Edge Version of Multiplicative Atom Bend Connectivity Index of Certain Nanotubes and Nanotorus

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1. Introduction

Let $G$ be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. A molecular graph is a finite simple graph such that its vertices correspond to the atoms and the edges to the bonds. A topological index is a numerical parameter mathematically derived from the graph structure. There are several topological indices that have some applications in theoretical chemistry, especially in QSPR/QSAR study [1, 2]. The degree $d_G(v)$ of a vertex $v$ is the number of vertices adjacent to $v$. The degree of an edge $e = uv$ in $G$ is defined by $d_G(e) = d_G(u) + d_G(v) - 2$. The line graph $L(G)$ of $G$ is the graph whose vertex set corresponds to the edges of $G$ such that two vertices of $L(G)$ are adjacent if the corresponding edges of $G$ are adjacent. We refer to [3] for undefined term and notation. The multiplicative atom bond connectivity index [4] of a graph $G$ is defined as

$$ABCII(G) = \prod_{uv \in E(G)} \sqrt{\frac{d_G(u) + d_G(v) - 2}{d_G(u) d_G(v)}}$$

This index was also studied, for example, in [5, 6]. We now define the edge version of multiplicative atom bond connectivity index of a graph $G$ as follows:

The edge version of multiplicative atom bond connectivity index of a molecular graph $G$ is defined as

$$ABCII_e(G) = \prod_{e \in E(L(G))} \sqrt{\frac{d_{L(G)}(e) + d_{L(G)}(f) - 2}{d_{L(G)}(e) d_{L(G)}(f)}}.$$
2. Results for Nanotubes

2.1. \( TUC_4C_6C_8[p, q] \) Nanotube

We consider the graph 2-D lattice of \( TUC_4C_6C_8[1, 1] \) nanotube as depicted in Figure 1(a). The line graph of \( TUC_4C_6C_8[1, 1] \) is shown in Figure 1(b). Also we consider the graph of \( TUC_4C_6C_8[p, q] \) nanotube with \( p \) columns and \( q \) rows. The graph of \( TUC_4C_6C_8[4, 5] \) is shown in Figure 1(c).

Figure 1.

We determine the edge version of multiplicative atom bond connectivity index for \( TUC_4C_6C_8[p, q] \) nanotube.

**Theorem 2.1.** The edge version of multiplicative atom bond connectivity index of \( TUC_4C_6C_8[p, q] \) nanotube is

\[
ABC_{II_e}(TUC_4C_6C_8[p, q]) = \left( \frac{2}{3} \right)^{2p} \left( \frac{5}{12} \right)^{4p} \left( \frac{3}{8} \right)^{9pq - 7p}.
\]

**Proof.** Let \( G \) be the graph of \( TUC_4C_6C_8[p, q] \) nanotube. By algebraic method, the line graph of \( TUC_4C_6C_8[p, q] \) has \( 18pq - 4p \) edges. Also we obtain that the edge set \( E(L(G)) \) can be divided into three partitions as given in Table 1.

<table>
<thead>
<tr>
<th>Number of edges</th>
<th>2p</th>
<th>8p</th>
<th>18pq - 14p</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_{L(G)}(e) )</td>
<td>( \left[ \frac{3 + 3 - 2}{3 \times 3} \right] )</td>
<td>( \left[ \frac{3 + 4 - 2}{3 \times 4} \right] )</td>
<td>( \left[ \frac{4 + 4 - 2}{4 \times 4} \right] )</td>
</tr>
<tr>
<td>( d_{L(G)}(f) )</td>
<td>( \left[ \frac{3 + 3 - 2}{3 \times 3} \right] )</td>
<td>( \left[ \frac{3 + 4 - 2}{3 \times 4} \right] )</td>
<td>( \left[ \frac{4 + 4 - 2}{4 \times 4} \right] )</td>
</tr>
<tr>
<td>( f \in E(L(G)) )</td>
<td>( \left[ \frac{3 + 3 - 2}{3 \times 3} \right] )</td>
<td>( \left[ \frac{3 + 4 - 2}{3 \times 4} \right] )</td>
<td>( \left[ \frac{4 + 4 - 2}{4 \times 4} \right] )</td>
</tr>
</tbody>
</table>

Table 1. Partitions of the edge set \( E(L(G)) \)

Thus, using the definition of edge version of multiplicative atom bond connectivity index, we obtain

\[
ABC_{II_e}(TUC_4C_6C_8[p, q]) = \left( \frac{2}{3} \right)^{2p} \left( \frac{5}{12} \right)^{4p} \left( \frac{3}{8} \right)^{9pq - 7p}.
\]

\( \square \)

2.2. \( TUSC_4C_8(S)[m, n] \) Nanotube

We consider the graph 2-D lattice of \( TUSC_4C_8(S)[1, 1] \) nanotube as depicted in Figure 2(a). The line graph of \( TUSC_4C_8(S)[1, 1] \) is shown in Figure 2(b). Also we consider the graph of \( TUSC_4C_8(S)[m, n] \) nanotube with \( m \) columns and \( n \) rows. The graph of \( TUSC_4C_8(S)[m, n] \) is shown in Figure 2(c).
In the following theorem, we determine the edge version of multiplicative atom bond connectivity index for $TUSC_4C_8(S)[m, n]$ nanotube.

**Theorem 2.2.** The edge version of multiplicative atom bond connectivity index of $TUSC_4C_8(S)[m, n]$ nanotube is

$$ABC_{II_e}(TUSC_4C_8(S)[m, n]) = \left(\frac{1}{2}\right)^{2m} \left(\frac{5}{12}\right)^{4m} \left(\frac{3}{8}\right)^{12mn - 4m}.$$ 

**Proof.** Let $G$ be the graph of $TUSC_4C_8(S)[m, n]$ nanotube. By algebraic method, the line graph of $TUSC_4C_8(S)[m, n]$ nanotube has $24mn + 4m$ edges. Also we obtain that the edge set $E(L(G))$ can be divided into three partitions as given in Table 2.

<table>
<thead>
<tr>
<th>$d_{L(G)}(e), d_{L(G)}(f) \in E(L(G))$</th>
<th>(2, 3)</th>
<th>(3, 4)</th>
<th>(4, 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of edges</td>
<td>4m</td>
<td>8m</td>
<td>$24mn - 8m$</td>
</tr>
</tbody>
</table>

Table 2. Partitions of the edge set $E(L(G))$

Thus using the definition of edge version of multiplicative atom bond connectivity index, we obtain

$$ABC_{II_e}(TUSC_4C_8(S)[m, n]) = \left(\frac{2 + 3 - 2}{2 \times 3}\right)^{4m} \left(\frac{3 + 4 - 2}{3 \times 4}\right)^{8m} \left(\frac{1 + 4 - 2}{4 \times 4}\right)^{24mn - 8m} = \left(\frac{1}{2}\right)^{2m} \left(\frac{5}{12}\right)^{4m} \left(\frac{3}{8}\right)^{12mn - 4m}.$$ 

\[\square\]

### 2.3. H-Naphtalenic $NPHX[m, n]$ Nanotube

We consider the graph 2-D lattice of H-Naphtalenic $NPHX[1, 1]$ nanotube as shown in Figure 3(a). The line graph of H-Naphtalenic $NPHX[1, 1]$ is shown in Figure 3(b). Also we consider the graph of H-Naphtalenic $NPHX[4, 3]$ nanotube as shown in Figure 3(c).

In the following theorem, we determine the edge version of multiplicative atom bond connectivity index for $NPHX[m, n]$ nanotube.
Theorem 2.3. The edge version of multiplicative atom bond connectivity index of NPHX[m, n] nanotube is

\[ \text{ABCII}_e(NPHX[m, n]) = \left(\frac{2}{3}\right)^{6m} \left(\frac{5}{12}\right)^{6m} \left(\frac{3}{8}\right)^{15mn - 13m}. \]

Proof. Let \( G \) be the graph of NPHX[m, n] nanotube. By algebraic method, the line graph of NPHX[m, n] nanotube has 30nm – 8m edges. Also we obtain that the edge set \( E(L(G)) \) can be divided into three partitions as given in Table 3.

<table>
<thead>
<tr>
<th>( d_{L(G)}(e) )</th>
<th>( d_{L(G)}(f) )</th>
<th>Number of edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3, 3)</td>
<td>(3, 4)</td>
<td>6m</td>
</tr>
<tr>
<td>(4, 4)</td>
<td></td>
<td>12m</td>
</tr>
<tr>
<td>( \quad )</td>
<td></td>
<td>(30mn – 26m)</td>
</tr>
</tbody>
</table>

Table 3. Partitions of edge set \( E(L(G)) \)

Thus using the definition of edge version of multiplicative atom bond connectivity index, we obtain

\[ \text{ABCII}_e(NPHX[m, n]) = \left(\sqrt{\frac{3 + 3 - 2}{3 \times 3}}\right)^{6m} \left(\sqrt{\frac{3 + 4 - 2}{3 \times 4}}\right)^{12m} \left(\sqrt{\frac{4 + 4 - 2}{4 \times 4}}\right)^{30mn - 26m}. \]

\( \quad \)

3. Results for Nanotorus

3.1. \( C_4C_6C_8[p, q] \) Nanotori

Consider the graph of 2-D lattice of \( C_4C_6C_8[2, 1] \) nanotori as shown in Figure 4 (a). The line graph of \( C_4C_6C_8[2, 1] \) nanotori is shown in Figure 4(b). Also we consider the graph of 2-D lattice of \( C_4C_6C_8[p, q] \) nanotori as shown in Figure 4(c).

Figure 4.

Theorem 3.1. The edge version of multiplicative atom bond connectivity index of \( C_4C_6C_8[p, q] \) is

\[ \text{ABCII}_e(C_4C_6C_8[p, q]) = \left(\frac{1}{2}\right)^p \left(\frac{2}{3}\right)^p \left(\frac{5}{12}\right)^{2p} \left(\frac{\sqrt{6}}{4}\right)^{18pq - 9p}. \]

Proof. Let \( G \) be the graph of \( C_4C_6C_8[p, q] \) nanotori. By algebraic method, the line graph of \( C_4C_6C_8[p, q] \) nanotori has 18pq – 2p edges. Also we obtain that the edge set \( E(L(G)) \) can be divided into four partitions as given in Table 4.

<table>
<thead>
<tr>
<th>( d_{L(G)}(e) )</th>
<th>( d_{L(G)}(f) \setminus e \in E(L(G)) )</th>
<th>Number of edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2, 4)</td>
<td>(3, 3)</td>
<td>2p</td>
</tr>
<tr>
<td>(3, 4)</td>
<td></td>
<td>( p )</td>
</tr>
<tr>
<td>(4, 4)</td>
<td></td>
<td>18pq – 9p</td>
</tr>
</tbody>
</table>

Table 4. Partitions of the edge set \( E(L(G)) \)

Thus using the definition of edge version of multiplicative atom bond connectivity index, we obtain

\[ \text{ABCII}_e(C_4C_6C_8[p, q]) = \left(\sqrt{\frac{3 + 3 - 2}{2 \times 4}}\right)^{2p} \left(\sqrt{\frac{3 + 3 - 2}{3 \times 3}}\right)^p \left(\sqrt{\frac{3 + 4 - 2}{3 \times 4}}\right)^{4p} \left(\sqrt{\frac{4 + 4 - 2}{4 \times 4}}\right)^{18pq - 9p}. \]
\[ \left( \frac{1}{2} \right)^{3p} \left( \frac{5}{12} \right)^{4p} \left( \frac{3}{8} \right)^{12pq-7p} \times \frac{\sqrt{6}}{4} \right)^{24pq-9p} \].

### 3.2. TC\(_4C_8(S)[p,q]\) Nanotori

Consider the graph of 2-D lattice of TC\(_4C_8(S)[1,1]\) nanotori as shown in Figure 5(a). The line graph of TC\(_4C_8(S)[1,1]\) nanotori is shown in Figure 5(b). Also we consider the graph of 2-D lattice of TC\(_4C_8(S)[5,3]\) nanotori as shown in Figure 5(c).

![Figure 5.](image)

In the following theorem, we determine the edge version of multiplicative atom bond connectivity index for TC\(_4C_8(S)[p,q]\) nanotori.

**Theorem 3.2.** The edge version of multiplicative atom bond connectivity index of TC\(_4C_8(S)[p,q]\) nanotori is

\[ ABC_{ILc}(TC_4C_8(S)[p,q]) = \left( \frac{1}{2} \right)^{3p} \left( \frac{5}{12} \right)^{4p} \left( \frac{3}{8} \right)^{12pq-7p} \times \frac{\sqrt{2} + 3}{2} \right)^{2p} \left( \frac{\sqrt{2} + 4}{2} \right)^{4p} \left( \frac{\sqrt{3} + 4}{3} \right)^{4p} \left( \frac{\sqrt{4} + 4}{4} \right)^{24pq-14p} \].

**Proof.** Let \( G \) be the graph of TC\(_4C_8(S)[p,q]\) nanotori. By algebraic method, the line graph of TC\(_4C_8(S)[p,q]\) nanotori has \( 24pq - 4p \) edges. Also we obtain that the edge set \( E(L(G)) \) can be divided into four partitions as given in Table 5.

<table>
<thead>
<tr>
<th>( d_{L(G)}(e) )</th>
<th>( d_{L(G)}(f) )</th>
<th>Number of edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>2p</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>4p</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>24pq - 14p</td>
</tr>
</tbody>
</table>

**Table 5.** Partitions of the edge set \( E(L(G)) \)

Thus, using the definition of edge version of multiplicative atom bond connectivity index, we obtain

\[ ABC_{ILc}(TC_4C_8(S)[p,q]) = \left( \frac{1}{2} \right)^{3p} \left( \frac{5}{12} \right)^{4p} \left( \frac{3}{8} \right)^{12pq-7p} \times \frac{\sqrt{2} + 3}{2} \right)^{2p} \left( \frac{\sqrt{2} + 4}{2} \right)^{4p} \left( \frac{\sqrt{3} + 4}{3} \right)^{4p} \left( \frac{\sqrt{4} + 4}{4} \right)^{24pq-14p} \].

References


