho\beta_\tau(v,\omega)-\theta\alpha_\tau(v,\omega)}} \tag{6}$$

Such as $\zeta\in y\left(v,\omega\right)\ \ \forall\ v,\omega$. Where

ρ,θ = Parameters given by the analyst.

$\alpha_\zeta\left(v,\omega\right)$ = The normalized value of the partial route length.

$\beta_\zeta\left(v,\omega\right)$ = The normalized value of the number of bees advertising the partial tour.

$y(v,\omega)$ = The set of partial tours that were visited by at least one bee.

ω = Iteration index ($\omega=1,2,3,\ldots,M$).

7. Fuzzy ANT System

Let us proposed a fuzzy Ant system that represents a combination of the Ant colony system and fuzzy logic. In other words, the ant system is combined with a non-quantitative approach. The basic modification is in the way in which transition probabilities are calculated. Fuzzy logic could be used to calculate an ant's utility to visit the next node. When deciding about the next node to be visited in the case of travelling salesman problem, the ant takes into account 'visibility' as well as pheromone trail intensity. We assume that the ant can perceive the particular distance between nodes as "small", "medium" or "large" and the trail intensity as "weak", "moderate" or "strong" etc. Possible membership functions of these fuzzy sets are shown in figure 2.
Depending on the distance to the next node, as well as the trail intensity, the ant will have a stronger or weaker utility to choose the considered link. These utilities can be described by approximate fuzzy sets. In the case of travelling salesman problem, the next link could be composed. The nodes with better utility values are more likely to be selected by ant. The performance numerical tests showed that the fuzzy Ant system could produce very good results when solving the problems. Recently Lucic and Teodorovic also successfully combined the bee system with fuzzy logic in order to solve vehicle routing problem with uncertain demands at (real value of demand at node is only known when the vehicle reaches the node).

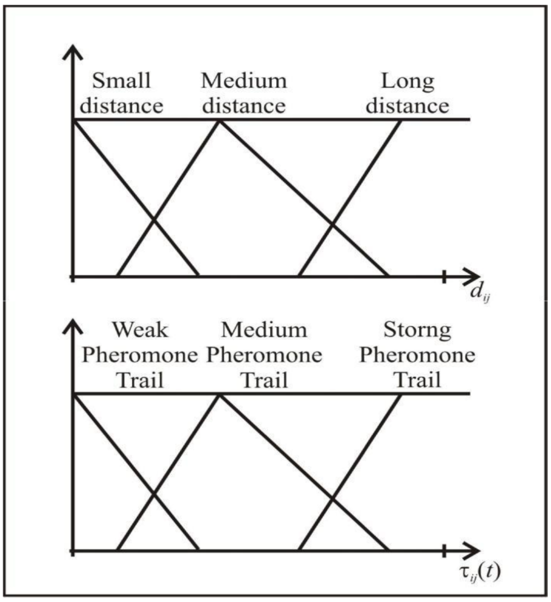

Figure 2. **Membership functions of the fuzzy sets describing distance to the next node and trail intensity**

8. Conclusion

The paper mainly aims confirm the applicability of fuzzy theory for route choice behaviour modeling, to estimate the driver's perception on the network attributes and to deal with several fuzzy inference methods to improve the fuzzy reasoning model are suggested in the paper. The bee system has been successfully applied to the classical travelling salesman problem. The results obtained are considered to be very good. A fuzzy ant system representing a combination of the ant colony system and fuzzy logic is successfully applied for solving schedule synchronization in public transit. These results show that the development of new models based on artificial life principles could significantly contribute to the solution of transportation problems.

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