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μ-α-Semi Generalized Open Sets in Generalized Topological Spaces

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Abstract: In this paper, I introduce a new class of sets in generalized topological spaces called μ-α-semi generalized open sets. Also I investigate some of their basic properties.

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1. Introduction

The concept of generalized topological spaces was introduced by A. Csaszar [1]. He also introduced many -open sets like μ-semi open sets, μ-pre open sets etc., in generalized topological spaces. In this paper, I introduce a new class of sets in generalized topological spaces called μ-α-semi generalized open sets. Also I investigate some of their basic properties.

2. Preliminaries

Definition 2.1 ([1]). *Let X be a nonempty set. A collection μ of subsets of X is a generalized topology (or briefly GT) on X if it satisfies the following:*

(1). $\phi, X \in \mu$ and

(2). If $\{M_i : i \in I\} \subseteq \mu$, then $\cup_{i\in I} M_i \in \mu$.

If μ is a GT on X, then (X, μ) is called a generalized topological space (or briefly GTS) and the elements of μ are called μ - open sets and their complement are called μ - closed sets.

Definition 2.2 ([1]). *Let (X, μ) be a GTS and let $A \subseteq X$. Then the μ-closure of A, denoted by $c_\mu(A)$, is the intersection of all μ-closed sets containing A.*

Definition 2.3 ([1]). *Let (X, μ) be a GTS and let $A \subseteq X$. Then the μ-interior of A, denoted by $i_\mu(A)$, is the union of all μ-open sets contained in A.*

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Definition 2.4 ([1]). *Let (X, μ) be a GTS. A subset A of X is said to be*

(1). μ-semi-open set if $A \subseteq c_\mu(i_\mu(A))$

(2). μ-pre-open set if $A \subseteq i_\mu(c_\mu(A))$

(3). μ-α-open set if $A \subseteq i_\mu(c_\mu(i_\mu(A)))$

(4). μ-β-open set if $A \subseteq c_\mu(i_\mu(c_\mu(A)))$

(5). μ-regular-open set if $A = i_\mu(c_\mu(A))$.

Definition 2.5 ([3]). *Let (X, μ) be a GTS. A subset A of X is said to be*

(1). μ-regular generalized open set if $i_\mu(A) \supseteq U$ whenever $A \supseteq U$, where U is μ-regular closed in X.

(2). μ-generalized open set if $i_\mu(A) \supseteq U$ whenever $A \supseteq U$, where U is μ-closed in X.

(3). μ-generalized-α-open set if $\alpha i_\mu(A) \supseteq U$ whenever $A \supseteq U$, where U is μ-closed in X.

Definition 2.6 ([4]). *Let (X, μ) be a GTS. Then a non-empty subset A is said to be a μ-α-semi generalized closed set (briefly μ-α-SGCS) if $sc_\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is μ-α-open in X.*

3. μ-α-Semi Generalized Open Sets

In this section I investigate μ-α-semi generalized open sets in generalized topological spaces and studied some of their basic properties.

Definition 3.1. *The complement A^c of a μ-α-semi generalized closed set (briefly μ-α-SGCS) in a GTS (X, μ) is called μ-α-semi generalized open set (briefly μ-α-SGOS) in X.*

Example 3.2. *Let $X = \{a, b, c\}$ and let $\mu = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then (X, μ) is a GTS. Now,*

$$\mu - \alpha O(X) = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}.$$

Then $A = \{b, c\}$ is a μ-α-semi generalized open set in (X, μ).

Theorem 3.3. *Every μ-open set in (X, μ) is a μ-α-semi generalized open set in (X, μ) but not conversely in general.*

Proof. Let A be a μ-open set in (X, μ). Then its complement A^c is a μ-closed set in (X, μ). Therefore A^c is a μ-α-semi generalized closed set in X and hence by Definition 3.1, A is a μ-α-semi generalized open set in (X, μ). □

Example 3.4. *Let $X = \{a, b, c, d\}$ and let $\mu = \{\phi, \{b\}, \{d\}, \{b, d\}, X\}$. Then (X, μ) is a GTS. Now,*

$$\mu - \alpha O(X) = \{\phi, \{b\}, \{d\}, \{b, d\}, X\}.$$

Then $A = \{a, c, d\}$ is a μ-α-semi generalized open set in (X, μ). But, as $i_\mu(A) = i_\mu(\{a, c, d\}) = \{d\} = A$, A is not a μ-open set in (X, μ).

Theorem 3.5. *Every μ-semi open set in (X, μ) is a μ-α-semi generalized open set in (X, μ).*

Proof. Let A be a μ-semi open set in (X, μ). Then its complement A^c is a μ-semi closed set in (X, μ). Therefore A^c is a μ-α-semi generalized closed set in (X, μ) and hence by Definition 3.1, A is a μ-α-semi generalized open set in (X, μ). □

Remark 3.6. *Every μ-α-semi generalized open sets and μ-pre open sets in X are independent in general.*

Example 3.7. *Let $X = \{a, b, c, \}$ and let $\mu = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then (X, μ) is a GTS. Now,*

$$\mu - \alpha O(X) = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}.$$

Then $A = \{b, c\}$ is a μ-α-semi generalized open set in (X, μ) but not a μ-pre open set as $i_\mu(c_\mu(A)) = i_\mu(c_\mu(\{b, c\})) = \{b\}$ and $A \nsubseteq \{b\}$.

Example 3.8. *Let $X = \{a, b, c, \}$ and let $\mu = \{\phi, \{a, b\}, X\}$. Then (X, μ) is a GTS. Now,*

$$\mu - \alpha O(X) = \{\phi, \{a, b\}, X\}.$$

Now let $A = \{a\}$. Then $i_\mu(c_\mu(A)) = i_\mu(c_\mu(\{b, c\})) = X$ and $A \subseteq X$. Therefore A is a μ-pre open set, but A is not a μ-α-semi generalized open set as $A^c \subseteq U = \{b, c\}$, where U is a μ-α-open set but $sc_\mu(A^c) = X \nsubseteq \{b, c\} = U$.

Theorem 3.9. *Every μ-α-open set in (X, μ) is a μ-α-semi generalized open set in (X, μ) but not conversely in general.*

Proof. Let A be a μ-α-open set in (X, μ). Then its complement A^c is a μ-α-closed set in (X, μ). Therefore A^c is a μ-α-semi generalized closed set in X and hence by Definition 3.1, A is a μ-α-semi generalized open set in (X, μ). □

Example 3.10. *Let $X = \{a, b, c\}$ and let $\mu = \{\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, X\}$. Then (X, μ) is a GTS. Now,*

$$\mu - \alpha O(X) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, X\}.$$

Then $A = \{a, b\}$ is a μ-α-semi generalized open set in (X, μ). But A is not a μ-α-open set as $i_\mu(c_\mu(i_\mu(A))) = i_\mu(c_\mu(i_\mu(\{a, b\}))) = \{a\}$ and $A \nsubseteq \{a\}$.

Remark 3.11. *Every μ-β-open set is not a and μ-α-semi generalized open set in (X, μ) in general.*

Example 3.12. *Let $X = \{a, b, c, \}$ and let $\mu = \{\phi, \{a, b\}, X\}$. Then (X, μ) is a GTS. Now,*

$$\mu - \alpha O(X) = \{\phi, \{a, b\}, X\}.$$

Now let $A = \{a\}$. Then, $c_\mu(i_\mu(c_\mu(A))) = c_\mu(i_\mu(c_\mu(\{a\}))) = X$ and $A \subseteq X$. Therefore A is a μ-β-open set in (X, μ), but not a μ-α-semi generalized open set as $A^c \subseteq U = \{b, c\}$, where U is a μ-α-open set but $sc_\mu(A^c) = X \nsubseteq \{b, c\} = U$.

Theorem 3.13. *Every $\mu-$regular-open set in (X, μ) is a μ-α-semi generalized open set in (X, μ) but not conversely in general.*

Proof. Let A be a μ-regular-open set in (X, μ). As every μ-regular open set is μ-open, A is μ-open in (X, μ). Then by Theorem 3.3, A is a μ-α-semi generalized open set in (X, μ). □

Example 3.14. *Let $X = \{a, b, c\}$ and let $\mu = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then (X, μ) is a GTS. Now,*

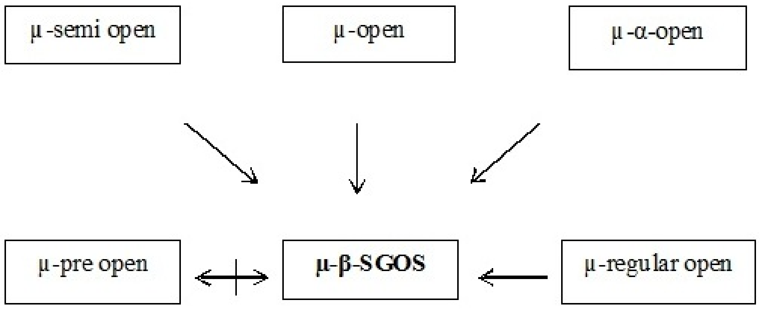
$$\mu - \alpha O(X) = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}.$$

Then $A = \{b, c\}$ is a μ-α-semi generalized open set but not a μ-regular open set as $i_\mu(c_\mu(A)) = i_\mu(c_\mu(\{b, c\})) = \{b\} \neq \{b, c\} = A$.

Theorem 3.15. *Every μ-regular-closed set in (X, μ) is a μ-α-semi generalized open set in (X, μ).*

Proof. Let A be a μ-regular-closed set in (X, μ). Then its complement A^c is a μ-regular-open set in (X, μ). Therefore A^c is a μ-α-semi generalized closed set in X and hence by Definition 3.1, A is a μ-α-semi generalized open set in (X, μ). □

In the following diagram, we have provided relations between various types of open sets.



Theorem 3.16. *A subset A of X is a μ-α-semi generalized open set iff $F \subseteq si_\mu(A)$ whenever $F \subseteq A$ and F is a μ-α-closed set in X.*

Proof. Necessity: Let A be a μ-α-semi generalized open set in X. Then A^c is a μ-α-semi generalized closed set in X. Let F be a μ-α-closed set and $F \subseteq A$. Then F^c is a μ-α-open set and $A^c \subseteq F^c$. Therefore $\text{sc}_\mu(A^c) \subseteq F^c$ by hypothesis. This implies that $(\text{si}_\mu(A))^c \subseteq F^c$. That is F $\subseteq si_\mu(A)$ as $sc_\mu(A^c) = (si_\mu(A))^c$.

Sufficiency: Let $A^c \subseteq U$, where U is a μ-α-open set in (X, μ). Then U^c is a μ-α-open set and $U^c \subseteq (A^c)^c = A$. Let $U^c = F$. Hence by hypothesis $F \subseteq si_\mu(A)$. This implies that $(si_\mu(A))^c \subseteq F^c$. That is $sc_\mu(A^c) \subseteq F^c = U$. Therefore A^c is a μ-α-semi generalized closed set and then A is a μ-α-semi generalized open set in X □

Remark 3.17. *Intersection of any two μ-α-semi generalized open sets in (X, μ) need not be a μ-α-semi generalized open set in X.*

Example 3.18. *Let $X = \{a, b, c\}$ and let $\mu = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$. Then (X, μ) is a GTS. Now,*

$$\mu - \alpha O(X) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}.$$

Then $A = \{b, c\}$ and $B = \{a, c\}$ are μ-α-semi generalized open sets in (X, μ). But, $A \cap B = \{c\}$ is not a μ-α-semi generalized open set as $si_\mu(\{c\}) = \{c\} \cap c_\mu(i_\mu(\{c\})) = \{c\} \cap \phi = \phi \not\supseteq \{c\} = U$ and $A \cap B \supseteq U$.

Theorem 3.19. *If $si_\mu(A) \subseteq B \subseteq A$ and A is a μ-α-semi generalized open set, then B is also a μ-α-semi generalized open set in $\mu - \alpha$.*

Proof. Let $si_\mu(A) \subseteq B \subseteq A$ and A is a μ-α-semi generalized open set in X. Then $(si_\mu(A))^c \supseteq B^c \supseteq A^c$ and so $A^c \subseteq B^c \subseteq sc_\mu(A^c)^c assc_\mu(A^c) = (si_\mu(A))^c$. If A is a μ-α- semi generalized open set then A^c is a μ-α-semi generalized closed set [4], we have B^c is a μ-α-semi generalized closed set and hence B is a μ-α-semi generalized open set in (X, μ). □

Theorem 3.20. *If a subset A of a GTS (X, μ) is both a μ-α-semi generalized open set and a μ-α-closed set then A is a μ-semi open set in X.*

Proof. Let A be a μ-α-closed set and a μ-α-semi generalized open set in (X, μ). Then, $si_\mu(A) \supseteq A$ as $A \supseteq A$. But always $A \supseteq si_\mu(A)$. Therefore, $A = si_\mu(A)$. Hence A is a μ-semi open set in (X, μ). □

Theorem 3.21. *If A is both a μ-pre closed set and a μ-semi open set in (X, μ) then, A is a μ-α-semi generalized open set in (X, μ).*

Proof. Assume that A is a μ-pre closed set and a μ-semi open set in (X, μ). Then $c_\mu(i_\mu(A)) \subseteq A$ and $A \subseteq c_\mu(i_\mu(A))$. Therefore $A = c_\mu(i_\mu(A))$. This implies A is a μ-regular closed set. Hence by Theorem 3.15, A is a μ-α-semi generalized open set in X. □

Theorem 3.22. *Every subset of X is a μ-α-semi generalized open set in X iff every μ-α-closed set is a μ-semi open set in X.*

Proof. Necessity: Let A be a μ-α-closed set in X, and by hypothesis, A is a μ-α-semi generalized open set in X. Hence by Theorem 3.20, A is a μ-semi open set in X.

Sufficiency: Let $A \supseteq U$ where U is a μ-α-closed set in X. Then by hypothesis, U is a μ-semi open set. This implies $si_\mu(U) = U$ and $si_\mu(A) \supseteq si_\mu(U) = U$. Hence $si_\mu(A) \supseteq U$. Thus A is a μ-α-semi generalized open set in X, by Theorem 3.16. □

Theorem 3.23. *Let A and B be μ-α- semi generalized open sets in (X, μ) such that $i_\mu(A) = si_\mu(A)$ and $i_\mu(B) = si_\mu(B)$, then $A \cap B$ is a μ-α-semi generalized open set in X.*

Proof. Let $A \cap B \supseteq U$, where U is a μ-α-closed set in X. Then $A \supseteq U$ and $B \supseteq U$. Since A and B are μ-α-semi generalized open sets, $si_\mu(A) \supseteq U$ and $si_\mu(B) \supseteq U$. Now $si_\mu(A \cap B) \supseteq i_\mu(A \cap B) = i_\mu(A) \cap i_\mu(B) = si_\mu(A) \cap si_\mu(B) \supseteq U \cap U = U$. This implies $si_\mu(A \cap B) \supseteq U$. Hence by Theorem 3.16, $A \cap B$ is a μ-α-semi generalized open set in X. □

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