

Diffusion-Thermo Effects on Heat and Mass Transfer of Non-Newtonian Power Law Fluid past a Porous Flat Plate with Chemical Reaction

B. Shashidar Reddy^{1,*}

¹ Department of Sciences and Humanities, Sreenidhi Institute of Science and Technology, Yamnampet, Hyderabad, Telangana, India.

Abstract: The diffusion-thermo effects on heat and mass transfer of a laminar boundary layer flow of a non-Newtonian power law fluid past a porous flat plate in the presence of magnetic field with chemical reaction effects have been considered. A suitable set of dimensionless variables is used and similar equations governing the flow are obtained. The resulting the non-linear ordinary differential equations are first linearised using Quasi-linearization technique and then an adequate implicit finite difference scheme is employed to obtain a numerical solution for the ordinary differential equations. The Representative results for the velocity, temperature and concentration profiles illustrating the influence of various governing parameters are presented and discussed. Temperature rises with increase in Dufour number and the effect of Chemical reaction parameter is to decrease the concentration of the fluid.

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1. Introduction

Many models consider only Newtonian fluids, for which strain rate is linearly related to applied shear stress. As non-Newtonian fluids classified as either pseudo-plastic or dilatants have become more common in industry, analysis of the boundary layer flow and heat transfer characteristics of these so called power-law fluids is now a priority. Dilatant fluids are liquids in which viscosity increases as the applied stress increases, whereas pseudo-plastic fluids are characterized by opposite relationship between viscosity and applied stress. In the existing literature a masterpiece of work has been done on heat and mass transfer of a power-law fluid. Schowalter [1] was the first one, who studied the application of a boundary layer to power-law pseudo-plastic fluids. Laminar boundary layer flow of non-Newtonian power-law fluid past a porous flat plate is examined by Jadhav [2]. Magneto-hydrodynamic flow of a power-law fluid over a stretching sheet investigated by Andersson and Bech [3]. Prasad [4] investigated the hydromagnetic flow and heat transfer of a non-Newtonian power-law fluid over a vertical stretching sheet. Recent additions considering magnetohydrodynamic flow were given Ghosh and Pop [5], Mahapatra [6] and Shashidar [7]. Ali [8, 9] investigated hydromagnetic flow and heat transfer over a non-isothermal permeable surface stretching with a power-law velocity with heat generation and suction/injection effects and in the presence of a non-uniform transverse magnetic field and boundary-layer flow of a viscous, incompressible, electrically conducting and heat-absorbing fluid along a semi-infinite vertical permeable moving plate in the presence of a uniform transverse magnetic

* E-mail: bsreddy_shashi@yahoo.com

field and thermal and concentration buoyancy effects respectively.

The study of hydro magnetic convective non-Newtonian fluid flows with heat and mass transfer in porous medium attracted many research due to its applications in many fields like, soil physics, geophysics, aerodynamics and aeronautics. The interest in these new problems generates from their importance in liquid metals, electrolytes and ionized gases. Many authors like Choudary and Das [10], and Reddy [11], investigated an unsteady free convective MHD non-Newtonian flow through a porous medium bounded by an infinite inclined porous plate. MHD effects on non-Newtonian power-law fluid past a continuously moving porous flat plate with heat flux and viscous dissipation was analyzed by Shashidar Reddy and Kishan [12].

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has been observed that an energy flux can be generated not only by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. Mohammad Mehdi Rashidi et. al [13] examined Heat and mass transfer for MHD viscoelastic fluid flow over a vertical stretching sheet with Soret and Dufour effects. Srinivasa chary and swamy [14] studied the effect of chemical reaction and radiation on mixed convection heat and mass transfer over a vertical plate in power-law fluid saturated porous medium. Kabeir [15] investigated the effects of radiation, chemical reaction, soret and dufour on heat and mass transfer by MHD stagnation point flow of a power-law fluid towards a stretching surface. Olajuwon [16] studied the effect of Thermo Diffusion and Chemical reaction on heat and mass transfer in a power-law fluid over a flat plate with heat generation. Recently Saritha [17] investigated heat and mass transfer of laminar boundary layer flow of non-Newtonian power-law fluid past a porous flat plate with Soret and Dufour effects.

This study extends earlier works by examining the combined effect of Chemical reaction and diffusion thermo effect on the heat and mass transfer of a laminar boundary layer flow of a non-Newtonian power law fluid past a porous flat plate in the presence of magnetic field and chemical reaction. Thermal diffusion effect is neglected in this study because it is of a smaller order of magnitude than the magnitude of rate of chemical reaction which exerts a stronger effect on the mass flux.

2. Mathematical Analysis

To construct the model, consider a steady, laminar, incompressible flow of a non-Newtonian power-law fluid past a semi infinite porous flat plate. The coordinate system is chosen such that X-axis is taken along the direction of the flow and y-axis normal to it. It is also assumed that a magnetic field with magnetic field intensity B_0 is applied along the Y-axis and that the magnetic Reynolds number is very small so that the induced magnetic field can be neglected. In addition, Hall Effect and electric field are assumed to be negligible. The fluid properties are assumed to be constant. We assumed that $T_w(x) = T_\infty + b_x$ and $C_w(x) = C_\infty + cx$, where b and c are constants such that the uniform wall temperature T_w and concentration C_w are higher than those of their full stream values T_∞, C_∞ . By considering the effects of Chemical reaction, Heat source/sink and Dufour, the governing equations for the flow in this investigation can be written as

$$\frac{\partial x}{\partial t} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial x}{\partial t} + v \frac{\partial u}{\partial y} = -\frac{K}{\rho} \frac{\partial}{\partial y} \left[-\frac{\partial u}{\partial y} \right]^n - \frac{\sigma B_0^2}{\rho} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = a \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_s c_p} \frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - K_1(C - C_\infty) \tag{4}$$

In the foregoing equations, u and v are the velocity components along the X and Y axes, n is the power-law index, K is the power-law fluid parameter, ρ is density, μ is the magnetic permeability, σ is the electrical conductivity of the fluid, $\alpha = k/\rho c_p$ is the thermal diffusivity, k is the thermal conductivity, c_p is the specific heat at a constant pressure, k_T is the thermal diffusion ratio, c_s is the concentration susceptibility, D_m is the coefficient of mass diffusivity, T is the temperature, K_1 is the reaction rate constant and C is the fluid concentration. The boundary conditions associated with the present problem are as follows

$$u = 0, v = V_w, T = T_w(x) \text{ and } C = C_w(x) \text{ at } y = 0 \tag{5}$$

$$u = U, v = 0, T = T_\infty, C = C_\infty \text{ as } y \rightarrow \infty$$

Where V_w is suction velocity. To facilitate the analysis, we follow the previous studies Jadhav [2] and Mohammad Mehdi Rashidi [13] and use the similarity variables

$$\left. \begin{aligned} \psi(\eta) &= (yxU^{2-n})^{\frac{1}{n+1}} f(\eta) \\ \eta &= y \left[\frac{U^{2-n}}{yx} \right]^{\frac{1}{n+1}} \\ \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty} \\ \varphi(\eta) &= \frac{C - C_\infty}{C_w - C_\infty} \end{aligned} \right\} \tag{6}$$

Where ψ is the stream function defined in the usual way and f is the reduced stream function for the flow. Then the velocity components are defined using the similarity variables as

$$\left. \begin{aligned} u &= \frac{\partial \psi}{\partial x} = U f'(\eta) \\ v &= -\frac{\partial \psi}{\partial y} = \frac{1}{n+1} \left(\gamma \frac{U^{2n-1}}{x^n} \right)^{\frac{1}{n+1}} (\eta f' - f) \end{aligned} \right\} \tag{7}$$

Introducing equation (6) and (7), the continuity equation is satisfied and the momentum, energy and concentration equations are transformed into ordinary differential equations as follows:

$$n(-f'')^{n-1} f'' + \frac{1}{n+1} f f'' - M f' = 0 \tag{8}$$

$$\theta'' + Pr \left(\frac{1}{n+1} \right) f \theta' - f' \theta + Du \phi'' = 0 \tag{9}$$

$$\frac{1}{Le} \phi'' + Pr \left(\frac{1}{n+1} \right) \phi' f - f' \phi - \gamma \phi = 0 \tag{10}$$

Where primes denote indicate differentiation with respect to η . The transformed boundary conditions in dimensionless form are

$$\left. \begin{aligned} f(0) &= f_w(\text{constant}) \\ f'(0) &= 0, \quad f'(\infty) = 1 \\ \theta(0) &= 1, \quad \theta(\infty) = 0 \\ \phi(0) &= 1, \quad \phi(\infty) = 0 \end{aligned} \right\} \tag{11}$$

Where $f_w = -\frac{(n+1)}{U} V_w Re_x^{\frac{1}{n+1}}$ is the suction/injection parameter ($f_w > 0$ for suction and $f_w < 0$ for injection), $M = \frac{\sigma B_0^2 x}{\rho U}$ is the Magnetic parameter, $Re_x = \frac{U^{2-n} x^n}{\mu}$ is the Reynolds number, $Pr = \frac{U x}{\alpha} Re_x^{-\frac{1}{n+1}}$ is the Prandtl number, $Le = \frac{\alpha}{D_m}$

is the Lewis number, $\gamma = \frac{K_1 x^2}{\alpha} Re_x^{-\frac{2}{n+1}}$ is the Chemical reaction parameter, $Du = \frac{D_m k T}{c_s c_p} \cdot \frac{(C_w - C_\infty)}{(T_w - T_\infty) \alpha}$ is the Dufour number. The physical quantities of engineering interest in this problem are the local Nusselt number and local Sherwood number, which are defined respectively by

$$Nu_x = \frac{q_w x}{k(T_w - T_\infty)} = -\theta'(0) Re_x^{\frac{1}{n+1}} \text{ and } Sh_x = \frac{J_w x}{D_m(C_w - C_\infty)} = -\phi'(0) Re_x^{\frac{1}{n+1}}$$

Where the rate of heat transfer q_w and rate of mass transfer J_w are defined as

$$q_w = -k \left[\frac{\partial T}{\partial y} \right]_{y=0} \text{ and } J_w = -D_m \left[\frac{\partial C}{\partial y} \right]_{y=0}$$

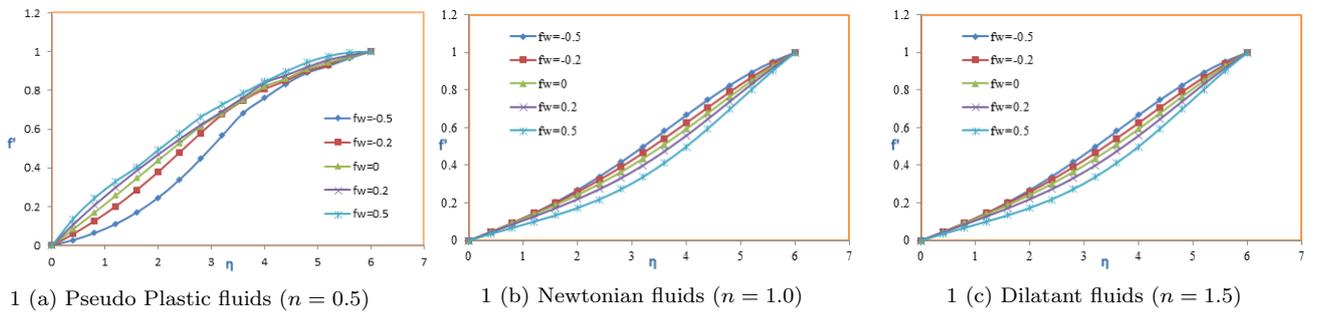


Figure 1. Velocity profiles for various values of Suction/injection parameter with $Pr = 1.0, M = 0.0, Du = 0.08, Le = 1.0, \gamma = 0.0$.

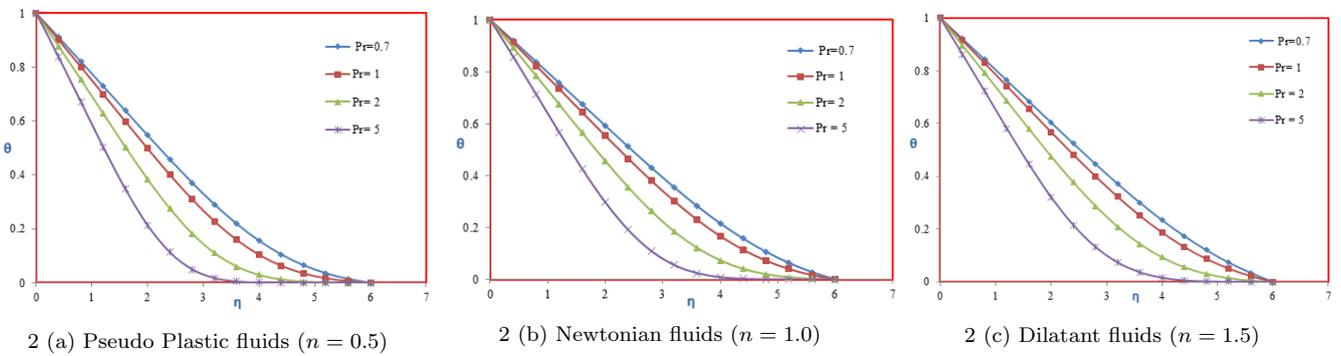


Figure 2. Temperature profiles for various values of Prandtl number with $M = 0.1, f_w = 0.0, Du = 0.08, Le = 1.0, \gamma = 0.0$.

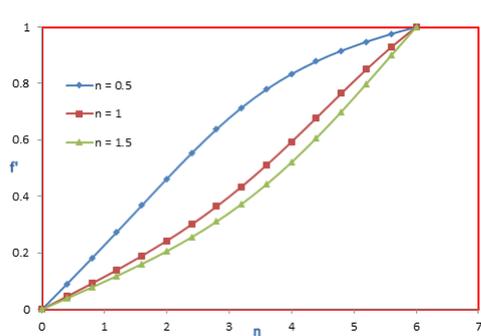


Figure 3. Velocity profiles for various values of power law index with $M = 0.1, Pr = 1.0, f_w = 0.0, Du = 0.08, Le = 1.0, \gamma = 0.0$.

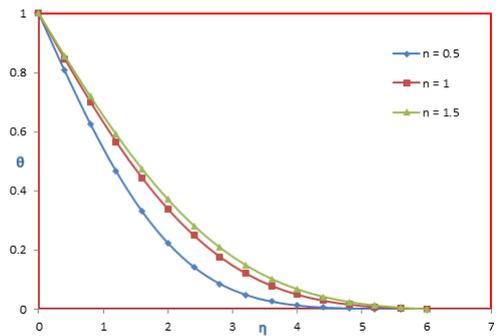


Figure 4. Temperature profiles for various values of power law index with $M = 0.1, Pr = 1.0, f_w = 0.0, Du = 0.08, \gamma = 0.0$.

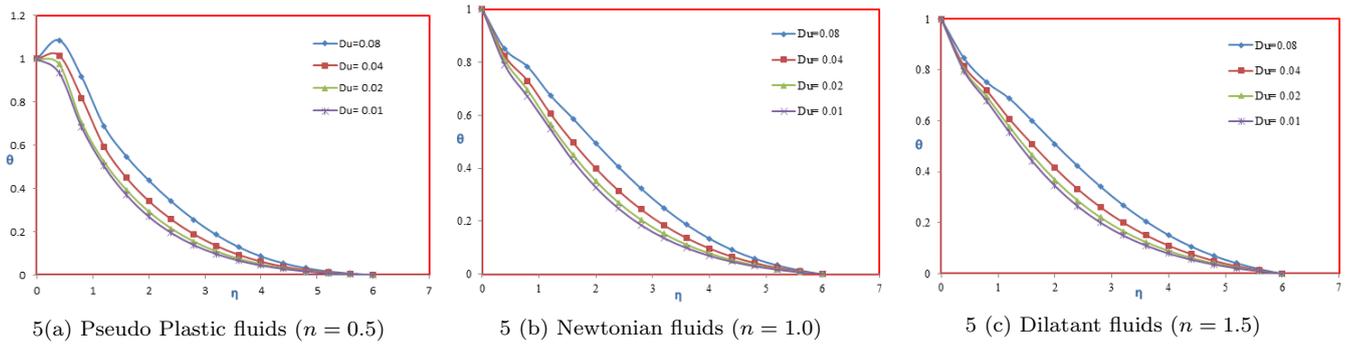


Figure 5. Temperature profiles for various values of Dufour numbers with $M = 0.1, Pr = 1.0, Le = 1.0, f_w = 0.0, \gamma = 0.0$.

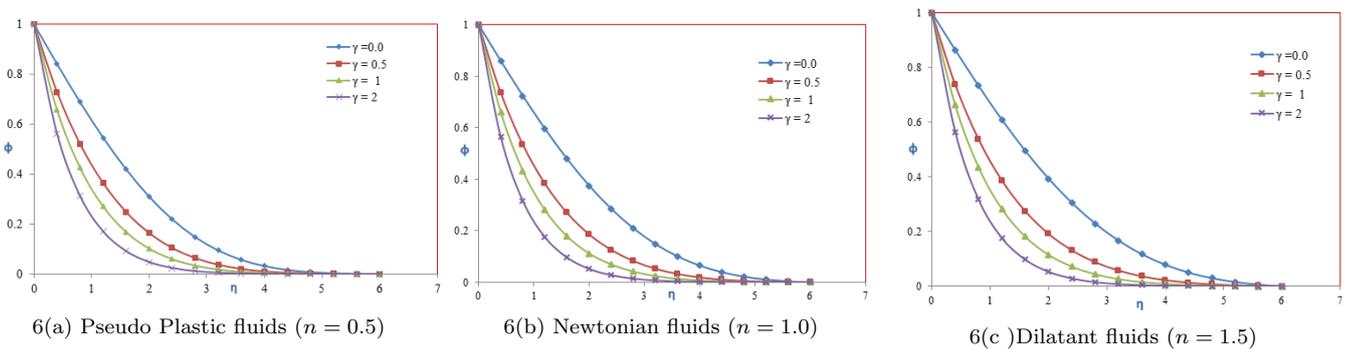


Figure 6. Concentration profiles for various values of Chemical reaction parameter with $M = 0.1, Pr = 1.0, Le = 1.0, f_w = 0.0, Du = 0.08$.

3. Numerical Procedure

By using similarity transformation, the governing equations of the problem are reduced to a system of coupled, non linear ordinary differential equations, which are solved numerically. The numerical solutions can be obtained in the following steps:

- Linearize Equation (8) using Quasi Linearization method [18].
- Write the difference equations using implicit finite difference scheme.
- Linearize the algebraic equations by Newtons method, and express them in matrix-vector form and
- Solve the linear system by Gauss Seidal Iteration method.

Since the equations governing the flow are nonlinear, iteration procedure is followed. For the sake of brevity, further details of the solution process are not presented here. The numerical computations were carried out C programming. It is also important to note that the computational time for each set of parametric values should be short. The physical domain of the problem is unbounded, whereas the computational domain in finite. The numerical solutions of f are considered as $(n + 1)^{th}$ order iterative solutions and F are the n^{th} order iterative solutions. To prove convergence of finite difference scheme, the computation is carried out for slightly changed value of h by running same program. No significant change was observed in the value. The convergence criterion used in this study is that the maximum change between the current and the previous iteration values in all the dependent variables satisfy 10^{-5} .

4. Results and Discussion

In this section, by applying the numerical values to different flow parameters, the effects on velocity, temperature and concentration fields are discussed. Figure (1) is to show the influence of suction/injection on velocity distribution. It is obvious from the graph that the effect of suction ($f_w > 0$) is to enhance the velocity for Pseudo plastic fluid whereas to

decrease the velocity for Newtonian and Dilatant fluids and the effect of injection ($f_w < 0$) is to slow down the velocity for Pseudo plastic fluid but to speed up the velocity for Newtonian and Dilatant fluids. Figure (2) is drawn for temperature profiles for different values of Prandtl number the cases of Newtonian and non-Newtonian fluids. The effect of Prandtl number is to reduce the temperature for both Newtonian and non-Newtonian fluids. Physically, fluids with smaller Prandtl number have larger thermal diffusivity. Figure (3) & (4) depicts the variation of Velocity profiles and Temperature profiles for Newtonian and non-Newtonian fluids with different values of power-law index. The effect of power-law index is to reduce the velocity and enhance the temperature distributions. The heat transfer caused by concentration gradient is called the diffusion thermo or Dufour effect. The effect of diffusion thermo on temperature for Newtonian and non-Newtonian fluids are plotted in Figure (5). Thermal diffusion effect is neglected in this study because it is of a smaller order of magnitude than the magnitude of rate of chemical reaction which exerts a stronger effect on the mass flux. It is evident from the figures that the increase in Dufour number increases the temperature distribution. The effect of Chemical reaction parameter on the concentration field is shown in figure (6). It can be noticed from the graphical presentation that the rise in the chemical reaction parameter will suppress the concentration of the fluid. This is due to the fact that the higher values of γ results in fall of chemical molecular diffusivity and therefore species concentration is suppressed. Hence the concentration distribution decreases at all points of the flow field with the increase in the reaction parameter.

References

- [1] W.R.Schowalter, *The application of boundary layer theory to power law pseudo-plastic: similar solutions*, AICHE Journal, 6(1960), 24-28.
- [2] B.P.Jadhav, *Laminar Boundary layer flow of non-Newtonian power-law fluid past a porous flat plate*, Journal of Global Research in Mathematical Archives, 1(10)(2013), 46-55.
- [3] H.I.Andersson, *Magnetohydrodynamic flow of a power-law fluid over a stretching sheet*, International Journal of Non-linear Mechanics, 27(1992), 929936.
- [4] K.V.Prasad, P.S.Datti and K.Vajravelu, *Hydromagnetic flow and heat transfer of a non-Newtonian power law fluid over a vertical stretching sheet*, International Journal of Heat and Mass Transfer, 53(2010), 879-888.
- [5] S.Ghosh and I.Pop, *A new approach on MHD natural convection boundary layer flow past a flat plate on finite dimensions*, Heat and Mass Transfer, 42(7)(2006), 587-595.
- [6] T.Mahapatra, S.Nandy and A.Gupta, *Analytical solution of magnetohydrodynamic stagnation-point flow of a power-law fluid towards a stretching surface*, Applied Mathematics and Computation, 215(5)(2009), 1696-1710.
- [7] B.Shashidar Reddy, N.Kishan and M.N.Rajasekhar, *MHD boundary layer flow of a non-Newtonian power-law on a moving flat plate*, Advances in Applied Science Research, 3(3)(2012), 1472-1481.
- [8] J.Chamkha Ali, *Hydromagnetic flow and Heat Transfer over a non-Isothermal power law stretched surface with heat generation*, International Journal of Fluid Mechanics Research, 28(4)(2001), 165185.
- [9] J.Chamkha Ali, *Unsteady MHD convective heat and mass transfer past a semi infinite vertical permeable moving plate with heat absorption*, International Journal of Fluid Mechanics Research, 31(2)(2004), 217-230.
- [10] R.Choudhury and S.K.Das, *Mixed Convective No-Newtonian MHD Flow With Ohmic Heating*, International Journal of Computer Applications, 68(10)(2013), 7-13.
- [11] R.Sivaraj and B.R.Kumar, *Unsteady MHD dusty viscoelastic fluid couette flow in an irregular channel with varying mass diffusion*, International Journal of Heat and Mass Transfer, 55(2012), 30763089.
- [12] B.Shashidar Reddy and N.Kishan, *MHD effects on non-Newtonian power-law fluid past a continuously moving porous*

- flat plate with heat flux and viscous dissipation*, International journal of Applied Mechanics and Engineering, 18(2)(2013), 425-445.
- [13] Mohammad Mehdi Rashidi, Mohamed Ali, Behnam Rostami, Peyman Rostami and Gong-Nan Xie, *Heat and mass transfer for MHD viscoelastic fluid flow over a vertical stretching sheet with Soret and Dufour effects*, Mathematical problems in Engineering. doi.org/10.1155?2015/861065.
- [14] Srinivasa Charya and G.Swamy Reddy, *Chemical reaction and Radiation effect on mixed convection heat and mass transfer over a vertical plate in power-law fluid saturated porous medium*, Journal of Egyptian mathematical society, 24(1)(2016), 108-115.
- [15] S.M.M.EL-Kabeir, J.Ali Chamkha and A.M.Rashad, *Heat and mass transfer by MHD stagnation-point flow of a power-law fluid towards a stretching surface with radiation, Chemical reaction, Soret and Dufour effect*, International Journal of Chemical Reactor Engineering, 8(2010), 1-20.
- [16] B.I.Olajuwon, *Effect of thermo diffusion and chemical reaction on heat and mass transfer in a power-law fluid over a flat plate with heat generation*, International Journal of Nonlinear Science, 15(2015), 117-127.
- [17] K.Saritha, M.N.Rajasekhar and B.Shashidar Reddy, *Heat and mass transfer of laminar boundary layer flow of non-Newtonian power-law fluid past a porous flat plate with soret and dufour effects*, Physical Science International Journal, 11(1)(2016), 1-13.
- [18] R.E.Bellman and R.E.Kalaba, *Quasi-Linearization and Non-linear boundary value problems*, NewYork, Elsevier, (1965).