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Compactness in Smooth Bitopological Spaces

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Abstract: The purpose of this paper is introducing the notion of smooth bitopological spaces. We define pairwise smooth-closure and pairwise smooth-interior of a fuzzy set and we study some of its properties. We give the definition for pairwise smooth compact set and show by an example that every pairwise smooth compact set need not be a smooth compact set. Finally we introduce the idea of pairwise smooth almost compact set and pairwise smooth nearly compact set.

MSC: 54A40.

Keywords: Smooth topological spaces, Smooth bitopological spaces, Pairwise smooth-closure, Pairwise smooth-interior, Pairwise smooth compact.

1. Introduction

A.A.Ramadan [6] introduced the concept of smooth topology. Let Y be a non empty set, a function $\tau : I^Y \to I = [0,1]$ satisfies the following properties:

(1). for all $\beta \in [0,1]$, $\tau(\bar{\beta}) = 1$.

(2). $\tau(C \wedge D) \geq \tau(C) \wedge \tau(D)$ for each $C, D \in I^Y$.

(3). $\tau\left(\bigvee_{i\in J} C_i\right) \geq \bigwedge_{i\in J} \tau(C_i)$ for each $\{C_i : i \in J\} \subseteq I^Y$.

Then we say that τ is a smooth topology on Y and the pair (Y, τ) is called a smooth topological space. The function $\tau^* : I^Y \to I$, defined by $\tau^*(C) = \tau(C')$ for every $C \in I^Y$, (where $C' = \bar{1} - C$), measures the grade of fuzzy subsets of Y being closed and the number $\tau^*(C)$ is called the degree of closedness of C. The function τ^* satisfies the properties similar to τ which are given in [8]. The definition of fuzzy bitopological spaces in Sostak's sense was given in [7]. The following functions τ_1 and τ_2 are the examples for smooth topologies on a non-empty set Y, which was explained in [8].

(1). $\tau_1 : I^Y \to I$ defined by

(i). $\tau_1(\bar{0}) = 1$.

(ii). $\tau_1(B) = \frac{\inf(B)}{\sup(B)}$ if $B \neq \bar{0}$.

(2). $\tau_2 : I^Y \to I$ defined by

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(i). $\tau_2(\bar{0}) = 1$.

(ii). $\tau_2(B) = \frac{1}{2}\frac{\inf(B)}{\sup(B)} + \frac{1}{2}$.

Throughout this paper, the triple (Y, τ_1, τ_2) is called as smooth bitopological space (briefly, sbts), where τ_1 and τ_2 are any two smooth topologies on Y. The notions of τ-smooth closure and τ-smooth interior were given in [8]. In this paper we denote the τ_i-smooth closure of A as $\bar{A}_i$ and the τ_i-smooth interior of A as A_i^0 for $i = 1, 2$. The concept of smooth compact set was discussed in [8].

Definition 1.1 ([7]). *A mapping $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ from a fuzzy bitopological space (X, τ_1, τ_2) to another fuzzy bitopological space (X, σ_1, σ_2) is said to be fuzzy pairwise continuous (fpc, for short) if and only if $\tau_i(f^{-1}(\mu)) \geq \sigma_i(\mu)$ for each $\mu \in I^Y$ and $i = 1, 2$.*

In this paper fuzzy pairwise continuous is termed as pairwise smooth continuous

2. Pairwise Smooth-closure and Pairwise Smooth-interior

Definition 2.1. *Let (X, τ_1, τ_2) be a sbts and $A \in I^X$. We define a function $\tau_{1,2} : I^X \to I$ as $\tau_{1,2}(A)$ = max{0, $\tau_1(A)$ + $\tau_2(A)$ - 1}.*

The mapping $\tau_{1,2} : I^X \to$ I measures the grade of fuzzy subsets of X being pairwise open and the number $\tau_{1,2}$(A) is called the degree of pairwise openness of A. The mapping $\tau_{1,2}^* : I^X \to$ I defined by $\tau_{1,2}^*$(A) = $\tau_{1,2}(A')$ for every A$\in I^X$, measures the grade of fuzzy subsets of X being pairwise closed and the number $\tau_{1,2}^*$(A) is called the degree of pairwise closedness of A.

Remark 2.2.

(1). As there is no guarantee for each $A \neq B \in I^X$, we would get that $\tau_{1,2}(A \bigvee B) \geq \tau_{1,2}(A)$ and $\tau_{1,2}(A \bigvee B) \geq \tau_{1,2}(B)$, $\tau_{1,2}$ need not be a smooth topology on X.

(2). If either $\tau_1(A) = 0$ or $\tau_2(A) = 0$ then $\tau_{1,2}(A) = 0$.

(3). If $\tau_{1,2}(A) \neq 0$ then $\tau_1(A) \neq 0$ and $\tau_2(A) \neq 0$. In this case $\tau_{1,2}(A) \leq \tau_1(A)$ and $\tau_{1,2}(A) \leq \tau_2(A)$.

Definition 2.3. *Let (X, τ_1, τ_2) be a sbts and $A \in I^X$, then We define pairwise smooth interior of A, denoted by $A_{1,2}^0$ as follows:*

(1). if $\tau_1(A) + \tau_2(A) = 2$ then $A_{1,2}^0 = A$.

(2). if $\tau_1(A) + \tau_2(A) < 2$ then $A_{1,2}^0 = \bigvee\{K \in I^X : \tau_1(K) + \tau_2(K) > \tau_1(A) + \tau_2(A), K \leq A\}$.

Definition 2.4. *Let (X, τ_1, τ_2) be a sbts and $A \in I^X$, then we define pairwise smooth closure of A, denoted by $\bar{A}_{1,2}$ as follows:*

(1). if $\tau_1^*(A) + \tau_2^*(A) = 2$ then $\bar{A}_{1,2} = A$.

(2). if $\tau_1^*(A) + \tau_2^*(A) < 2$ then $\bar{A}_{1,2} = \bigwedge\{K \in I^X : \tau_1^*(K) + \tau_2^*(K) > \tau_1^*(A) + \tau_2^*(A), A \leq K\}$.

Theorem 2.5. *Let (X, τ_1, τ_2) be a sbts and $A, B \in I^X$, then*

(a). $tau_{1,2}^*(\bar{A}_{1,2}) < \frac{\tau_1^*(\bar{A}_{1,2}) + \tau_2^*(\bar{A}_{1,2})}{2}$

(b). $\tau_{1,2}(A^0_{1,2}) < \frac{\tau_1(A^0_{1,2})+\tau_2(A^0_{1,2})}{2}$

(c). $A \leq B$ *and* $\tau_1^*(A) + \tau_2^*(A) \leq \tau_1^*(B) + \tau_2^*(B) \Rightarrow \bar{A}_{1,2} \leq \bar{B}_{1,2}$

(d). $A \leq B$ *and* $\tau_1(B) + \tau_2(B) \leq \tau_1(A) + \tau_2(A) \Rightarrow A^0_{1,2} \leq B^0_{1,2}$

provided that $\tau_i(A) \neq 0$*, where* $i \in \{1, 2\}$*.*

Proof.

(a). If $\tau_1^*(A) + \tau_2^*(A) = 2$ then $\bar{A}_{1,2} = A$. Therefore

$$\tau^*_{1,2}(\bar{A}_{1,2}) = \tau^*_{1,2}(A) \tag{1}$$

if $\tau_1^*(A) + \tau_2^*(A) < 2$ and $\bar{A}_{1,2} < A$ then by definition of

$$\bar{A}_{1,2} we get \tau_1^*(\bar{A}_{1,2}) + \tau_2^*(\bar{A}_{1,2}) > \tau_1^*(A) + \tau_2^*(A) \tag{2}$$

Also we know that $\tau^*_{1,2}(A) \leq \tau_1^*(A)$ and $\tau^*_{1,2}(A) \leq \tau_2^*(A)$, that is $2\tau^*_{1,2}(A) \leq \tau_1^*(A) + \tau_2^*(A) < \tau_1^*(\bar{A}_{1,2}) + \tau_2^*(\bar{A}_{1,2})$. Therefore

$$\tau^*_{1,2}(A) < \frac{\tau_1^*(\bar{A}_{1,2}) + \tau_2^*(\bar{A}_{1,2})}{2} \tag{3}$$

From (1) and (3) we get $\tau^*_{1,2}(\bar{A}_{1,2}) < \frac{\tau_1^*(\bar{A}_{1,2})+\tau_2^*(\bar{A}_{1,2})}{2}$.

(b). If $\tau_1(A) + \tau_2(A) = 2$ then $A^0_{1,2} = A$. Therefore

$$\tau_{1,2}(A^0_{1,2}) = \tau_{1,2}(A) \tag{4}$$

if $\tau_1(A) + \tau_2(A) < 2$ and $A^0_{1,2} < A$ then by definition of $A^0_{1,2}$ we get

$$\tau_1(A^0_{1,2}) + \tau_2(A^0_{1,2}) > \tau_1(A) + \tau_2(A) \tag{5}$$

Also we know that $\tau_{1,2}(A) \leq \tau_1(A)$ and $\tau_{1,2}(A) \leq \tau_2(A)$, that is $2\tau_{1,2}(A) \leq \tau_1(A) + \tau_2(A) < \tau_1(A^0_{1,2}) + \tau_2(A^0_{1,2})$. Therefore

$$\tau_{1,2}(A) < \frac{\tau_1(A^0_{1,2}) + \tau_2(A^0_{1,2})}{2} \tag{6}$$

From (4) and (6) we get $\tau_{1,2}(A^0_{1,2}) < \frac{\tau_1(A^0_{1,2})+\tau_2(A^0_{1,2})}{2}$.

(c). $\bar{A}_{1,2} = \bigwedge\{K \in I^X : \tau_1^*(K) + \tau_2^*(K) > \tau_1^*(A) + \tau_2^*(A), A \leq K\}$. As $A \leq B$ and $\tau_1^*(A) + \tau_2^*(A) \leq \tau_1^*(B) + \tau_2^*(B)$, we get

$$\begin{aligned}\bar{A}_{1,2} &\leq \bigwedge\{K \in I^X : \tau_1^*(K) + \tau_2^*(K) > \tau_1^*(B) + \tau_2^*(B) \geq \tau_1^*(A) + \tau_2^*(A); A \leq B \leq K\}\\ &= \bigwedge\{K \in I^X : \tau_1^*(K) + \tau_2^*(K) > \tau_1^*(B) + \tau_2^*(B), B \leq K\} = \bar{B}_{1,2}\end{aligned}$$

(d). $A^0_{1,2} = \bigvee\{K \in I^X : \tau_1(K) + \tau_2(K) > \tau_1(A) + \tau_2(A), K \leq A\}$. As $A \leq B$ and $\tau_1(B) + \tau_2(B) \leq \tau_1(A) + \tau_2(A)$, we get

$$\begin{aligned}A^0_{1,2} &\leq \bigvee\{K \in I^X : \tau_1(K) + \tau_2(K) > \tau_1(A) + \tau_2(A) \geq \tau_1(B) + \tau_2(B); K \leq A \leq B\}\\ &= \bigvee\{K \in I^X : \tau_1(K) + \tau_2(K) > \tau_1(B) + \tau_2(B), K \leq B\} = B^0_{1,2}\end{aligned}$$

□

One can easily obtain the following theorem.

Theorem 2.6. *Let (X, τ_1, τ_2) be a sbts and $A \in I^X$, then*

(a). $(\overline{A}_{1,2})' = (A')^0_{1,2}$.

(b). $\overline{A}_{1,2} = ((A')^0_{1,2})'$.

(c). $(A^0_{1,2})' = (\overline{A'})_{1,2}$.

(d). $A^0_{1,2} = ((\overline{A'})_{1,2})'$.

Theorem 2.7. *Let (X, τ_1, τ_2) be a sbts and $A, B \in I^X$, then*

(a). $(\overline{0}_X)_{1,2} = 0_X$.

(b). $A \leq \bar{A}_{1,2}$.

(c). $\bar{A}_{1,2} \leq \overline{(\bar{A}_{1,2})}_{1,2}$.

(d). $\bar{A}_{1,2} \bigvee \bar{B}_{1,2} \leq (\overline{A \bigvee B})_{1,2}, if \tau_i^*(A) = \tau_i^*(B) where i \in \{1, 2\}$.

Proof. We prove (b) and (d) only.

(b). $\bar{A}_{1,2} = \bigwedge\{K \in I^X / \tau_1^*(K) + \tau_2^*(K) > \tau_1^*(A) + \tau_2^*(A); A \leq K\} \geq A$. Therefore $A \leq \bar{A}_{1,2}$.

(d). $(\overline{A \bigvee B})_{1,2} = \bigwedge\{K \in I^X : \tau_1^*(K) + \tau_2^*(K) > \tau_1^*(A \bigvee B) + \tau_2^*(A \bigvee B); A \bigvee B \leq K\}$

$= \bigwedge\{K \in I^X : \tau_1^*(K) + \tau_2^*(K) \geq (\tau_1^*(A) \bigwedge \tau_1^*(B)) + (\tau_2^*(A) \bigwedge \tau_2^*(B)); A \bigvee B \leq K\}$.

As $\tau_i^*(A) = \tau_i^*(B)$ where $i \in \{1, 2\}$, we get

$$(\overline{A \bigvee B})_{1,2} \geq \bigwedge\{K \in I^X : \tau_1^*(K) + \tau_2^*(K) > \tau_1^*(A) + \tau_2^*(A); A \leq K\} = \bar{A}_{1,2}.$$

That is $\bar{A}_{1,2} \leq (\overline{A \bigvee B})_{1,2}$. Similarly we can get $\bar{B}_{1,2} \leq (\overline{A \bigvee B})_{1,2}$. Hence $\bar{A}_{1,2} \bigvee \bar{B}_{1,2} \leq (\overline{A \bigvee B})_{1,2}$, if $\tau_i^*(A) = \tau_i^*(B)$ where $i \in \{1, 2\}$. □

Theorem 2.8. *Let (X, τ_1, τ_2) be a sbts and $A, B \in I^X$, then*

(a). $(1_X)^0_{1,2} = 1_X$.

(b). $A^0_{1,2} \leq A$.

(c). $(A^0_{1,2})^0_{1,2} \leq A^0_{1,2}$.

(d). $(A \bigwedge B)^0_{1,2} \leq A^0_{1,2} \bigvee B^0_{1,2}$, *if* $\tau_i(A) = \tau_i(B)$, *where* $i \in \{1, 2\}$.

Proof.

(a). $(1_X)^0_{1,2} = \bigvee\{K \in I^X / \tau_1(K) + \tau_2(K) > \tau_1(1) + \tau_2(1); 1 \leq K\}$

$= \bigvee\{K \in I^X / \tau_1(K) + \tau_2(K) > 2; 1 \leq K\}$

As $\tau_i(K) \leq 1$ there is no $K \in I^X$ with $\tau_1(K) + \tau_2(K) > 2$. Therefore $(1_X)^0_{1,2} = 1_X$.

(b). $A^0_{1,2} = \bigvee\{K \in I^X / \tau_1(K) + \tau_2(K) > \tau_1(A) + \tau_2(A); K \leq A\} \leq A$. Therefore $A^0_{1,2} \leq A$.

(c). From (b), $(A^0_{1,2})^0_{1,2} \leq A^0_{1,2}$.

(d). $(A\bigwedge B)^0_{1,2} = \bigvee\{K\in I^X : \tau_1(K)+\tau_2(K) > \tau_1(A\bigwedge B)+\tau_2(A\bigwedge B); K\leq A\bigwedge B\}$

$= \bigvee\{K\in I^X : \tau_1(K)+\tau_2(K) \geq (\tau_1(A)\bigwedge \tau_1(B))+(\tau_2(A)\bigwedge \tau_2(B)); K\leq A\bigwedge B\}$.

$\leq \bigvee\{K\in I^X : \tau_1(K)+\tau_2(K) > \tau_1(A)+\tau_2(A); K\leq A\}$

or $\bigvee\{K\in I^X : \tau_1(K)+\tau_2(K) > \tau_1(B)+\tau_2(B); K\leq B\}\{As\ \ \tau_i(A)=\tau_i(B)\ \ where\ \ i\in\{1,2\}\} = A^0_{1,2}\bigvee B^0_{1,2}$. Therefore $(A\bigwedge B)^0_{1,2}\leq A^0_{1,2}\bigvee B^0_{1,2}$. □

The concept of fuzzy pairwise compact was given in [7]. In this paper we give a modified definition for fuzzy pairwise compact which is termed as pairwise smooth compact.

Definition 2.9. *Let (X,τ_1,τ_2) be a sbts. A fuzzy set $A\in I^X$ is said to be pairwise smooth compact if for every collection of fuzzy sets $\{W_\alpha\}_{\alpha\in J}$ with*

(i). $A\leq \bigvee_\alpha W_\alpha$.

(ii). $\tau_1(A)+\tau_2(A) < \tau_1(W_\alpha)+\tau_2(W_\alpha)\ \ \forall\ \ \alpha\ \ if\ \ 0\leq \tau_1(A)+\tau_2(A) < 2$

$\tau_1(A)+\tau_2(A) \leq \tau_1(W_\alpha)+\tau_2(W_\alpha)\ \ \forall\ \ \alpha\ \ if\ \ 0\leq \tau_1(A)+\tau_2(A) = 2$

there exists a finite subset J_0 of J such that $A\leq \bigvee_{\alpha\in J_0} W_\alpha$.

Theorem 2.10. *Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a pairwise smooth continuous function. If $A\in I^X$ with $\tau_i(A)\leq\sigma_i(f(A))$ is pairwise smooth compact then $f(A)$ is pairwise smooth compact.*

Proof. Let $A\in I^X$ be a pairwise smooth compact set. Then

$$f(A)(y) = \begin{cases} \bigvee\{A(x)/f(x)=y\} \\ 0\ \ if\ \ there\ \ is\ \ no\ \ x\ \ such\ \ that\ \ f(x)=y \end{cases}$$

Let $f(A)\leq \bigvee_{\alpha\in J} W_\alpha$ such that

$$\sigma_1(f(A))+\sigma_2(f(A)) < \sigma_1(W_\alpha)+\sigma_2(W_\alpha)\ \forall\,\alpha\ if\ \sigma_1(f(A))+\sigma_2(f(A)) < 2$$

$$\sigma_1(f(A))+\sigma_2(f(A)) = \sigma_1(W_\alpha)+\sigma_2(W_\alpha)\ \forall\,\alpha\ if\ \sigma_1(f(A))+\sigma_2(f(A)) = 2$$

Therefore $f(A)(y)\leq \bigvee_{\alpha\in J} W_\alpha(y)$. Therefore to each $x\in X, A(x)\leq f(A)(y)\leq \bigvee_{\alpha\in J} f^{-1}(W_\alpha)(x)$ where $f(x)=y$. That is $A\leq \bigvee_{\alpha\in J} f^{-1}(W_\alpha)$. As f is pairwise smooth continuous, for each $\alpha\in J$, we have $\tau_1(f^{-1}(W_\alpha))+\tau_2(f^{-1}(W_\alpha))\geq \sigma_1(W_\alpha)+\sigma_2(W_\alpha) > \sigma_1(f(A))+\sigma_2(f(A))\geq \tau_1(A)+\tau_2(A)$ if $0<\sigma_1(f(A))+\sigma_2(f(A))<2$ and $\tau_1(f^{-1}(W_\alpha))+\tau_2(f^{-1}(W_\alpha)) = \sigma_1(W_\alpha)+\sigma_2(W_\alpha) = \sigma_1(f(A))+\sigma_2(f(A))\geq\tau_1(A)+\tau_2(A)$ if $\sigma_1(f(A))+\sigma_2(f(A))=2$. As A is pairwise smooth compact, $A\leq \bigvee_{\alpha\in J_0\subset J} f^{-1}(W_\alpha)$ where J_0 is a finite subset of J. That is $f(A)\leq \bigvee_{\alpha\in J_0} f(f^{-1}(W_\alpha))$. That is $f(A)\leq \bigvee_{\alpha\in J_0} W_\alpha$. [$\because f(f^{-1}(B))\leq B$ for any fuzzy set B in Y] $\Rightarrow f(A)$ is pairwise smooth compact. □

The next example asserts that a pairwise smooth compact set need not be a smooth compact set.

Example 2.11. *Let (X,τ_1,τ_2) be a sbts where τ_1 and τ_2 are the two smooth topologies given in the introductory section. Consider the fuzzy set A on the real line $\mathbb{R}$ given by*

$$A(x) = \begin{cases} 0.5x+0.5; & if\ x\in[0,1] \\ \frac{1}{x}; & if\ x\in\{1,1.1,1.2,1.3,\dots,1.9\} \\ \frac{1}{x}; & if\ x\in\{2,3,4,5,\dots\} \\ 0; & otherwise \end{cases}$$

We show that A is pairwise smooth compact with respect to τ_1 and τ_2, but it is not smooth compact with respect to τ_1. Let $\mathscr{F} = \{U_\alpha\}_{\alpha \in J}$ be a collection of fuzzy sets with

(i). $A \leq \bigvee_\alpha U_\alpha$

(ii). $\tau_1(A) + \tau_2(A) < \tau_1(U_\alpha) + \tau_2(U_\alpha)$ *for all* $\alpha \in J$.

Every element U of this collection is such that $A(x) \leq U(x)$ for all but finitely many of the points $x \in \{1, 1.1, 1.2, 1.3, \ldots, 1.9\} \cup \{2, 3, 4, 5, \ldots\}$ [Clearly $\tau_2(U) \neq 0$. If $\tau_1(U) > 0$ then $\inf(U) \neq 0$. Let $\inf(U) = \alpha > 0$. Then $A(x) \leq U(x)$ for all $x \geq \frac{1}{\alpha}$. So Then $A(x) \leq U(x)$ for all but finitely many points of $x \in (\{1, 1.1, 1.2, 1.3, \ldots, 1.9\} \cup \{2, 3, 4, \ldots\}) \cap [1, \frac{1}{\alpha}))$]. Choose for each $y \in \{1, 1.1, 1.2, 1.3, \ldots, 1.9\} \cup \{2, 3, 4, 5, \ldots\}$ with $A(y) \not\leq U(y)$, an element $U_\alpha \in \mathscr{F}$ such that $A(y) \leq U_\alpha(y)$. Thus we get a finite subcollection $J_0 \subset \mathscr{F}$ such that $A \leq U \bigvee \{U_\alpha\}_{\alpha \in J_0}$. This implies that A is pairwise smooth compact. Let

$$U_k(x) = \begin{cases} 0.5x + 0.5; & \textit{if } x \in [0, 1] \\ \frac{1}{x}; & \textit{if } x \in \{1, 1.1, 1.2, 1.3, \ldots, 1.9\} \\ \frac{1}{x}; & \textit{if } x \in \{2, 3, 4, \ldots, k\} \\ 0; & \textit{otherwise} \end{cases}$$

Then $A(x) \leq \bigvee_{k=2}^{\infty} U_k(x)$. But there is no m such that $A(x) \leq \bigvee_{k=2}^{m} U_k(x)$. Therefore A is not smooth compact.

Definition 2.12. *Let (X, τ_1, τ_2) be a sbts. A fuzzy set $A \in I^X$ is said to be pairwise smooth nearly compact if for every collection of fuzzy sets $\{W_\alpha\}_{\alpha \in J}$ with*

(i). $A \leq \bigvee_\alpha W_\alpha$

(ii). $\tau_1(A) + \tau_2(A) < \tau_1(W_\alpha) + \tau_2(W_\alpha) \; \forall \, \alpha \; if \; 0 \leq \tau_1(A) + \tau_2(A) < 2$

$\tau_1(A) + \tau_2(A) \leq \tau_1(W_\alpha) + \tau_2(W_\alpha) \; \forall \, \alpha \; if 0 \leq \tau_1(A) + \tau_2(A) = 2$

there exists a finite subset J_0 of J such that $A \leq \bigvee_{\alpha \in J_0} (\overline{W_\alpha})^0$.

Definition 2.13. *Let (X, τ_1, τ_2) be a sbts. A fuzzy set $A \in I^X$ is said to be pairwise smooth almost compact if for every collection of fuzzy sets $\{W_\alpha\}_{\alpha \in J}$ with*

(i). $A \leq \bigvee_\alpha W_\alpha$

(ii). $\tau_1(A) + \tau_2(A) < \tau_1(W_\alpha) + \tau_2(W_\alpha) \; \forall \, \alpha \; if \; 0 \leq \tau_1(A) + \tau_2(A) < 2$

$\tau_1(A) + \tau_2(A) \leq \tau_1(W_\alpha) + \tau_2(W_\alpha) \; \forall \, \alpha \; if \; 0 \leq \tau_1(A) + \tau_2(A) = 2$

there exists a finite subset J_0 of J such that $A \leq \bigvee_{\alpha \in J_0} \overline{W_\alpha}$.

Theorem 2.14. *Let (X, τ_1, τ_2) be a sbts, then a pairwise smooth nearly compact set A is pairwise smooth almost compact.*

Proof. Let (X, τ_1, τ_2) be a sbts.

(i). $A \leq \bigvee_\alpha W_\alpha$

(ii). $\tau_1(A) + \tau_2(A) < \tau_1(W_\alpha) + \tau_2(W_\alpha) \; \forall \, \alpha \; if \; 0 \leq \tau_1(A) + \tau_2(A) < 2$

$\tau_1(A) + \tau_2(A) \leq \tau_1(W_\alpha) + \tau_2(W_\alpha) \; \forall \, \alpha \; if \; 0 \leq \tau_1(A) + \tau_2(A) = 2$

there exists a finite subset J_0 of J such that $A \leq \bigvee_{\alpha \in J_0} (\overline{W_\alpha})^0$. Since $(\overline{W_\alpha})^0 \leq \overline{W_\alpha}$ for each $\alpha \in J$, by Theorem 2.8, $A \leq \bigvee_{\alpha \in J_0} (\overline{W_\alpha})^0 \leq \bigvee_{\alpha \in J_0} \overline{W_\alpha}$. That is $A \leq \bigvee_{\alpha \in J_0} \overline{W_\alpha}$. Hence A is pairwise smooth almost compact. □

3. Conclusion

In this paper we have introduced the idea of smooth bitopological spaces in Sostak's sense. We have investigated about compactness in smooth bitopological spaces.

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