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b-coloring of Central Graph of Triangular Snake

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Abstract: A given k-coloring c of a graph $G = (V, E)$ is a b-coloring if for every color class c_i, $1 \leq i \leq k$, there is a vertex colored i whose neighborhood intersect every other color class c_j, $1 \leq i \leq k$, of c. The b-chromatic number of G is the greatest integer k such that G admits a b-coloring with k colors. In this paper, the authors find the b-chromatic number of central graph of the triangular snake.

MSC: 05C15, 05C76.

Keywords: Graph coloring, b-coloring, central graph, triangular snake.

1. Introduction

Let $G = (V, E)$ be a simple undirected connected graph without loops or multiple edges. Definitions not given here may be found in [1]. A proper k coloring is a function $c : V(G) \to \{1, 2, \ldots, k\}$ such that $c(u) \neq c(v)$ for all $uv \in E(G)$. The chromatic number $\chi(G)$ is the minimum integer k for which G admits proper k-coloring. Given a k-coloring c, a vertex v is called a b-vertex of color i, if $c(v) = i$ and v has at least one neighbor in each of the other color classes c_j, $i \neq j$. Then proper coloring c of graph G is a b-coloring if every color class has a b-vertex. The b-chromatic number of a graph G, denoted $\varphi(G)$, is the largest integer k such that G may have a b-coloring by k colors. The concept of b-chromatic number $\varphi(G)$ was first introduced by Irving and Manlove [6]. They also proved the upper bound of $\varphi(G)$, is $\varphi(G) \leq m(G)$, where $m(G)$ is the largest integer m such that G has m vertices of degree at least $m - 1$. The central graph $C(G)$ of a graph G is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G. The b-chromatic number of central graph of some general graphs are obtained in [5]. A triangular snake is obtained from a path by identifying each of the path with an edge of the cycle C_3. It is denoted by T_n. (See figure 1).

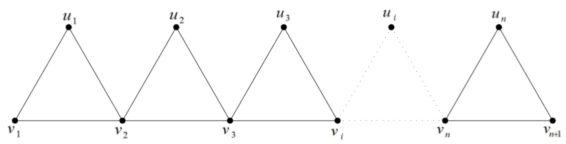

Figure 1. **Triangular Snake T_n**

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2. A Brief Review of b-Coloring

The parameter $\varphi(G)$, introduced by Irving and Manlove [6] has received atypical consideration by many authors. In 2005, Hoang and Kouider [4] characterized all bipartite graphs G and all P_4-sparse graphs G such that each induced subgraph H of G satisfies $\varphi(H) = \chi(H)$. They also prove that every $2K_2$-free and $\overline{P_5}$-free graph is b-perfect. Kratochvl, Tuza and Voigt [10] evaluated the asymptotic behavior for the b-chromatic number of random graphs and proved that it is NP-completeness of the problem to decide whether there is a dominating proper k-coloring even for connected bipartite graphs and $k = \Delta(G) + 1$. Kouider and Zaker [9] proposed some upper bounds for the b-chromatic number of several classes of graphs in function of other graph parameters. In [12] it is proved that the b-chromatic number of any d-regular graph of girth 5 that contains no cycle of order 6 is $d + 1$.

So many authors have studied the behavior of the b-chromatic number, e.g. Kara et al. [7] proved that chordal graphs and some planar graphs are b-continuous. In [2] it is proved that P_4-sparse graphs (and, in particular, cographs) are b-continuous and b-monotonic. Besides, they describe a dynamic programming algorithm to compute the b-chromatic number in polynomial time within these graph classes.

Maffray et al. [3] proved that, if G is a connected cactus and $m(G) = 7$ then the difference between $\varphi(G)$ and $m(G)$ is at most one and we can obtain $\varphi(G)$ in polynomial time. Several related concepts concerning b-colorings of graphs have been studied in [8, 11, 13]. In this paper we study the b-coloring of central graph of triangular snake, which is denoted by $C(T_n)$ and we will evaluate the b-chromatic number for it.

3. Main Result

Theorem 3.1. *If $n \geq 2$, then the b-chromatic number of central graph of triangular snake graph is $\varphi\{C(T_n)\} = 2n + 1$.*

Proof. Let T_n be the triangular snake graph with $2n + 1$ vertices and $3n$ edges. Let

$$\{v_1, v_2, \ldots, v_{n+1}, u_1, u_2, u_3, \ldots, u_n\}$$

be the vertices of the triangular snake T_n (See figure 1). Now by definition of central graph, each edge of graph is subdivided by a new vertex. Therefore assume that each edge (v_i, v_{i+1}) and the line joining v_i and v_{i+1} to a vertex u_i, $i = 1, 2, 3, \ldots, n$ are subdivided by the vertices e_i, e_i', and $e_i^{''}$, $i = 1, 2, 3, \ldots, n$ respectively. Now assign the following $(2n + 1)$ coloring to $C(T_n)$ as b-chromatic:

$$\begin{aligned}
c(u_i) &= 2n + 2 - i; && 1 \leq i \leq n, \\
c(v_i) &= i; && 1 \leq i \leq n + 1, \\
c(e_i) &= 2n + 2 - i; && 1 \leq i \leq n, \\
c(e_i') &= i + 1; && 1 \leq i \leq n, \\
c(e_i'') &= i; && 1 \leq i \leq n.
\end{aligned}$$

It follows that $\varphi\{C(T_n)\} \geq 2n + 1$. It remains to show that the upper bound of $\varphi\{C(T_n)\} \leq 2n + 1$. Suppose if we assign new color $2n + 2$ to the vertices of $C(T_n)$; $n \geq 2$, it will not produce a b-coloring. Since for $n \geq 2, C(T_n)$ has at least $2n + 1$

vertices of degree $2n$. Thus

$$\varphi\{C(T_n)\} \leq m\{C(T_n)\} = 2n + 1.$$

Hence $\varphi\{C(T_n)\} = 2n + 1$; $n \geq 2$ (See figure 2).

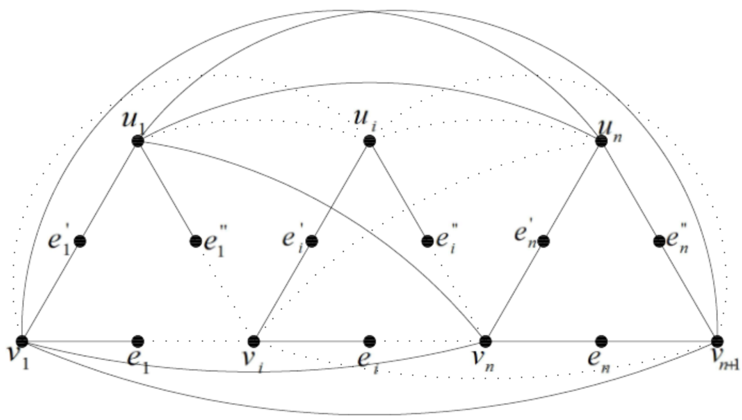

Figure 2. **Central graph of Triangular Snake** $C(T_n)$

□

4. Conclusion

b-coloring play important role in clustering, automatic reading system and distributed system. We have investigated b-chromatic number of central graph of triangular snake graph. The investigation of similar results for different graphs as well in the context of various graph coloring problems is an open area of research.

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