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Production System with Repair and Optimal Maintenance Policy

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Abstract: The objective of this paper is to present the efficient and effective importance in productivity with the help of system repair and optimal maintenance. In fact the repair and maintenance is more concerning issue in the prevention of the losses due to breakdowns with in an enterprise. Queueing model is employed to analyze the system repair. Which describes the repair rate of the state is less or more. It is necessary to plan the activities to be carried out to ensure that the machines and plants of enterprises (production units) are working continuously. Therefore the gain is more important particularly when the production system grows as well as production quantity increases. Such study emphasis on optimal maintenance policy which is discussed in different cases. Availability of the system is examined by varying the various parameters.

MSC: 15A09, 15B05,15A99,15B99

Keywords: Optimal maintenance policy, Reliability and Availability function, Repair system, Queueing model.

1. Introduction

Due to lot of competitions among industries would make an effort to keep them agile in the global market. So companies realize that their competitive performance and their future are sturdily linked to the effectiveness and efficiency of maintenance policy. Maintenance of production units are responsible for keeping the equipment healthy, safe to operate and suitably configured to perform their assign tasks. During their operational life, industrial systems are subject to repair when a failure occurs. A repair activity is aimed to reduce the failure rate of the system and to extend its useful life time. Thus maintenance processes come into existence with effective repair of the system in short term, long term and continuous monitoring. From last many decades maintenance and replacement issues have been extensively investigated in literature. Balagursamy [1] studied of maintenance policy for m-order energy systems with s-dependent units. Balagursamy and Mishra [2] investigated of availability and failure frequency of repairable m order systems. Proctor and Wang [4] obtained the optimal maintenance policy for the system that experience state degration points. Wang and Lin [5] studied of the reliability model for the dependent the failure in parallel redundant systems also Beran [6] find out the outage frequency of repairable parallel unit system under availability. Sharma and Mishra [7] investigated the reliability optimization of a series system with active and standby redundancy. Nakagawa and Mizutani [8] discussed maintenance policies for a finite interval. Kitagawa et al [9] a system comprising non identical units in series where only minimal repair are performed when unit failures are detected. Wang [10] detail structure of maintenance analysis which covers every aspect of maintenance has been focused. Yevkin & Krivtsov [11] gave comparative study of optimal maintenance policies along with repair by considering Weibull distribution.

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Gilardoni et al [12] proposed maintenance policies which have a lower expected cost than a periodical one which does not take into account the failure history.

2. Notation

$A(t)$ = Availability function.

$R(t)$ = Reliability function.

$M(t)$ = Maintenance function.

Tf = Expected time to repair the system after an in-service failure.

Ts = Expected time to complete the schedule main on the system.

$F(t)$ = Commutative-distribution function (cdf).

$f(t)$ = Probability-density function (pdf).

$R(T)$ = Probability that the system will not fail before time T.

$N(T)$ = Number of system renewals.

λ = Timeindependent failure rate of units.

μ = Constant repair rate of units.

ρ = Operability ratio.

λ_r = Timeindependent failure rate at rated stress.

ρ_s = Service factor.

k = Minimum number of units to be good to keep the system operative.

m = Total number of parallel redundant units in the system.

r = Number of repair facilities available.

h_x = System failure rate at the state where x units have failed.

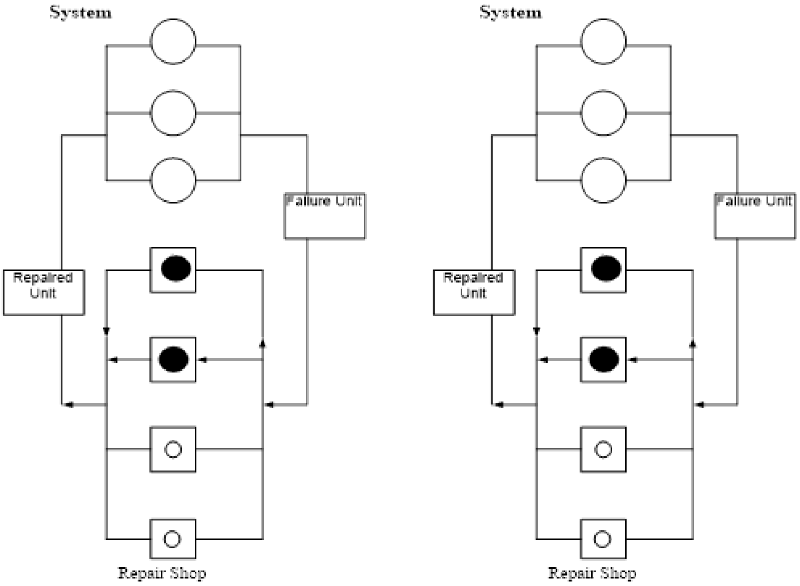
$Pr(x)$ = Probability of the event x occurring.

Pr = Reliability of units at rated load.

$A(k, m)$ = Availability (steady-state) of k-out of m.

3. Systems with Repair

The importance of allowing the repair of failed units in a system should be obvious when considering systems with redundant units. If repair is possible in a failed unit without affecting the overall system operation, then it is desirable to know what the chances are of returning this unit to either operation or an operable state before its lack of operation causes complete system failure. Consequently, reliability of the unit is not meaningful when repair is allowed and need some additional measures of system effectiveness that considers the effects of repair. Analyze the systems with failure and repair, which are statistically dependent and exponentially distributed. Whenever a unit fail, immediately it require repair, if not then the failed unit waits in the queue for getting the first opportunity for repair. As soon as $m - k + 1$ units have failed, system failure is said to have occurred. If the system is in state $m - k + 1$, then the only way it can leave this state is for a repair to take place, thus passing to state $m - k$.



(a) $x \leq r$ (b) $x > r$

Figure 1: Figure (a) & (b) System with fractional repair facilities

Let x denote the state in which exactly $x(x = 0, 1, \ldots, m-k+1)$ units have failed. If the system is in state x at any time t, then

$$Prob.(x \to x+1, 0 \leq x \leq m-k,)\ in\ t,\ t+\Delta t = 1 - exp(-h_x \Delta t) = h_x \Delta t + 0(\Delta t) \tag{1}$$

and

$$Prob.(x \to x+1, 0 \leq x \leq m-k,)\ in\ t,\ \Delta t + t = 1 - exp(-\mu_{x\Delta t}) = \mu_x \Delta t + 0(\Delta t) \tag{2}$$

Where h_x and μ_x are failure and repair rates of the system in the state x. The following events occur as

$$P'_x(t) = h_{x-1}P_{x-1}(t) - (h_x + \mu_x)P_x(t) + \mu_{x+1}P_{x+1}(t) = 0,\ for\ x = 1, 2, \ldots, m-k \tag{3}$$

$$P'_0(t) = -h_0 P_0(t) + \mu_1 P_1(t) = 0,\ for\ x = 0 \tag{4}$$

$$P'_x(t) = h_{n-1}P_{n-1} - \mu_n P_n(t) = 0,\ for\ x = m-k+1. \tag{5}$$

Apply steady-state conditions,

$$h_{x-1}P_{x-1} - (h_x + \mu_x)P_x + \mu_{x+1}P_{x+1} = 0, \quad x = 1, 2, \ldots, m-k. \tag{6}$$

$$-h_0 P_0 + \mu_1 P_1 = 0, \quad x = 0 \tag{7}$$

$$h_{m-k}P_{m-k+1} - \mu_{m-k+1}P_{m-k+1} = 0, \quad x = m-k+1. \tag{8}$$

The above equations can be solved for the ratios P_x/P_0 recursively as follows:

$$\frac{P_{x+1}}{P_0} = \begin{cases} \frac{h_0}{\mu_1} & \text{for } x = 0 \\ \frac{(h_x/\mu_x)(P_x/P_0) - h_{x-1}(P_{x-1}/P_0)}{\mu_{x+1}} & \text{for } 1 \leq x \leq m-k \end{cases} \tag{9}$$

Substituting for P_x and P_{x-1},

$$\frac{P_{x+1}}{P_0} = \prod_{i=0}^{x} \frac{h_i}{\mu_{i+1}}$$

or,

$$P_x = \prod_{i=0}^{x} \frac{h_i}{\mu_{i+1}}, \quad 1 \le x \le m-k+1, \quad \text{since} \sum_{j=0}^{m-k+1} P_j = 1 \tag{10}$$

Then

$$P_0 = 1 - \sum_{j=1}^{m-k+1} P_j = 1 - P_0 \sum_{j=1}^{m-k+1} \prod_{i=0}^{j-1} \frac{h_i}{\mu_{i+1}}$$

Hence

$$P_0 = \frac{1}{1 + \sum\limits_{j=1}^{m-k+1} \prod\limits_{i=0}^{j-1} \frac{h_i}{\mu_{i+1}}} \tag{11}$$

Then

$$P_x = \frac{\prod\limits_{i=0}^{x-1} \frac{h_i}{\mu_{i+1}}}{1 + \sum\limits_{j=1}^{m-k+1} \prod\limits_{i=0}^{j-1} \frac{h_i}{\mu_{i+1}}} \tag{12}$$

The system steady-state availability is

$$A(k,m) = \lim_{t\to\infty} A(k,m;t) = \lim_{t\to\infty} \sum_{x-0}^{m-k} P_x(t) = \sum_{x-0}^{m-k} P_x == P_0 + \frac{\sum\limits_{x=1}^{m-k} \prod\limits_{i=0}^{x-1} \frac{h_i}{\mu_{i+1}}}{1 + \sum\limits_{j=1}^{m-k+1} \prod\limits_{i=0}^{j-1} \frac{h_i}{\mu_{i+1}}} \tag{13}$$

Eliminating P_0 from (13), it gives

$$A(k,m) = \frac{1 + \sum\limits_{j=1}^{m-k} \prod\limits_{i=0}^{j-1} \frac{h_i}{\mu_{i+1}}}{1 + \sum\limits_{j=1}^{m-k+1} \prod\limits_{i=0}^{j-1} \frac{h_i}{\mu_{i+1}}} \tag{14}$$

The system unavailability is

$$\bar{A}(k,m) = P_{m-k+1} = \frac{\prod\limits_{i=0}^{m-x} \frac{h_i}{\mu_{i+1}}}{1 + \sum\limits_{j=1}^{m-k+1} \prod\limits_{i=0}^{j-1} \frac{h_i}{\mu_{i+1}}} \tag{15}$$

4. Optimal Maintenance Policy

Maintenance optimization model is a kind of problems in which a tradeoff between the maintenance cost and the reliability motivates maintenance mangers to choose the best solution among all feasible solutions for variety systems.

Let T_s be the expected time to complete the scheduled maintenance and T_f the expected time to repair the system after an in service failure. If $T(t)$ is the mean down-time in the time interval $(0,t)$, then

$$T(t) = [T_s R(T) + T_f(1 - R(T))]N(T) \tag{16}$$

The limiting availability of the system is

$$A = \lim_{t\to\infty} \frac{t}{t + T(t)} = \lim_{t\to\infty} \frac{t}{t + [T_s R(T) + T_f(1 - R(T))]N(T)} \tag{17}$$

Apply steady-state condition,

$$\lim_{t\to\infty} \frac{t}{N(t)} = \int_0^T R(x)dx \quad and \quad A = \frac{\int_0^T R(x)dx)}{\int_0^T R(x)dx + (T_s + T_f)R(T) + T_f} \tag{18}$$

The necessary condition for the optimum maintenance interval T_0 is obtained by setting the derivative of A with respect to T equal to zero, i.e.,

$$\lambda_s(T)\int_0^T R(t)dt + R(T) = \frac{\rho_s}{\rho_{s^{-1}}} \tag{19}$$

Where $\rho_s = T_f/T_s$ is the service factor, obviously when $\rho_s \leq 1$, no finite value of T will maximize A. Hence, a policy of repair maintenance is optimum. Generally, when $T_f > T_s$, then the additional time involved in the unexpected event of failure and $\lambda_s(t)$ is a continuous and strictly increasing function of t, (19) give a unique and finite solution, thus, availability of the system is maximum, by (18) and (19)

$$A_{max} = \frac{1}{1+T_s(\rho_{s^{-1}})\lambda(T_0)}, \qquad \rho_s > 1 \tag{20}$$

Substituting for $\lambda_s(t)$ and $R(t)$ in (19) and (20), it give

$$\frac{k\lambda_r T)^{m-k}}{(m-k)!}\Big[(m-k+1) - \sum_{i=0}^{m-k}\sum_{j=0}^{i}\frac{(k\lambda_r T)^j}{j!}exp(-k\lambda_r T)\Big] + exp(-k\lambda_r T)\Big[\sum_{i=0}^{m-k}\frac{(k\lambda_r T)^i}{i!}\Big]^2 - \frac{\rho_s}{\rho_{s^{-1}}}\sum_{i=0}^{m-k}\frac{(k\lambda_r T)^i}{i!} = 0 \tag{21}$$

and

$$A_{max} = \frac{\sum\limits_{i=0}^{m-k}(k\lambda_r T_0)^i}{\sum\limits_{i=0}^{m-k}\frac{(k\lambda_r T_0)^i}{i!} + k\lambda_r T_s(\rho_{s^{-1}})\frac{(k\lambda_r T_0)^{m-k}}{(m-k)!}} \tag{22}$$

The following two cases for optimal policy which describe the performance of maintenance.

Case I: When $m-k$=0. The reliability of the system is

$$R(t) = 1 - F(t) = \sum_{i=0}^{m-k}\frac{(k\lambda_r T_0)^i}{i!}exp(-k\lambda_r t) \tag{23}$$

and the system failure rate is

$$\lambda_s(t) = \frac{f(t)}{1-F(t)} = \frac{k\lambda_r}{(m-k)!\sum\limits_{i=0}^{m-k}\frac{1}{i!(k\lambda_r t)^{m-k-i}}} \tag{24}$$

From (23) and (24),

$$R(t) = exp(-k\lambda_r t)$$

$$\lambda_s(t) = k\lambda_r \qquad (Constant)$$

$$\lambda_s(T)\int_0^T R(t)dt + R(T) = 1 \tag{25}$$

Now, (18) has no solution for any finite T. The system availability is described by (17)

$$A = [1 + k\lambda_r T_s(\rho_s + \frac{exp(-k\lambda_r T)}{1-exp(-k\lambda_r T)})]^{-1} \tag{26}$$

Thus, A is a monotonically increasing function of T, when $T_0 = \infty$, i.e., the optimal policy is to perform maintenance at failure only.

$$A = \frac{1}{1+k\lambda_r T_s\rho_s} \tag{27}$$

Case II: When $m-k > 0$. Failure rate of the system, when $m > k$ is

$$\lambda_s(t) = \frac{k\lambda_r}{1+(m-k)!\sum\limits_{i=0}^{m-k}\frac{1}{i!(k\lambda_r t)^{m-k-i}}} \tag{28}$$

and is continuous and strictly increasing function of t. Assume that $\lambda_s(0) = 0$, $\lambda_s(\infty) = k\lambda_r$ and

$$\int_0^T R(t)dt = \begin{cases} \frac{(m-k+1)}{k\lambda_r}, & \text{for } T = \infty \\ 0, & \text{for } T = 0 \end{cases}$$

Therefore (18) has a solution for T_0 if

$$1 < \frac{\rho_s}{\rho_{s^{-1}}} < m - k + 1 \tag{29}$$

Since $\rho_s > 1$, the left-hand inequality is always satisfied. For the right-hand inequality,

$$\rho_s > \frac{m-k+1}{m-k} \tag{30}$$

Let, us consider $m - k = 1$, then

$$k\lambda_r(\rho_s - 2)T + (\rho_s - 1)exp(-k\lambda_r T) - \rho_s = 0 \tag{31}$$

Since $k\lambda_r T \geq 0$, obviously (31) has a solution only when $\rho_s > 2$ which also confirms the inequality condition provided (30). It provides the solution of (31) for various value of ρ_s and k, for small values of $k\lambda_r T$ for which

$$exp(-k\lambda_r T) \approx 1 - k\lambda_r T + \frac{(k\lambda_r T)^2}{2}$$

After simplification of (31), it gives

$$\left(\frac{\rho_{s-1}}{2}\right)(k\lambda_r T)^2 - k\lambda_r T - 1 = 0 \tag{32}$$

and therefore

$$T_0 = \frac{1 + \sqrt{2\rho_{s-1}}}{k\lambda_r(\rho_{s-1})}, \qquad \rho_s > 2 \tag{33}$$

The following cases discuss below

- **Single Repair Point:** Failure rate of the system at any state i is $h_i = (m - i)\lambda_i$, where λ_i is the failure rate of units at state i, then

  $$\lambda_s = \lambda_i\left(\frac{k}{m-k}\right)^\beta$$

  Then, after get

  $$\lambda_i = \lambda_r\left(\frac{k}{m-i}\right)$$

  Hence,

  $$h_i = k\lambda_r, \; for \; i = 0, 1, 2, \ldots, m - k. \tag{34}$$

  If the repair shop has only one repair facility, then the repair rate is constant

  $$\mu_i = \mu, \; for \; i = 1, 2, \ldots, m - k + 1 \tag{35}$$

  where μ is the repair rate of each unit. Then from (14) and (15), gives

  $$A(k, m) = \frac{\sum_{j=1}^{m-k}\left(\frac{k\lambda_r}{\mu}\right)^j}{1 + \sum_{j=1}^{m-k+1}\left(\frac{k\lambda_r}{\mu}\right)^j} \tag{36}$$

➢ **Parallel Repair Points:** When parallel repair facilities exist, the failed unit does not wait in the queue and therefore

$$\mu_i = i\mu, \forall i \tag{37}$$

and the failure rate is given by (34), it gives

$$A(k,m) = \frac{1 + \sum_{j=1}^{m-k} \prod_{i=0}^{j-1} \frac{k\lambda_r}{(i+1)\mu}}{1 + \sum_{j=1}^{m-k+1} \prod_{i=0}^{j-1} \frac{k\lambda_r}{(i+1)\mu}} \tag{38}$$

➢ **Non-Linear stress-model and $1 < r < m-k+1$:** When the number of repair facilities is more than one but less than $m-k+1$, at a particular instant, it may have two situations:

(a). The number of failed units is less than or equal to the number of repair facilities.

(b). The number of failed units is more than the repair points,

$$h_x = k^a (m-k)^{1-a} \lambda_r \ and\ \mu_x = x\mu, x = 1, 2, \ldots, r.$$

Then,

$$\prod_{i=0}^{x-1} \frac{h_i}{\mu_{i+1}} = \frac{(k^a \lambda_r) \prod_{i=0}^{x-1} (m-k)^{1-a}}{x!\mu^x} = \frac{B(m,x)\left[\frac{k^a \lambda_r}{\mu}\right]^x}{[m(m-1)\ldots(m-x+1]^a} \tag{39}$$

The number of failure unit is more than the repair facilities. If $x > r$, therefore $\mu_r = r\mu$ and hence

$$\prod_{i=0}^{x-1} \frac{h_i}{\mu_{i+1}} = \frac{(k^a \lambda_r)^x \prod_{i=0}^{x-1} (m-k)^{1-a}}{(r!\mu^r)(r\mu)^{x-r}} = \frac{[m(m-1)\ldots(m-x+1]^{1-a}}{(r!r^r)} \left[\frac{k^a \lambda_r}{r\mu}\right]^x \tag{40}$$

Thus the system availability is

$$A(k,m) = \frac{1 + \sum_{j=1}^{r} D_1(j) + \sum_{j=r+1}^{m-k+1} D_2(j)}{(1 + \sum_{j=1}^{r} D_1(j) + \sum_{j=r+1}^{m-k} D_2(j)} \tag{41}$$

Where

$$D_1(j) \equiv \frac{B(m,j)\left[\frac{k^a \lambda_r)}{\mu}\right]^j}{[m(m-1)\ldots(m-x+1]^a}$$

and

$$D_2(j) \equiv \frac{B[m(m-1)\ldots(m-x+1]^{1-a}}{(r!r^r)} \times \left[\frac{k^a \lambda_r)}{r\mu}\right]^j$$

The above mentioned results describe in equations 36, 38, 41 with different values of k and other various dependent conditions are provided in Table 1. In table 2 & 3 the numerical values for $\frac{\mu}{\lambda_r} = 1$ & 2 respectively are assumed for comparison purpose.

Conditions	Availability		
	$k = 1$	$K = 2$	k = 3
$a = 0$ & $r = 1$	$\frac{\mu(\mu^2+3\mu\lambda_r+6\lambda_r^2)}{\mu^3+3\mu^2\lambda_r+6\mu\lambda_r^2+6\lambda_r^3}$	$\frac{\mu(\mu+3\lambda_r)}{\mu^2+3\mu\lambda_r+6\lambda_r^2}$	$\frac{\mu}{\mu+3\lambda_r}$
$a = 1$ & $r = 1$	$\frac{\mu(\mu^2+\mu\lambda_r+\lambda_r^2)}{\mu^3+\mu^2\lambda_r+\mu\lambda_r^2+\lambda_r^3}$	$\frac{\mu(\mu+2\lambda_r)}{(\mu^2+2\mu\lambda_r+4\lambda_r^2}$	$\frac{\mu}{\mu+3\lambda_r}$
$a = 1$ & $r \geq 4-k$	$\frac{3\mu(2\mu^2+2\mu\lambda_r+\lambda_r^2)}{6\mu^3+6\mu^2\lambda_r+3\mu\lambda_r^2+\lambda_r^3}$	$\frac{\mu(\mu+2\lambda_r)}{(\mu^2+2\mu\lambda_r+2\lambda_r^2}$	$\frac{\mu}{\mu+3\lambda_r}$

Conditions	Availability		
	$k=1$	$K=2$	k $=3$
$a=1$ & $r=2$	$\frac{27\mu(2\mu^2+2\mu\lambda_r+\lambda_r^2)}{54\mu^3+54\mu^2\lambda_r+27\mu\lambda_r^2+4\lambda_r^3}$	$\frac{\mu(\mu+2\lambda_r)}{\mu^2+2\mu\lambda_r+2\lambda_r^2}$	$\frac{\mu}{\mu+3\lambda_r}$
$a=2$ & $r=1$	$\frac{\mu(6\mu^2+2\mu\lambda_r+\lambda_r^2)}{6\mu^3+2\mu^2\lambda_r+\mu\lambda_r^2+\lambda_r^3}$	$\frac{\mu(3\mu+4\lambda_r)}{3\mu^2+4\mu\lambda_r+8\lambda_r^2}$	$\frac{\mu}{\mu+3\lambda_r}$
$a=0$ & $r\geq 4-k$	$\frac{\mu(\mu^2+3\mu\lambda_r+3\lambda_r^2)}{\mu^3+3\mu^2\lambda_r+3\mu\lambda_r^2+\lambda_r^3}$	$\frac{\mu(\mu+3\lambda_r)}{\mu^2+3\mu\lambda_r+3\lambda_r^2}$	$\frac{\mu}{\mu+3\lambda_r}$

Table 1:

Conditions	Availability		
	$k=1$	$K=2$	k $=3$
$a=0$ & $r=1$	0.625	0.40	0.25
$a=1$ & $r=1$	0.75	0.43	0.25
$a=1$ & $r\geq 4-k$	0.94	0.60	0.25
$a=1$ & $r=2$	0.97	0.60	0.25
$a=2$ & $r=1$	0.90	0.47	0.25
$a=0$ & $r\geq 4-k$	0.86	0.57	0.25

Table 2:

Conditions	Availability		
	$k=1$	$K=2$	k $=3$
$a=0$ & $r=1$	0.84	0.63	0.4
$a=1$ & $r=1$	0.93	1.5	0.4
$a=1$ & $r\geq 4-k$	0.98	0.91	0.4
$a=1$ & $r=2$	0.99	0.91	0.4
$a=2$ & $r=1$	0.98	0.71	0.4
$a=0$ & $r\geq 4-k$	0.96	0.77	0.4

Table 3:

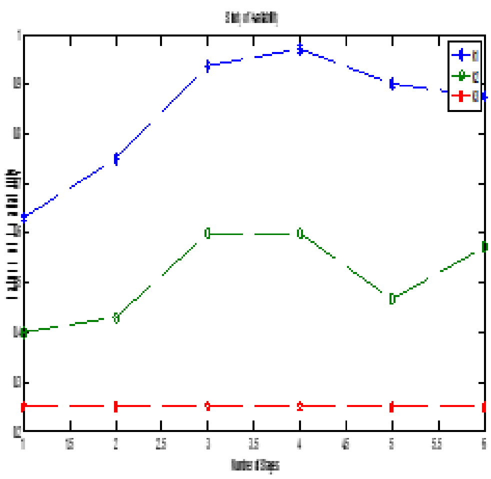


Figure 2:

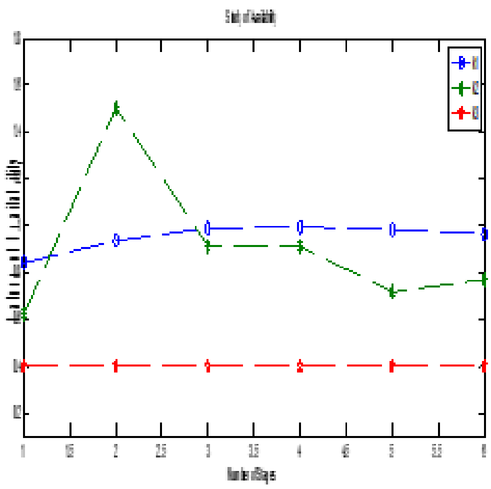


Figure 3:

5. Conclusion

In this investigation, system repair and optimal maintenance policy of production units were discussed with different cases. Availability of the system is carried out with the help of numerical illustration by varying the various parameters and it is found that the system availability is appearing in up and down fashion for $K = 1, 2$ and $\frac{\mu}{\lambda_r} = 1$. However it is noticed that the availability of the system is constant for $K = 3$ and $\frac{\mu}{\lambda_r} = 2$. Hence it concludes that when the ratio of $\frac{\mu}{\lambda_r}$ is increasing then availability of the system becomes stable.

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