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A Study on Q-Intuitionistic L-Fuzzy Submerging of a Semiring Under Homomorphism and Anti-Homomorphism

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Abstract: In this paper, we made an attempt to study the algebraic nature of Q-intuitionistic L-fuzzy subsemiring of a semiring under homomorphism and anti-homomorphism.

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1. Introduction

After the introduction of fuzzy sets by L.A.Zadeh [27], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T.Atanassov [5, 6], as a generalization of the notion of fuzzy set. The notion of fuzzy subnearrings and ideals was introduced by S.Abou Zaid [1]. A.Solairaju and R.Nagarajan [23, 24] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. In this paper, we introduce the some theorems in Q-intuitionistic L-fuzzy subsemiring of a semiring under Homomorphism and Anti-homomorphism and established some results.

1.1. Preliminaries

Definition 1.1 ([27])**.** *Let X be a non-empty set. A fuzzy subset A of X is a function $A : X \to [0, 1]$.*

Definition 1.2 ([23, 24])**.** *Let X be a non-empty set and $L = (L, \leq)$ be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A (Q, L)-fuzzy subset A of X is a function $A : X \times Q \to L$.*

Definition 1.3 ([18])**.** *Let $(R, +, \cdot)$ be a semiring and Q be a non empty set. $A(Q, L)$-fuzzy subset A of R is said to be a (Q, L)-fuzzy subsemiring (QLFSSR) of R if the following conditions are satisfied:*

(1). $A(x + y, q) \geq A(x, q) \wedge A(y, q)$,

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(2). $A(xy,q) \geq A(x,q) \wedge A(y,q)$, for all x and y in R and q in Q.

Example 1.4. *Let $(N,+,\cdot)$ be a semiring and $Q=\{p\}$, Then the (Q,L)-Fuzzy Set A of N is defined by*

$$A(x) = \begin{cases} 0.63 & if\ x\ is\ even \\ 0.27 & if\ x\ is\ odd \end{cases}$$

Clearly A is an (Q,L)-Fuzzy subsemiring.

Definition 1.5 ([5, 6]). *An intuitionistic fuzzy subset (IFS) A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.*

Definition 1.6. *Let $(L, \leq)$ be a complete lattice with an involutive order reversing operation $N : L \to L$ and Q be a nonempty set. A Q-intuitionistic L-fuzzy subset (QILFS) A in X is defined as an object of the form $A = \{\langle (x,q), \mu_A(x,q), \nu_A(x,q)\rangle /$ x in X and q in Q $\}$, where $\mu_A : X \times Q \to L$ and $\nu A : X \times Q \to L$ define the degree of membership and the degree of non-membership of the element $x \epsilon X$ respectively and for every $x \in X$ satisfying $\mu_A(x) \leq N(\nu_A(x))$.*

Definition 1.7. *Let A and B be any two Q-intuitionistic L-fuzzy subsets of a set X. We define the following operations:*

(1). $A \cap B = \{\langle x, \mu_A(x,q) \wedge \mu_B(x,q), \nu_A(x,q) \vee \nu_B(x,q)\rangle\}$, for all $x \in X$ and q in Q.

(2). $A \cup B = \{\langle x, \mu_A(x,q) \vee \mu_B(x,q), \nu_A(x,q) \wedge \nu_B(x,q)\rangle\}$, for all $x \in X$ and q in Q.

(3). $\Box A = \{\langle x, \mu_A(x,q), 1 - \mu_A(x,q)\rangle / x \in X\}$, for all x in X and q in Q.

(4). $\diamond A = \{\langle x, 1 - \nu_A(x,q), \nu_A(x,q)\rangle / x \in X\}$, for all x in X and q in Q.

Definition 1.8 ([20]). *Let R be a semiring. A Q-intuitionistic L-fuzzy subset A of R is said to be a Q-intuitionistic L-fuzzy subsemiring (QILFSSR) of R if it satisfies the following conditions:*

(1). $\mu_A(x+y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$,

(2). $\mu_A(xy,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$,

(3). $\nu_A(x+y,q) \leq \nu_A(x,q) \vee \nu_A(y,q)$,

(4). $\nu_A(xy,q) \leq \nu_A(x,q) \wedge \nu_A(y,q)$, for all x and y in R and q in Q.

Definition 1.9. *Let $(Z_3,+,\cdot)$ be a semiring . Then Q-intuitionistic L-fuzzy subset $A = \{((x,q), \mu_A(x,q), \nu_A(x,q))/x \in Z_3$ and q in Q $\}$ of Z_3 , where*

$$\mu_A(x) = \begin{cases} 0.6 & if\ x\ =\ 0 \\ 0.3 & if\ x\ =\ 1,2 \end{cases}$$

and

$$\nu_A(x) = \begin{cases} 0.2 & if\ x\ =\ 0 \\ 0.4 & if\ x\ =\ 1,2 \end{cases}$$

Clearly A is a Q-Intuitionistic L-fuzzy subsemiring.

Definition 1.10. *Let A and B be any two Q-intuitionistic L-fuzzy subsemiring of a semiring G and H, respectively. The product of A and B, denoted by $A \times B$, is defined as $A \times B = \{\langle (x,y), q), \mu_{A\times B}((x,y),q), \nu_{A\times B}((x,y),q)\rangle /$ for all x in G and y in H and q in Q }, where $\mu_{A\times B}((x,y),q) = \mu_A(x,q) \wedge \mu_B(y,q)$ and $\nu_{A\times B}((x,y),q) = \nu_A(x,q) \vee \nu_B(y,q)$.*

Definition 1.11. *Let A be an Q-intuitionistic L-fuzzy subset in a set S, the strongest Q-intuitionistic L-fuzzy relation on S, that is a Q-intuitionistic L-fuzzy relation on A is V given by $\mu_V((x,y),q) = \mu_A(x,q) \wedge \mu_A(y,q)$ and $\nu_V((x,y),q) = \nu_A(x,q) \wedge \nu_A(y,q)$, for all x and y in S and q in Q.*

Definition 1.12. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two semirings. Let $f: R \to R'$ be any function and A be an Q-intuitionistic L-fuzzy subsemiring in R, V be an Q-intuitionistic L-fuzzy subsemiring in $f(R) = R'$, defined by $\mu_V(y,q) = \sup_{x\in f^{-1}(y)} \mu_A(x,q)$ and $\nu_V(y,q) = \inf_{x\in f^{-1}(y)} \nu_A(x,q)$, for all x in R and y in R'. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.*

Definition 1.13. *Let A be an Q-intuitionistic L-fuzzy subsemiring of a semiring $(R,+,\cdot)$ and a in R. Then the pseudo Q-intuitionistic L-fuzzy coset $(aA)^p$ is defined by $((a\mu_A)^p)(x,q) = p(a)\mu_A(x,q)$ and $((a\nu_A)^p)(x,q) = p(a)\nu_A(x,q)$, for every x in R and for some p in P and q in Q.*

Example 1.14. *Let $(Z_3,+,\cdot)$ be a semiring. Then Q-intuitionistic L-fuzzy subset $A = \{((x,q), \mu_A(x,q), \nu_A(x,q))/x \in Z_3$ and q in Q } of Z_3, where*

$$\mu_A(x) = \begin{cases} 0.6 & if\ x\ =\ 0 \\ 0.3 & if\ x\ =\ 1,2 \end{cases}$$

and

$$\nu_A(x) = \begin{cases} 0.2 & if\ x\ =\ 0 \\ 0.4 & if\ x\ =\ 1,2 \end{cases}$$

Clearly A is a Q-Intuitionistic L-fuzzy subsemiring. Now taking $p(a) = 0.1$ for every a in Z_3. Then the pseudo Q-intuitionistic L-fuzzy coset $(aA)^p$ is defined by

$$\mu_A(x) = \begin{cases} 0.06 & if\ x\ =\ 0 \\ 0.03 & if\ x\ =\ 1,2 \end{cases}$$

and

$$\nu_A(x) = \begin{cases} 0.02 & if\ x\ =\ 0 \\ 0.04 & if\ x\ =\ 1,2 \end{cases}$$

Clearly $(aA)p$ is a Q-Intuitionistic L-fuzzy subsemiring.

Definition 1.15. *Let A be a Q-intuitionistic L-fuzzy subset of X. For α, β in L, the Q-level subset of A is the set $A_{(\alpha,\beta)} = \{x \in X : \mu_A(x,q) \geq \alpha, \nu_A(x,q) \leq \beta\}$.*

2. Properties of Q-Intuitionistic L-Fuzzy Subsemiring of A Semiring Under Homomorphism and Anti-Homomorphism

Theorem 2.1. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two semirings and Q be a non-empty set. The homomorphic image of a Q-intuitionistic L-fuzzy subsemiring of R is an Q-intuitionistic L-fuzzy subsemiring of R'.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two semirings Q be a non-empty set. Let $f: R \to R'$ be a homomorphism. Then $f(x+y) = f(x)+f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let A be a Q-intuitionistic L-fuzzy subsemiring of R. We have to prove that V is a Q-intuitionistic L-fuzzy subsemiring of R'. Now, for $f(x), f(y)$ in R' and q in Q, $\mu_v(f(x)+f(y), q) = \mu_v(f(x+y), q) \geq \mu_A(x+y, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$ which implies that $\mu_v(f(x)+f(y), q) \geq \mu_v(f(x, q)) \wedge \mu_v(f(y, q))$. Again, $\mu_v(f(x)f(y), q) = \mu_v(f(xy), q) \geq \mu_A(xy, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$ which implies that $\mu_v(f(x)f(y), q) \geq \mu_v(f(x, q)) \wedge \mu_v(f(y, q))$. Now, for$f(x), f(y)$ in R', $\nu_v(f(x)+f(y), q) = \nu_v(f(x+y), q) \leq \nu_A(x+y, q) \leq \nu_A(x, q) \vee \nu_A(y, q)$ which implies that $\nu_v(f(x)+f(y), q) \leq \nu_v(f(x), q) \vee \nu_v(f(y, q))$. Again, $\nu_v(f(x)f(y), q) = \nu_v(f(xy), q) \leq \nu_A(xy, q) \leq \nu_A(x, q) \vee \nu_A(y, q)$ which implies that $\nu_v(f(x)f(y), q) \leq \nu_v(f(x, q)) \vee \nu_v(f(y, q))$, for all $f(x)$ and $f(y)$ in R' and q in Q. Hence V is a Q-intuitionistic L-fuzzy subsemiring of R'. □

Theorem 2.2. *Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two semirings and Q be a non-empty set. The homomorphic preimage of a Q-intuitionistic L-fuzzy subsemiring of $f(R) = R'$ is a Q-intuitionistic L-fuzzy subsemiring of R.*

Proof. Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two semirings Q be a non-empty set. Let $f : R \to R'$ be a homomorphism. Then i) $f(x+y) = f(x)+f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let V be a Q-intuitionistic L-fuzzy subsemiring of $f(R) = R'$. We have to prove that A is an Q-intuitionistic L-fuzzy subsemiring of R. Let x and y in R. Then, $\mu_A(x+y, q) = \mu_v(f(x+y), q)$, since $\mu_v(f(x, q)) = \mu_A(x, q) = \mu_v(f(x)+f(y), q) \geq \mu_v(f(x, q)) \wedge \mu_v(f(y, q)) = \mu_A(x, q) \wedge \mu_A(y, q)$, since $\mu_v(f(x, q)) = \mu_A(x, q)$ which implies that $\mu_A(x+y, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$. Again, $\mu_A(xy, q) = \mu_v(f(xy), q)$, since $\mu_v(f(x)) = \mu_A(x, q) = \mu_v(f(x)f(y), q) \geq \mu_v(f(x, q)) \wedge \mu_v(f(y, q)) = \mu_A(x, q) \wedge \mu_A(y, q)$, since $\mu_v(f(x, q)) = \mu_A(x, q)$ which implies that $\mu_A(xy, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$. Let x and y in R. Then, $\nu_A(x+y, q) = \nu_v(f(x+y), q)$, since $\nu_v(f(x, q)) = \nu_A(x, q) = \nu_v(f(x)+f(y), q) \leq \nu_v(f(x, q)) \vee \nu_v(f(y, q)) = \nu_A(x, q) \vee \nu_A(y, q)$, since $\nu_v(f(x, q)) = \nu_A(x, q)$ which implies that $\nu_A(x+y, q) \leq \nu_A(x, q) \vee \nu_A(y, q)$. Again, $\nu_A(xy, q) = \nu_v(f(xy, q))$, since $\nu_v(f(x, q)) = \nu_A(x, q) = \nu_v(f(x)f(y), q) \leq \nu_v(f(x, q)) \vee \nu_v(f(y, q)) = \nu_A(x, q) \vee \nu_A(y, q)$, since $\nu_v(f(x, q)) = \nu_A(x, q)$ which implies that $\nu_A(xy, q) \leq \nu_A(x, q) \vee \nu_A(y, q)$. Hence A is a Q-intuitionistic L-fuzzy subsemiring of R. □

Theorem 2.3. *Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two semirings Q be a non-empty set. The anti-homomorphic image of a Q-intuitionistic L-fuzzy subsemiring of R is an Q-intuitionistic L-fuzzy subsemiring of R'.*

Proof. Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two semirings Q be a non-empty set. Let $f : R \to R'$ be an anti-homomorphism. Then $f(x+y) = f(y)+f(x)$ and $f(xy) = f(y)f(x)$, for all $x, y \in R$. Let A be a Q-intuitionistic L-fuzzy subsemiring of R. We have to prove that V is an Q-intuitionistic L-fuzzy subsemiring of $f(R) = R'$. Now, for $f(x), f(y)$ in R' and q in Q, $\mu_v(f(x)+f(y), q) = \mu_v(f(y+x), q) \geq \mu_A(y+x, q) \geq \mu_A(y, q) \wedge \mu_A(x, q) = \mu_A(x, q) \wedge \mu_A(y, q)$ which implies that $\mu_v(f(x)+f(y), q) \geq \mu_v(f(x, q)) \wedge \mu_v(f(y, q))$. Again, $\mu_v(f(x)f(y), q) = \mu_v(f(yx, q)) \geq \mu_A(yx, q) \geq \mu_A(y, q) \wedge \mu_A(x, q) = \mu_A(x, q) \wedge \mu_A(y, q)$, which implies that $\mu_v(f(x)f(y), q) \geq \mu_v(f(x, q)) \wedge \mu_v(f(y, q))$. Now, for $f(x), f(y)$ in R' and q in Q, $\nu_v(f(x)+f(y), q) = \nu_v(f(y+x), q) \leq \nu_A(y+x, q) \leq \nu_A(y, q) \vee \nu_A(x, q) = \nu_A(x, q) \vee \nu_A(y, q)$ which implies that $\nu_v(f(x)+f(y), q) \leq \nu_v(f(x, q)) \vee \nu_v(f(y, q))$. Again, $\nu_v(f(x)f(y), q) = \nu_v(f(yx, q)) \leq \nu_A(yx, q) \leq \nu_A(y, q) \vee \nu_A(x, q) = \nu_A(x, q) \vee \nu_A(y, q)$ which implies that $\nu_v(f(x)f(y), q) \leq \nu_v(f(x, q)) \vee \nu_v(f(y, q))$. Hence V is a Q-intuitionistic L-fuzzy subsemiring of R'. □

Theorem 2.4. *Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two semirings Q be a non-empty set. The anti-homomorphic preimage of a Q-intuitionistic L-fuzzy subsemiring of R' is an Q-intuitionistic L-fuzzy subsemiring of R.*

Proof. Let$(R, +, \cdot)$ and $(R', +, \cdot)$ be any two semirings Q be a non-empty set. Let $f : R \to R'$ be an anti-homomorphism. Then $f(x+y) = f(y)+f(x)$ and$f(xy) = f(y)f(x)$, for all x and y in R. Let V be a Q-intuitionistic L-fuzzy subsemiring of$f(R) = R'$. We have to prove that A is an Q-intuitionistic L-fuzzy subsemiring of R. Let x and y in R. Then $\mu_A(x+y, q) = \mu_v(f(x+y), q)$, since $\mu_v(f(x, q)) = \mu_A(x, q) = \mu_v(f(y)+f(x), q) \geq \mu_v(f(y, q)) \wedge \mu_v(f(x, q)) = \mu_v(f(x, q)) \wedge \mu_v(f(y, q)) =$

$\mu_A(x,q) \wedge \mu_A(y,q)$, since $\mu_v(f(x,q)) = \mu_A(x,q)$ which implies that $\mu_A(x+y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$. Again, $\mu_A(xy,q) = \mu_v(f(xy),q)$, since $\mu_v(f(x,q)) = \mu_A(x,q) = \mu_v(f(y)f(x),q) \geq \mu_v(f(y,q)) \wedge \mu_v(f(x,q)) = \mu_v(f(x,q)) \wedge \mu_v(f(y,q)) = \mu_A(x,q) \wedge \mu_A(y,q)$, since $\mu_v(f(x,q)) = \mu_A(x,q)$ which implies that $\mu_A(xy,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$. Then $\nu_A(x+y,q) = \nu_v(f(x+y),q)$, since $\nu_v(f(x,q)) = \nu_A(x,q) = \nu_v(f(y)+f(x),q) \leq \nu_v(f(y,q)) \vee \nu_v(f(x,q)) = \nu_v(f(x,q)) \vee \nu_v(f(y,q)) = \nu_A(x,q) \vee \nu_A(y,q)$, since $\nu_v(f(x,q)) = \nu_A(x,q)$ which implies that $\nu_A(x+y) \leq \nu_A(x,q) \vee \nu_A(y,q)$. Again, $\nu_A(xy,q) = \nu_v(f(xy),q)$, since $\nu_v(f(x,q)) = \nu_A(x,q) = \nu_v(f(y)f(x),q) \leq \nu_v(f(y,q)) \vee \nu_v(f(x,q)) = \nu_v(f(x,q)) \vee \nu_v(f(y,q)) = \nu_A(x,q) \vee \nu_A(y,q)$, since $\nu_v(f(x,q)) = \nu_A(x,q)$ which implies that $\nu_A(xy,q) \leq \nu_A(x,q) \vee \nu_A(y,q)$. Hence A is a Q-intuitionistic L-fuzzy subsemiring of R. □

In the following Theorem $\circ$ is the Composition Operation of Functions:

Theorem 2.5. *Let A be a Q-intuitionistic L-fuzzy subsemiring of a semiring H and f is an isomorphism from a semiring R onto H. Then $A \circ f$ is a Q-intuitionistic L-fuzzy subsemiring of R.*

Proof. Let x and y in R and A be a Q-intuitionistic L-fuzzy subsemiring of a semiring H and Q be a non-empty set. Then we have, $(\mu_A \circ f)(x+y,q) = \mu_A(f(x+y),q) = \mu_A(f(x)+f(y),q) \geq \mu_A(f(x,q)) \wedge \mu_A(f(y,q)) \geq (\mu_A \circ f)(x,q) \wedge (\mu_A \circ f)(y,q)$, which implies that $(\mu_A \circ f)(x+y,q) \geq (\mu_A \circ f)(x,q) \wedge (\mu_A \circ f)(y,q)$. And $(\mu_A \circ f)(xy,q) = \mu_A(f(xy,q)) = \mu_A(f(x,q)f(y,q)) \geq \mu_A(f(x,q)) \wedge \mu_A(f(y,q)) \geq (\mu_A \circ f)(x,q) \wedge (\mu_A \circ f)(y,q)$, which implies that $(\mu_A \circ f)(xy,q) \geq (\mu_A \circ f)(x,q) \wedge (\mu_A \circ f)(y,q)$. Then we have, $(\nu_A \circ f)(x+y,q) = \nu_A(f(x+y),q) = \nu_A(f(x)+f(y),q) \leq \nu_A(f(x),q \vee \nu_A(f(y),q) \vee (\nu_A \circ f)(x,q) \vee (\nu_A \circ f)(y,q)$, which implies that $(\nu_A \circ f)(x+y,q) \leq (\nu_A \circ f)(x,q) \vee (\nu_A \circ f)(y,q)$. And $(\nu_A \circ f)(xy,q) = \nu_A(f(xy),q) = \nu_A(f(x)f(y),q) \leq \nu_A(f(x),q) \vee \nu_A(f(y),q) \leq (\nu_A \circ f)(x,q) \vee (\nu_A \circ f)(y,q)$, which implies that $(\nu_A \circ f)(xy,q) \leq (\nu_A \circ f)(x,q) \vee (\nu_A \circ f)(y,q)$, for all x and y in R and q in Q. Therefore $(A \circ f)$ is a Q-intuitionistic L-fuzzy subsemiring of a semiring R. □

Theorem 2.6. *Let A be an Q-intuitionistic L-fuzzy subsemiring of a semiring H and f is an anti-isomorphism from a semiring R onto H. Then $A \circ f$ is a Q-intuitionistic L-fuzzy subsemiring of R.*

Proof. Let x and y in R and A be a Q-intuitionistic L-fuzzy subsemiring of a semiring H and Q be a non-empty set. Then we have, $(\mu_A \circ f)(x+y,q) = \mu_A(f(x+y),q) = \mu_A(f(y)+f(x),q) \geq \mu_A(f(x,q)) \wedge \mu_A(f(y,q)) \geq (\mu_A \circ f)(x,q) \wedge (\mu_A \circ f)(y,q)$, which implies that $(\mu_A \circ f)(x+y,q) \geq (\mu_A \circ f)(x,q) \wedge (\mu_A \circ f)(y,q)$. Again $(\mu_A \circ f)(xy,q) = \mu_A(f(xy),q) = \mu_A(f(y)f(x),q) \geq \mu_A(f(x,q)) \wedge \mu_A(f(y,q)) \geq (\mu_A \circ f)(x,q) \wedge (\mu_A \circ f)(y,q)$, which implies that $(\mu_A \circ f)(xy,q) \geq (\mu_A \circ f)(x,q) \wedge (\mu_A \circ f)(y,q)$. Then we have, $(\nu_A \circ f)(x+y,q) = \nu_A(f(x+y),q) = \nu_A(f(y)+f(x),q) \leq \nu_A(f(x,q)) \vee \nu_A(f(y,q)) \leq (\nu_A \circ f)(x,q) \vee (\nu_A \circ f)(y,q)$, which implies that $(\nu_A \circ f)(x+y,q) \leq (\nu_A \circ f)(x,q) \vee (\nu_A \circ f)(y,q)$. Again, $(\nu_A \circ f)(xy,q) = \nu_A(f(xy),q) = \nu_A(f(y)f(x),q) \leq \nu_A(f(x,q)) \vee \nu_A(f(y,q)) \leq (\nu_A \circ f)(x,q) \vee (\nu_A \circ f)(y,q)$, which implies that $(\nu_A \circ f)(xy,q) \leq (\nu_A \circ f)(x,q) \vee (\nu_A \circ f)(y,q)$. Therefore $A \circ f$ is a Q-intuitionistic L-fuzzy subsemiring of a semiring R. □

Theorem 2.7. *Let A be a Q-intuitionistic L-fuzzy subsemiring of a semiring $(R,+,\cdot)$, then the pseudo Q-intuitionistic L-fuzzy coset $(aA)^p$ is a Q-intuitionistic L-fuzzy subsemiring of a semiring R, for every a in R and p in P.*

Proof. Let A be a Q-intuitionistic L-fuzzy subsemiring of a semiring R. For every x and y in R and q in Q, we have, $((a\mu_A)^p)(x+y,q) = p(a)\mu_A(x+y,q) \geq p(a)\{(\mu_A(x,q) \wedge \mu_A(y,q)\} = \{p(a)\mu_A(x,q) \wedge p(a)\mu_A(y,q)\} = \{((a\mu_A)^p)(x,q) \wedge ((a\mu_A)^p)(y,q)\}$. Therefore, $((a\mu_A)^p)(x+y,q) \geq \{((a\mu_A)^p)(x,q) \wedge ((a\mu_A)^p)(y,q)\}$. Now, $((a\mu_A)^p)(xy,q) = p(a)\mu_A(xy,q) \geq p(a)\{\mu_A(x,q) \wedge \mu_A(y,q)\} = \{p(a)\mu_A(x,q) \wedge p(a)\mu_A(y,q)\} = \{((a\mu_A)^p)(x,q) \wedge ((a\mu_A)^p)(y,q)\}$. Therefore, $((a\mu_A)^p)(xy,q) \geq \{((a\mu_A)^p)(x,q) \wedge ((a\mu_A)^p)(y,q)\}$.For every x and y in R and q in Q, we have, $((a\nu_A)^p)(x+y,q) = p(a)\nu_A(x+y,q) \leq p(a)\{(\nu_A(x,q) \vee \nu_A(y,q)\} = \{p(a)\nu_A(x,q) \vee p(a)\nu_A(y,q)\} = \{((a\nu_A)^p)(x,q) \vee ((a\nu_A)^p)(y,q)\}$. Therefore, $((a\nu_A)^p)(x+y,q) \leq \{((a\nu_A)^p)(x,q) \vee ((a\nu_A)^p)(y,q)\}$. Now, $((a\nu_A)^p)(xy,q) = p(a)\nu_A(xy,q) \leq p(a)\{\nu_A(x,q) \vee \nu_A(y,q)\} = \{p(a)\nu_A(x,q) \vee$

$p(a)\nu_A(y,q)\} = \{((a\nu_A)^p)(x,q) \vee ((a\nu_A)^p)(y,q)\}$. Therefore, $((a\nu_A)^p)(xy,q) \leq \{((a\nu_A)^p)(x,q) \vee ((a\nu_A)^p)(y,q)\}$. Hence $(aA)^p$ is a Q-intuitionistic L-fuzzy subsemiring of a semiring R. □

Theorem 2.8. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two semirings and Q be a non-empty set. If $f : R \to R'$ is a homomorphism, then the homomorphic image of a Q-level subsemiring of an Q-intuitionistic L-fuzzy subsemiring of R is a Q-level subsemiring of a Q-intuitionistic L-fuzzy subsemiring of R'*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two semirings and Q be a non-empty set. $f : R \to R'$ be a homomorphism. That is, $f(x+y) = f(x)+f(y)$, for all x and y in R and $f(xy) = f(x)f(y)$, for all x and y in R. Let $V = f(A)$, where A is an Q-intuitionistic L-fuzzy subsemiring of R. Clearly V is a Q-intuitionistic L-fuzzy subsemiring of R'. If x and y in R, then $f(x)$ and $f(y)$ in R'. Let $A_{(\alpha,\beta)}$ be a Q-level subsemiring of A. Suppose x and y in$A_{(\alpha,\beta)}$, then x+y and xy in $A_{(\alpha,\beta)}$. That is, $\mu_A(x,q) \geq \alpha$ and $\nu_A(x,q) \leq \beta, \mu_A(y,q) \geq \alpha$ and $\nu_A(y,q) \leq \beta, \mu_A(x+y,q) \geq \alpha, \mu_A(xy,q) \geq \alpha$ and $\nu_A(x+y,q) \leq \beta$, $\nu_A(xy,q) \leq \beta$. We have to prove that $f(A_{(\alpha,\beta)})$ is a Q-level subsemiring of V. Now, $\mu_V(f(x),q) \geq \mu_A(x,q) \geq \alpha$, implies that $\mu_V(f(x),q) \geq \alpha; \mu_V(f(y),q) \geq \mu_A(y,q) \geq \alpha$, implies that $\mu_V(f(y),q) \geq \alpha, \mu_V(f(x)+f(y),q) = \mu_V(f(x+y),q) \geq \mu_A(x+y,q) \geq \alpha$, which implies that $\mu_V(f(x)+f(y),q) \geq \alpha$, for all $f(x)$ and f(y) in R'. $\mu_V(f(x)f(y),q) = \mu_V(f(xy),q) \geq \mu_A(xy,q) \geq \alpha$, which implies that $\mu_V(f(x)f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R' . And, $\nu_V(f(x),q) \leq \nu_A(x,q) \leq \beta$, implies that $\nu_V(f(x),q) \leq \beta; \nu_V(f(y),q) \leq \nu_A(y,q) \leq \beta$, implies that $\nu_V(f(y),q) \leq \beta, \nu_V(f(x)+f(y),q) = \nu_V(f(x+y),q) \leq \nu_A(x+y,q) \leq \beta$, which implies that $\nu_V(f(x)+f(y),q) \leq \beta$, for all $f(x)$ and $f(y)$ in R'. $\nu_V(f(x)f(y),q) = \nu_V(f(xy),q) \leq \nu_A(xy,q) \leq \beta$, which implies that $\nu_V(f(x)f(y),q) \leq \beta$, for all $f(x)$ and $f(y)$ in R'. Therefore, $\mu_V(f(x)+f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R' and $\mu_V(f(x)f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R' and $\nu_V(f(x)+f(y),q) \leq \beta$, for all $f(x)$ and $f(y)$ in R' and $\nu_V(f(x)f(y),q) \leq \beta$, for all $f(x)$ and $f(y)$ in R'. Hence $f(A_{(\alpha,\beta)})$ is a Q-level subsemiring of a Q-intuitionistic L-fuzzy subsemiring V of a semiring R'. □

Theorem 2.9. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two semirings and Q be a non-empty set. If $f : R \to R'$ is a homomorphism, then the homomorphic pre-image of a Q-level subsemiring of a Q-intuitionistic L-fuzzy subsemiring of R' is a Q-level subsemiring of a Q-intuitionistic L-fuzzy subsemiring of R.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two semirings and Q be a non-empty set. $f : R \to R'$ be a homomorphism. That is,$f(x+y) = f(x)+f(y)$, for all x and y in R and $f(xy) = f(x)f(y)$, for all x and y in R. Let $V = f(A)$, where V is an Q-intuitionistic L-fuzzy subsemiring of R'. Clearly A is an Q-intuitionistic L-fuzzy subsemiring of R. Let x and y in R and q in Q. Let $f(A_{(\alpha,\beta)})$ be a Q-level subsemiring of V. Suppose $f(x)$ and $f(y)$ in $f(A_{(\alpha,\beta)})$, then f(x)+ f(y) and f(x)f(y) in $f(A_{(\alpha,\beta)})$. That is, $\mu_V(f(x),q) \geq \alpha$ and $\nu_V(f(x),q) \leq \beta$; $\mu_V(f(y),q) \geq \alpha$ and $\nu_V(f(y),q) \leq \beta; \mu_V(f(x)+f(y),q) \geq \alpha, \mu_V(f(x)f(y),q) \geq \alpha$ and $\nu_V(f(x)+f(y),q) \leq \beta, \nu_V(f(x)f(y),q) \leq \beta$. We have to prove that $A_{(\alpha,\beta)}$ is a Q-level subsemiring of A. Now, $\mu_A(x,q) = \mu_V(f(x),q) \geq \alpha$, implies that $\mu_A(x,q) \geq \alpha; \mu_A(y,q) = \mu_V(f(y),q) \geq \alpha$, implies that $\mu_A(y,q) \geq \alpha$, we have $\mu_A(x+y,q) = \mu_V(f(x+y),q) = \mu_V(f(x)+f(y),q) \geq \alpha$, which implies that $\mu_A(x+y,q) \geq \alpha$, for all x and y in R and q in Q. And $\mu_A(xy,q) = \mu_V(f(xy),q) = \mu_V(f(x)f(y),q) \geq \alpha$, which implies that $\mu_A(xy,q) \geq \alpha$, for all x and y in R and q in Q. And, $\nu_A(x,q) = \nu_V(f(x),q) \leq \beta$, implies that $\nu_A(x,q) \leq \beta; \nu_A(y,q) = \nu_V(f(y),q) \leq \beta$, implies that $\nu_A(y,q) \leq \beta$, we have $\nu_A(x+y,q) = \nu_V(f(x+y),q) = \nu_V(f(x)+f(y),q) \leq \beta$ which implies that $\nu_A(x+y,q) \leq \beta$, for all x and y in R and q in Q. And $\nu_A(xy,q) = \nu_V(f(xy),q) = \nu_V(f(x)f(y),q) \leq \beta$ which implies that $\nu_A(xy,q) \leq \beta$, for all x and y in R and q in Q. Therefore, $\mu_A(x+y,q) \geq \alpha$, for all x and y in R and $\mu_A(xy,q) \geq \alpha$, for all x and y in R and $\nu_A(x+y,q) \leq \beta$, for all x and y in R and $\nu_A(xy,q) \leq \beta$, for all x and y in R and q in Q. Hence $A_{(\alpha,\beta)}$ is a Q-level subsemiring of a Q-intuitionistic L-fuzzy subsemiring A of R. □

Theorem 2.10. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two semirings and Q be a non-empty set. If $f : R \to R'$ is an anti-homomorphism, then the anti-homomorphic image of a Q-level subsemiring of a Q-intuitionistic L-fuzzy subsemiring of R is a Q-level subsemiring of a Q-intuitionistic L-fuzzy subsemiring of R'.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two semirings and Q be a non-empty set and $f : R \to R'$ be an anti-homomorphism. That is, $f(x+y) = f(y)+f(x)$, for all x and y in R and $f(xy) = f(y)f(x)$, for all x and y in R. Let $V = f(A)$, where A is an Q-intuitionistic L-fuzzy subsemiring of R. Clearly V is a Q-intuitionistic L-fuzzy subsemiring of R'. If x and y in R, then $f(x)$ and $f(y)$ in R'. Let $A_{(\alpha,\beta)}$ be a Q-level subsemiring of A. Suppose x and y in $A_{(\alpha,\beta)}$, then $y+x$ and yx in $A_{(\alpha,\beta)}$. That is, $\mu_A(x,q) \geq \alpha$ and $\nu_A(x,q) \leq \beta, \mu_A(y,q) \geq \alpha$ and $\nu_A(y,q) \leq \beta, \mu_A(y+x,q) \geq \alpha, \mu_A(yx,q) \geq \alpha$ and $\nu_A(y+x,q) \leq \beta, \nu_A(yx,q) \leq \beta$. We have to prove that $f(A_{(\alpha,\beta)})$ is a Q-level subsemiring of V. Now, $\mu_V(f(x),q) \geq \mu_A(x,q) \geq \alpha$, implies that $\mu_V(f(x),q) \geq \alpha; \mu_V(f(y),q) \leq \mu_A(y,q) \geq \alpha$, implies that $\mu_V(f(y),q) \geq \alpha, \mu_V(f(x)+f(y),q) = \mu_V(f(y+x),q) \geq \mu_A(y+x,q) \geq \alpha$, which implies that $\mu_V(f(x)+f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R'. $\mu_V(f(x)f(y),q) = \mu_V(f(yx),q) \geq \mu_A(yx,q) \geq \alpha$, which implies that $\mu_V(f(x)f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R'. And, $\nu_V(f(x),q) \leq \nu_A(x,q) \leq \beta$, implies that $\nu_V(f(x),q) \leq \beta; \nu_V(f(y),q) \leq \nu_A(y,q) \leq \beta$, implies that $\nu_V(f(y),q) \leq \beta, \nu_V(f(x)+f(y),q) = \nu_V(f(y+x),q) \leq \nu_A(y+x,q) \leq \beta$, which implies that $V(f(x)+f(y),q) \leq \beta$, for all $f(x)$ and $f(y)$ in R'. $V(f(x)f(y),q) = \nu_V(f(yx),q) \leq \nu_A(yx,q) \leq \beta$, which implies that $\nu_V(f(x)f(y),q) \leq \beta$, for all $f(x)$ and $f(y)$ inR'. Therefore, $\mu_V(f(x)+f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R' and $\mu_V(f(x)f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R' and $\nu_V(f(x)+f(y),q) \leq \beta$, for all $f(x)$ and $f(y)$ in R' and $\nu_V(f(x)f(y),q) \leq \beta$, for all $f(x)$ and $f(y)$ in R'. Hence f ($A_{(\alpha,\beta)}$) is a Q-level subsemiring of a Q-intuitionistic L-fuzzy subsemiring V of a semiring R'. □

Theorem 2.11. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two semirings and Q be a non-empty set. If $f : R \to R'$ is an anti-homomorphism, then the anti-homomorphic pre-image of a Q-level subsemiring of a Q-intuitionistic L-fuzzy subsemiring of R' is a Q-level subsemiring of a Q-intuitionistic L-fuzzy subsemiring of R.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two semirings and Q be a non-empty set and $f : R \to R'$ be an anti-homomorphism. That is, $f(x+y) = f(y)+f(x)$, for all x and y in R and $f(xy) = f(y)f(x)$, for all x and y in R. Let $V = f(A)$, where V is a Q-intuitionistic L-fuzzy subsemiring of R'. Clearly A is a Q-intuitionistic L-fuzzy subsemiring of R. Let x and y in R and q in Q. Let $f(A_{(\alpha,\beta)})$ be a Q-level subsemiring of V. Suppose $f(x)$ and $f(y)$ in $f(A_{(\alpha,\beta)})$, then $f(y)+f(x)$ and $f(y)f(x)$ in $f(A_{(\alpha,\beta)})$. That is, $\mu_V(f(x),q) \geq \alpha \textit{ and } \nu_V(f(x),q) \leq \beta; \mu_V(f(y),q) \geq \alpha$ and $\nu_V(f(y),q) \leq \beta; \mu_V(f(y)+f(x),q) \geq \alpha, \mu_V(f(y)f(x),q) \geq \alpha$ and $\nu_V(f(y)+f(x),q) \leq \beta, \nu_V(f(y)f(x),q) \leq \beta$. We have to prove that $A_{(\alpha,\beta)}$ is a Q-level subsemiring of A. Now, $\mu_A(x,q) = \mu_V(f(x),q) \geq \alpha$, implies that $\mu_A(x,q) \geq \alpha; \mu_A(y,q) = \mu_V(f(y),q) \geq \alpha$, implies that $\mu_A(y,q) \geq \alpha$, we have $\mu_A(x+y,q) = \mu_V(f(x+y),q) = \mu_V(f(y)+f(x),q) \geq \alpha$, which implies that $\mu_A(x+y,q) \geq \alpha$, for all x and y in R. And $\mu_A(xy,q) = \mu_V(f(xy),q) = \mu_V(f(y)f(x),q) \geq \alpha$, which implies that $\mu_A(xy,q) \geq \alpha$, for all x and y in R. And, $\nu_A(x,q) = \nu_V(f(x),q) \leq \beta$, implies that $\nu_A(x,q) \leq \beta; \nu_A(y,q) = \nu_V(f(y),q) \leq \beta$, implies that $\nu_A(y,q) \leq \beta$, we have $\nu_A(x+y,q) = \nu_V(f(x+y),q) = \nu_V(f(y)+f(x),q) \leq \beta$ which implies that $\nu_A(x+y,q) \leq \beta$, for all x and y in R. And $\nu_A(xy,q) = \nu_V(f(xy),q) = \nu_V(f(y)f(x),q) \leq \beta$ which implies that $\nu_A(xy,q) \leq \beta$, for all x and y in R. Therefore, $\mu_A(x+y,q) \geq \alpha$, for all x and y in R and q in Q and $\mu_A(xy,q) \geq \alpha$, for all x and y in R and $\nu_A(x+y,q) \leq \beta$, for all x and y in R and $\nu_A(xy,q) \leq \beta$, for all x and y in R and q in Q. Hence $A_{(\alpha,\beta)}$ is a Q-level subsemiring of a Q-intuitionistic L-fuzzy subsemiring A of R. □

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