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Snakes Related Families of Closed Neighbourhood Prime Graphs

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Abstract: In this paper we prove that double snake of C_3, double snake of C_4, snake of K_4 and snake of wheel W_4 are Closed Neighbourhood Prime Graphs.

MSC: 05C78.

Keywords: Closed neighborhood, prime, double snake, flag, labeling, snakes, double snake.

1. Introduction and Preliminaries

The graphs considered in this paper are finite, simple, undirected and connected. For definitions and terminology we depend on Gallian [3] and Harary [4]. Patel and Shrimali [6] has introduced neighbourhood prime labeling of graph.We have introduced closed neighbourhood prime labeling [3]. Let G be a (p, q) graph. Define a bijective function $f : V(G) \to \{1, 2, 3, \ldots, p\}$ such that gcd of all labels of vertices incident with v including label of v is 1. This is true for every vertex v in $V(G)$ except the isolated vertices. The graph for which such a function f is defined is called as Closed Neighbourhood Prime graph. And the function f is called as Closed Neighbourhood Prime function.We have show that stars, bistars, flags, snakes, pathunions are some families of Closed Neighbourhood Prime graphs.We have also shown that different nonisomorphic structures available on path union of flag of C_3 are cnp graphs. [3] Following observations play important role in deciding the gcd of collection of positive numbers. (i). G.C.D. of any two consecutive integers is one. (ii). If the set of numbers contains the number one the G.C.D. is equal one. (iii). If the set contain a prime and no multiple of it then G.C.D. is one. (iv). If the set contains an even number say m and all other numbers are odd numbers which does not have common divisor with m then G.C.D. is 1. In this paper we show that (i). $S(2-C_3, n)$ (ii). $S(2-C_4, n)$ (iii). $S(K_4, n)$ (iv) $S(w_4, n)$ are c-nhd graphs.

Definition 1.1. *Snake on C_4 i.e. $S(C_4, n)$: Take a path on n vertices given by $(v_1, v_2, \ldots, v_n)$ and between every pair of consecutive vertices v_i and v_{i+1} of P_n take a new pair of vertices w_i and w_{i+1} and take new edges (v_iw_i), (w_iw_{i+1}) and $(w_{i+1}v_{i+1})$. A $S(C_4, n)$ has $4(n-1)$ edges and $3(n-1)+1$ vertices.*

Definition 1.2. *Double snake of on C_3 i.e. $S(2-C_3, n)$: Take a path on n vertices given by $(v_1, e_1, v_2, \ldots, e_{n-1}, v_n)$ and between every pair of consecutive vertices v_i and v_{i+1} of P_n take two new vertices $w_{i,1}$ and $w_{i,2}$ and take new edges*

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$c_{i,1} = (v_i w_{i,j})$, $j = 1, 2$ *and* $c'_{i,1} = (w_{i,1} v_{i+1})$ *and* $c'_{i,2} = (w_{i,2} v_{i+1})$ $(i = 1, 2, ..., n-1)$. *A* $S(2 - C_3, n)$ *has* $5(n-1)$ *edges and* $3(n-1) + 1$ *vertices.*

2. Main Results

Theorem 2.1. *$S(2 - C_3, n)$ is c-nhd prime graph.*

Proof. Define a function $f : V \to \{1, 2, \ldots, p\}$ as follows:

$$f(vi) = i, \qquad i = 1, 2, ..., n$$

$$f(w_{i,j}) = n + (i-1)2 + j, \quad j = 1, 2$$

The resultant graph is C-nhd prime. The following figure elaborates the theme.

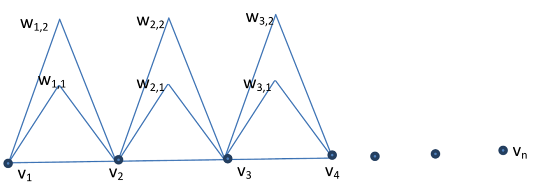

Figure 1: Ordinary labeling of $S(2 - C_3, n)$

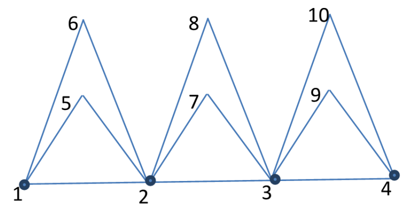

Figure 2: c-nhd prime labeling of $S(2 - C_3, 3)$

□

Theorem 2.2. *$G = S(2 - C_4, n)$ is c-nhd prime graph.*

Proof. We define G in terms of vertex set V and Edge set E as

$$V(G) = \{v_1, v_2, \ldots, v_n\} \cup \{u_{i1}, u_{i2}, w_{i,1}, w_{i,2}/i = 1, 2, .., n-1\}$$

$$E(G) = \{e_i = (v_i v_{i+1})/I = 1, 2, ..., n-1.\} \cup \{(v_i u_{i,1})(u_{i,1} u_{i,2})(u_{i,2} v_{i+1}/i = 1, 2, .., n-1)\}$$

$$\cup \{(v_i w_{i,1}), (w_{i,1} w_{i,2}), (w_{i,2} v_{i+1})/i = 1, 2, ..., n-1\}$$

Define a function $f : V \to \{1, 2, \ldots, p\}$ as follows:

$$f(v_i) = i, \qquad i = 1, 2, \ldots, n$$

$$f(u_{i,j}) = n + (i-1)2 + j, \qquad j = 1, 2; i = 1, 2, \ldots, n-1$$

$$f(w_{i,j}) = 3n - 2 + (i-1)2 + j, \quad j = 1, 2; i = 1, 2, .., n-1.$$

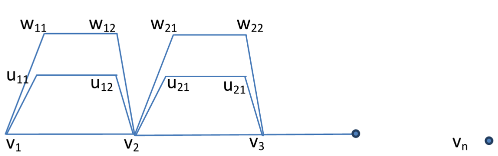

Figure 3: Ordinary labeling of $S(2 - C_4, n)$

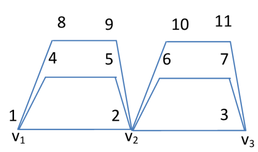

Figure 4: c-nhd p labeling of labeling of $S(2 - C_4, 3)$

□

Theorem 2.3. *The graph $G = S(K_4, n)$ is c-nhd cordial prime graph.*

Proof. We define G in terms of vertex set V and Edge set E as given by $V = \{v_1, v_2, ..., v_n\} \cup \{u_{i,1}, u_{i,2}\}$ and edge set as $E = \{e_i = (v_i v_{i+1}), i = 1, 2, \ldots, n-1\} \cup \{(v_i u_{i,1}), (u_{i,1} u_{i,2}), (u_{i,2} v_{i+1}), (u_{i,1} v_{i+1}), (v_i u_{i,2}) / i = 1, 2, ..., n-1 \text{ and } j = 1, 2\}$. Define a function $f : V \to \{1, 2, \ldots, p\}$ as follows: $f(vi) = i$ and $f(u_{i,j}) = n + 2(i-1) + j$ for $i = 1, 2, \ldots, n$ and $j = 1, 2$.

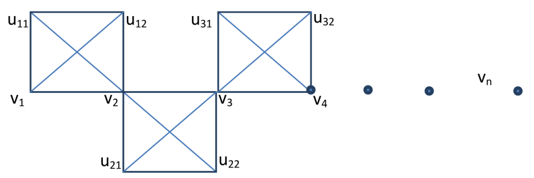

Figure 5: Ordinary c-nhd prime labeling of $S(K_4, n)$

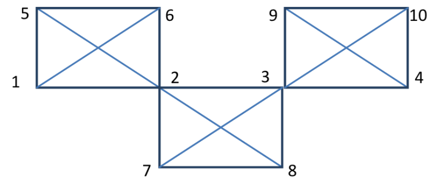

Figure 6: c-nhd prime labeling of $S(K_4, n)$

□

Theorem 2.4. *The graph $G = S(S_4, n)$ is c-nhd cordial prime graph.*

Proof. We define G in terms of vertex set V and Edge set E as given by $V = \{v_1, v_2, ..., v_n\} \cup \{u_{i,1}, u_{i,2}\}$ and edge set as $E = \{e_i = (v_i v_{i+1}),\ i = 1, 2, \ldots, n-1\} \cup \{(v_i u_{i,1}), (u_{i,1} u_{i,2}), (u_{i,2} v_{i+1}), (u_{i,1} v_{i+1}) / i = 1, 2, .., n-1 \text{ and } j = 1, 2\}$. Define a function $f : V \to \{1, 2, \ldots, p\}$ as follows $f(v_i) = i$ and $f(u_{i,j}) = n + 2(i-1) + j$ for $i = 1, 2, \ldots, n$ and $j = 1, 2$. Instead of defining snake on edge (xy) as we have done above if we take snake on (uy) we will get double C_3 snake (refer figure 7). That explains structural invariance of S_4 under c-nhd primary labeling.

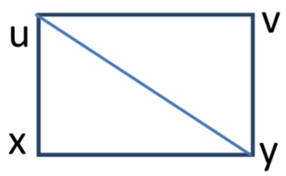

Figure 7: copy of S_4

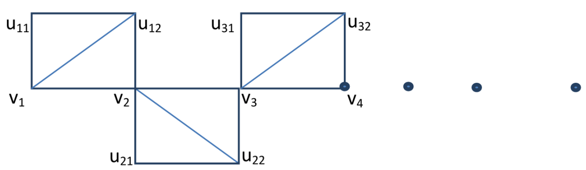

Figure 8: Ordinary c-nhd prime labeling of $S(S_4, n)$

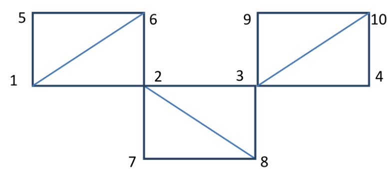

Figure 9: c-nhd prime labeling of $S(S_4, 3)$

□

Theorem 2.5. *The graph $G = S(w_4, n)$ is c-nhd cordial prime graph.*

Proof. We define The graph G in terms of vertex set V and Edge set E as follows: $V = \{u_{i,1}, u_{i,2}\} \cup \{w_i / i = 1, 2, .., n-1\} \cup \{v_1, v_2, ..., v_n\}$ and $E = \{e_i = (v_i v_{i+1}), i = 1, 2, ..., n-1.\} \cup \{(w_i v_j) / i = 1, ..., n-1 \text{ and } j = i, i+1\} \cup \{(w_i ui, j) / i = 1, 2, ..., n-1 \text{ and } j = 1, 2\} \cup \{(w_i v_{i+1} / i = 1, 2, ..., n-1\} \cup \{(w_i u_{i,2}) / i = 1, 2, ..., n-1\}$. Define a function $f : V \to \{1, 2, \ldots, p\}$ as follows $f(v_i) = i$ and $f(u_{i,j}) = n + 2(i-1) + j$ for $i = 1, 2, \ldots, n$ and $j = 1, 2$, $f(w_i) = 3n - 2 + i$, $i = 1, 2, ..., n-1$.

□

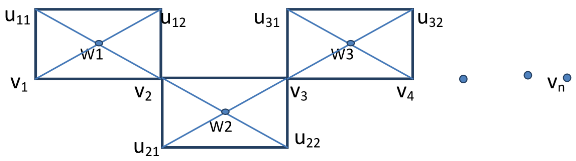

Figure 10: $S(w_4, n)$ ordinary labeling

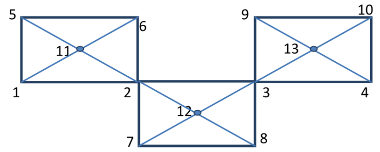

Figure 11: $S(w_4, n), n$-nhd prime labeling

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