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K-Idempotent Centro Symmetric Matrices

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Abstract: The basic concepts and theorems of k-Idempotent Centro symmetric matrices are introduced with examples.

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1. Introduction and Preliminaries

Centrosymmetric matrix have practical applications are in information theory, linerar system theory, linear estimate theory and numerical analysis. The concept of centrosymmetric matrices introduced in [1, 3] and properties of K-centrosymmetric matrix are given in [2]. The concept of k-Idempotent matrices was introduced in [4]. In this paper, our intention is to define k-Idempotent Centrosymmetric matrix and also we discussed some results on k-Idempotent Centrosymmetric matrix. A is idempotent matrix, A^T is called Transpose of A. Let k be a fixed product of disjoint transposition in S_n and 'K' be the permutation matrix associated with KI. Clearly K satisfies the following properties, $K^2 = I$, $K^T = K$.

Definition 1.1. *A Symmetric matrix $A = \langle a_{ij} \rangle$ in $C^{n\times n}$ is idempotent if $A^2 = A$.*

Definition 1.2. *If a matrix $A = \langle a_{ij} \rangle$ in $C^{n\times n}$ is said to be k-Idempotent if $KA^2K = A$, where k is the associated permutation matrix of 'K'.*

Definition 1.3. *If a matrix $A = \langle a_{ij} \rangle$ in $C^{n\times n}$ is said to be Centro symmetric matrix if $A = A^T$.*

Definition 1.4. *If a matrix $A = \langle a_{ij} \rangle$ in $C^{n\times n}$ is said to be K-Idempotent Centro symmetric matrix if $K(A^2)^TK = A^T$.*

2. Main Results

Theorem 2.1. *Let $A \in C^{n\times n}$ is K-idempotent Centrosymmetric matrix then $(A^2)^T = KA^TK$.*

Proof. Given A be k-idempotent Centrosymmetric matrix.

$$KAK = A^2$$

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$$K(A^T)K = (A^T)^2$$
$$K(A^T)K = (A^2)^T$$
$$(A^2)^T = K(A^T)K$$

□

Theorem 2.2. *Let A^T be a k-idempotent Centro symmetric matrix then $(I - A)^T$ is k-idempotent Centro symmetric if and only if A^T is idempotent.*

Proof. Assume that, $(I - A)^T$ is k-idempotent Centro symmetric Matrix. Prove that, A^T is idempotent.

$$(I - A)^T = K((I - A)^2)^T K$$
$$= K(I - 2A + A^2)^T K$$
$$= (I - 2A^2 + A)^T$$
$$\Rightarrow -2(A^2)^T + A^T + A^T = 0$$
$$\Rightarrow 2A^T 2(A^2)^T = 0$$
$$\Rightarrow 2(A^T - (A^2)^T) = 0$$

Hence $2(A^T - (A^2)^T) = 0$, which implies that A^T is idempotent.

Conversely, If A^T is idempotent then A^T commutes with the permutation matrix k.

$$K((I - A)^2)^T K = K((I^2 - 2A + A^2)^T)K$$
$$= K(I - A)^T K$$
$$K((I - A)^2)^T K = (I - A)^T$$

□

Theorem 2.3. *Let A^T and B^T be k-idempotent Centrosymmetric matrix then $(A+B)^T$ is also k-idempotent centrosymmetric matrix.*

Proof. Given A^T and B^T be two k-idempotent Centrosymmetric matrix. Therefore,

$$A + B = KA^2K + KB^2K$$
$$K(A + B)K = KA^2K + KB^2K$$
$$K(A^T + B^T)K = K(A^T)^2K + K(B^T)^2K$$
$$(A + B)^T = K(A^2)^T K + K(B^2)^T K$$
$$= A^T + B^T$$
$$K(A + B)^T K = A^T + B^T$$

□

Theorem 2.4. *Let A^T and B^T be two k-idempotent Centro symmetric matrix then $(A - B)^T$ is also k-idempotent Centro symmetric matrix.*

Proof. Given A^T and B^T be two k-idempotent Centro symmetric matrix. Therefore,

$$\begin{aligned}
A - B &= KA^2K - KB^2K \\
K(A-B)K &= KA^2K - KB^2K \\
K(A^T - B^T)K &= K(A^T)^2K - K(B^T)^2K \\
(A-B)^T &= K(A^2)^TK - K(B^2)^TK \\
&= A^T - B^T \\
K(A-B)^TK &= A^T - B^T
\end{aligned}$$

□

Theorem 2.5. *Let A^T be a k-idempotent Centro symmetric matrix then $(A*)^T$ is also k-idempotent Centro symmetric matrix.*

Proof. Given A^T be a k-idempotent Centro symmetric matrix.

$$\begin{aligned}
A^* &= (KA^2K)^* \\
(A^T)^* &= (K(A^T)^2K)^* \\
&= (K(A^2)^TK)^* \\
(A^*)^T &= (K(A^2)^*K)^T
\end{aligned}$$

Therefore, $(A^*)^T$ is also k-idempotent centrosymmetric matrix. □

Theorem 2.6. *Let A^T be a k-idempotent Centrosymmetric matrix then $(A^T)^4$ is also k-idempotent centrosymmetric matrix.*

Proof. Given A^T is k-idempotent centrosymmetric matrix.

$$\begin{aligned}
A^4 &= A^2A^2 \\
&= KAKKAK \\
(A^T)^4 &= K(A^T)KK(A^T)K \\
&= K(A^T)(A^T)K \\
&= K(A^T)^2K \\
&= K(A^2)^TK \\
&= A^T \\
(A^T)^4 &= A^T
\end{aligned}$$

$(A^T)^4$ is also k-idempotent centrosymmetric matrix. □

Theorem 2.7. *Let A^T be k-idempotent centrosymmetirc matrix then KA^T and A^TK are tripotent centrosymmetric matrix.*

Proof.

$$\begin{aligned}
(KA)^3 &= KAKAKA \\
&= KAA^2A
\end{aligned}$$

$$(KA^T)^3 = KA^T(A^T)^2A^T$$
$$= K(A^T)^2(A^T)^2$$
$$= K(A^T)^4$$
$$(KA^T)^3 = KA^T$$

Similarly, $(A^TK)^3 = A^TK$. □

Example 2.8. *Let* $A = \begin{pmatrix} 4 & -1 \\ 12 & -3 \end{pmatrix}$ *and* $B = \begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix}$ *;* $K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$*, then*

(1). $K(A+B)^TK = A^T + B^T$

(2). $K(A-B)^TK = A^T - B^T$

Solution.

(1). $K(A+B)^TK = A^T + B^T$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\left[\begin{pmatrix} 4 & -1 \\ 12 & -3 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix}\right]^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 12 & -3 \end{pmatrix}^T + \begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix}^T$$
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 14 & -2 \end{pmatrix}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 12 \\ -1 & -3 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 4 & -1 \\ 14 & -2 \end{pmatrix}^T = \begin{pmatrix} 4 & 14 \\ -1 & -2 \end{pmatrix}$$
$$\begin{pmatrix} 4 & 14 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 4 & 14 \\ -1 & -2 \end{pmatrix}$$

(2). $K(A-B)^TK = A^T - B^T$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\left[\begin{pmatrix} 4 & -1 \\ 12 & -3 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix}\right]^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 12 & -3 \end{pmatrix}^T - \begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix}^T$$
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 10 & -4 \end{pmatrix}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 12 \\ -1 & -3 \end{pmatrix} - \begin{pmatrix} 0 & 2 \\ 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 4 & -1 \\ 10 & -4 \end{pmatrix}^T = \begin{pmatrix} 4 & 10 \\ -1 & -4 \end{pmatrix}$$
$$\begin{pmatrix} 4 & 10 \\ -1 & -4 \end{pmatrix} = \begin{pmatrix} 4 & 10 \\ -1 & -4 \end{pmatrix}$$

Example 2.9. *Let* $A = \begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix}$ *and* $K = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$*, then prove that* $(A^2)^T = KA^TK$.

Solution. $KA^TK = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$KA^TK = \begin{pmatrix} 0 & 2 \\ 0 & 1 \end{pmatrix} \tag{1}$$

$$A^2 = \begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix}$$

$$(A^2)^T = \begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix}^T$$

$$(A^2)^T = \begin{pmatrix} 0 & 2 \\ 0 & 1 \end{pmatrix} \tag{2}$$

From (1) and (2) we get, $(A^2)^T = KA^TK$.

Example 2.10. *Let* $A = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ *then prove that* $(A^T)^4 = A^T$

Solution. Given $A = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$

$$A^T = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \tag{3}$$

$$(A^T)^2(A^T)^2 = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}^2 \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}^2$$

$$= \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$$

$$(A^T)^4 = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \tag{4}$$

from (3) and (4) we get, $(A^T)^4 = A^T$.

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