Int. J. Math. And Appl., 6(1)(2018), 117–124. (Special Issue)
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Hexagonal Fuzzy Numbers and its Corresponding Matrices

J. Jhonson Savarimuthu[1,*] and N. Karthika[1]

1 PG & Research Department of Mathematics, St.Joseph's College of Arts and Science, Cuddalore, Tamil Nadu, India.

Abstract: In this paper, we define Hexagonal fuzzy number (HFN) in continuation with the other defined fuzzy numbers. We also include basic arithmetic operations of Hexagonal fuzzy numbers. Finally we define Hexagonal fuzzy matrix with some matrix properties.

MSC: 15B15.

Keywords: Fuzzy Numbers, Hexagonal Fuzzy Numbers, Hexagonal Fuzzy Matrices.

 Accepted on: 13th April 2018

1. Introduction

In 1965, fuzzy sets were introduced by Zadeh [1]. Fuzzy sets have been introduced by Lotfi.A.Zadeh Fuzzy set. Dubois and Prade has defined any of the fuzzy Number as a Fuzzy subset. Thomanson introduced Fuzzy matrices and discussed about the convergence of powers of fuzzy matrix. A fuzzy matrix is a matrix with elements having values in the fuzzy interval. In this paper the unit interval $[0,1]$ and the interval $[-1,1]$ are called fuzzy interval. Fuzzy matrices play on important role in scientific development. In this paper, we recall the definition of Hexagonal fuzzy Number and some operations on Hexagonal Fuzzy Number (HFNs). We presented some properties of Hexagonal Fuzzy Matrices (HFMs). Finally we presented Conclusion is included.

2. Preliminaries

Definition 2.1. *A fuzzy set is characterized by its membership function, taking values from the domain, space or universe of discourse mapped into the unit interval $[0,1]$. A fuzzy set A in the universal set X is defined as $A = (X, \mu(X); x \in X)$. Here, $\mu_A : A \to [0,1]$ is the grade of the membership function and $\mu_A(X)$ is the grade value of $x \subset X$ in the fuzzy set A.*

Definition 2.2. *A fuzzy set A is called normal if there exists an element $x \subset X$ whose membership value is one, i.e., $\mu_A(X) = 1$.*

Definition 2.3. *A fuzzy number A is a subset of real line R, with the membership function μ_A satisfying the following properties:*

[*] *E-mail: johnson22970@gmail.com*

(1). $\mu_A(X)$ is piecewise continuous in its domain.

(2). A is normal, i.e., there is a $x_0 \in A$ such that $\mu_A(x_0) = 1$.

(3). A is convert, i.e., $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$. For all $x_1, x_2 \in X$.

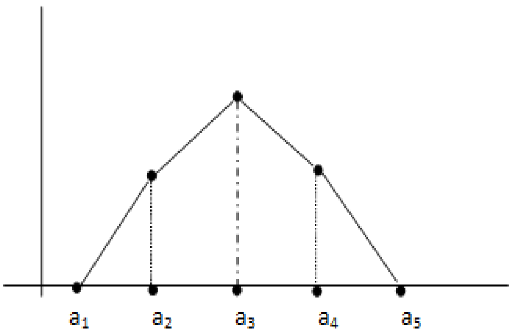
Due to wide applications of the fuzzy number, two types of fuzzy number, namely, triangular fuzzy number and trapezoidal fuzzy number, are introduced in the field of fuzzy algebra.

Definition 2.4. *A fuzzy number $A = (a_1, a_2, a_3, a_4)$ is said to be trapezoidal fuzzy number if its membership function is given by where $a_1 \leq a_2 \leq a_3 \leq a_4$*

$$\mu_A(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \dfrac{(x-a_1)}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\ \dfrac{(a_4-x)}{a_4-a_3} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{for } x < a_4 \end{cases}$$

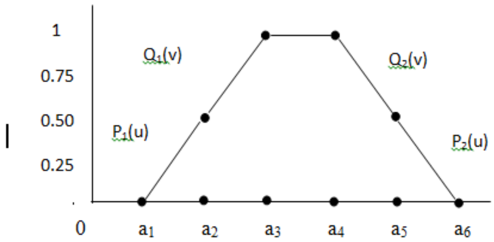
Definition 2.5. *A pentagonal fuzzy number defied as $A = (a_1, a_2, a_3, a_4, a_5)$, and its the membership function is given by,*

$$\mu_A(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \dfrac{(x-a_1)}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\ \dfrac{(x-a_2)}{a_3-a_2} & \text{for } a_2 \leq x \leq a_3 \\ 1 & \text{for } x = a_3 \\ \dfrac{(a_4-x)}{a_4-a_3} & \text{for } a_2 \leq x \leq a_3 \\ \dfrac{(a_4-x)}{a_4-a_3} & \text{for } a_4 \leq x \leq a_5 \\ 0 & \text{for } x < a_5 \end{cases}$$



Definition 2.6. *A fuzzy number A_H is a hexagonal fuzzy number denoted by $A_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ where $a_1, a_2, a_3, a_4, a_5, a_6$ are real number and its membership function $\mu_{AH}(x)$ is given by*

$$\mu_A(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \frac{1}{2}\frac{(x-a_1)}{(a_2-a_1)} & \text{for } a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2}\frac{(x-a_2)}{a_3-a_2} & \text{for } a_2 \leq x \leq a_3 \\ 1 & \text{for } a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2}\frac{(a_4-x)}{a_4-a_3} & \text{for } a_4 \leq x \leq a_5 \\ \frac{1}{2}\frac{(a_6-x)}{a_6-a_5} & \text{for } a_5 \leq x \leq a_6 \\ 0 & \text{for } x > a_6 \end{cases}$$



Definition 2.7. *Hexagonal fuzzy number A_H is the ordered quadruple $P_1(u), Q_1(v), Q_2(v), P_2(u)$ for $u \in [0, 0.5]$ and $v \in [0.5, w]$ where,*

$$P_1(u) = \frac{1}{2}\frac{(u - a_1)}{(a_2 - a_1)}$$
$$Q_1(v) = \frac{1}{2} + \frac{1}{2}\frac{(v - a_2)}{(a_3 - a_2)}$$
$$Q_1(v) = 1 - \frac{1}{2}\frac{(v - a_4)}{a_4 - a_3}$$
$$P_2(u) = \frac{1}{2}\frac{(a_6 - u)}{a_6 - a_5}$$

Definition 2.8. *The classical set A_α called alpha cut set is the set of elements whose degree of membership is the set of element whose degree of membership in $A_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ is no less than, α it is defined as*

$$A_\alpha = \{x \in X/\mu_{A_H}(x) \geq \alpha\} = \begin{cases} [P_1(\alpha), P_2(\alpha)] & for \quad \alpha \in [0, 0.5) \\ [Q_1(\alpha), Q_2(\alpha)] & for \quad \alpha \in [0.5, 1] \end{cases}$$

Definition 2.9. *The α-cut of as normal hexagonal fuzzy number $\tilde{A_H} = (a_1, a_2, a_3, a_4, a_5, a_6)$ given by the definition(i.e) $w = 1$ for all $\alpha \in [0, 1]$ is*

$$A_\alpha = \begin{cases} [2\alpha(a_2 - a_1) + a_1, -2\alpha(a_6 - a_5) + a_6] & \alpha \in [0, 0.5] \\ [2\alpha(a_3 - a_2) - a_3 + 2a_2 - 2\alpha(a_5 - a_4) + 2a_5 - a_4] & \alpha \in [0.5, 1] \end{cases}$$

3. Arithmetic operation of Hexagonal Fuzzy Number

The following are three operations that can be performed on hexagonal fuzzy number, suppose $\tilde{A_H} = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B_H} = (b_1, b_2, b_3, b_4, b_5, b_6)$ are two hexagonal fuzzy numbers then,

Addition: $\tilde{A_H}(+)\tilde{B_H} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$

Subtraction: $\tilde{A_H}(-)\tilde{B_H} = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6)$

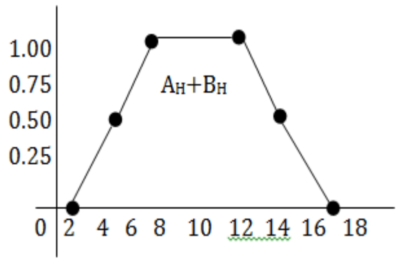
Multiplication: $\tilde{A_H}(*)\tilde{B_H} = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6)$

3.1. Addition of two hexagonal fuzzy number

If $\tilde{A_H} = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B_H} = (b_1, b_2, b_3, b_4, b_5, b_6)$ are two hexagonal fuzzy numbers for all $\alpha \in [0, 1]$. Let us add to the alpha cuts A_α and B_α of $\tilde{A_H}$ and $\tilde{B_H}$ using interval arithmetic

$$A_\alpha + B_\alpha = \begin{cases} [2\alpha(a_2 - a_1) + a_1, -2\alpha(a_6 - a_5) + a_6] + [2\alpha(b_2 - b_1) + b_1, -2\alpha(b_6 - b_5) + b_6] & \alpha \in [0, 0.5] \\ [2\alpha(a_3 - a_2) - a_3 + 2a_2, -2\alpha(a_5 - a_4) + 2a_5 - a_4] + [2\alpha(b_3 - b_2) - b_3 + 2b_2, -2\alpha(b_5 - b_4) + 2b_5 - b_4] & \alpha \in [0.5, 1] \end{cases}$$

Let the example 1 such that, $\tilde{A_H} = (1, 2, 3, 6, 7, 8)$ and $\tilde{B_H} = (1, 2, 3, 5, 7, 9)$. For $\alpha \in [0, 0.5)$, $A_\alpha = [4\alpha + 2, -6\alpha + 17]$, $B_\alpha = [4\alpha + 2, -6\alpha + 17] \Rightarrow A_\alpha + B_\alpha = [8\alpha + 4, -6\alpha + 34]$. For $\alpha \in [0.5, 1]$, $A_\alpha = [4\alpha + 2, -6\alpha + 17]$, $B_\alpha = [4\alpha + 2, -6\alpha + 17] \Rightarrow A_\alpha + B_\alpha = [8\alpha + 4, -6\alpha + 34]$. Since the both $\alpha \in [0, 0.5)$ and For $\alpha \in [0.5, 1]$ arithmetic intervals are same. Where $\alpha = 0$ $A_0 + B_0 = [4, 34]$. Like wise $\alpha = 0.5$ $A_{0.5} + B_{0.5} = [8, 28]$. And for $\alpha = 1$, $A_1 + B_1 = [12, 22]$. Hence $A_\alpha + B_\alpha = [2, 4, 6, 11, 14, 17]$ all the points coincides with the sum of the two hexagonal fuzzy number. Hence the addition of two α-cuts lies within the interval.

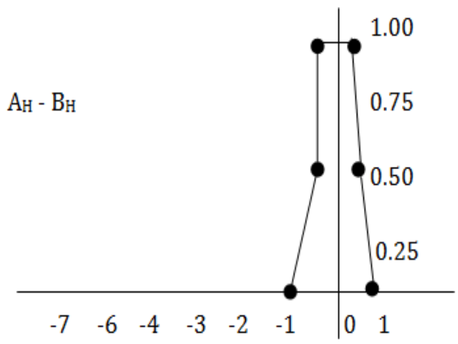


3.2. Subtraction of two hexagonal fuzzy number

If $\tilde{A_H} = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B_H} = (b_1, b_2, b_3, b_4, b_5, b_6)$ are two hexagonal fuzzy numbers for all $\alpha \in [0, 1]$. Let us subtract to the alpha cuts A_α and B_α of $\tilde{A_H}$ and $\tilde{B_H}$ using interval arithmetic

$$A_\alpha - B_\alpha = \begin{cases} [2\alpha(a_2 - a_1) + a_1, -2\alpha(a_6 - a_5) + a_6] - [2\alpha(b_2 - b_1) + b_1, -2\alpha(b_6 - b_5) + b_6] & \alpha \in [0, 0.5] \\ [2\alpha(a_3 - a_2) - a_3 + 2a_2, -2\alpha(a_5 - a_4) + 2a_5 - a_4] - [2\alpha(b_3 - b_2) - b_3 + 2b_2, -2\alpha(b_5 - b_4) + 2b_5 - b_4] & \alpha \in [0.5, 1] \end{cases}$$

Let the example 2 such that, $\tilde{A_H} = (1, 2, 3, 6, 7, 8)$ and $\tilde{B_H} = (1, 2, 3, 5, 7, 9)$. For $\alpha \in [0, 0.5)$, $A_\alpha = [0, 2\alpha - 1]$, $B_\alpha = [0, 2\alpha - 1] \Rightarrow A_\alpha - B_\alpha = [0, 4\alpha - 1]$. For $\alpha \in [0.5, 1]$, $A_\alpha = [0, 2\alpha - 1]$, $B_\alpha = [0, 2\alpha - 1] \Rightarrow A_\alpha - B_\alpha = [0, 4\alpha - 1]$. Since the both $\alpha \in [0, 0.5)$ and For $\alpha \in [0.5, 1]$ arithmetic intervals are same. Where $\alpha = 0$ $A_0 + B_0 = [0, -2]$. Like wise $\alpha = 0.5$, $A_{0.5} + B_{0.5} = [0, 0]$. And for $\alpha = 1$ $A_1 + B_1 = [0, 2]$. Hence $A_\alpha + B_\alpha = [0, 0, 0, 1, 0, -1]$ all the points coincides with the different of the two hexagonal fuzzy number. Hence the addition of two α-cuts lies within the interval.

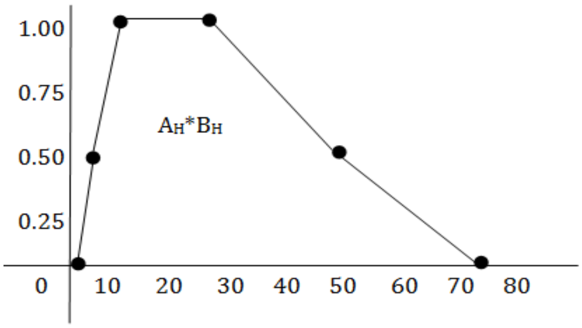


3.3. Multiplication of two hexagonal fuzzy number

If $\tilde{A_H} = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B_H} = (b_1, b_2, b_3, b_4, b_5, b_6)$ are two hexagonal fuzzy numbers for all $\alpha \in [0, 1]$. Let us Multiply to the alpha cuts A_α and B_α of $\tilde{A_H}$ and $\tilde{B_H}$ using interval arithmetic

$$A_\alpha * B_\alpha = \begin{cases} [2\alpha(a_2 - a_1) + a_1, -2\alpha(a_6 - a_5) + a_6] * [2\alpha(b_2 - b_1) + b_1, -2\alpha(b_6 - b_5) + b_6] & \alpha \in [0, 0.5] \\ [2\alpha(a_3 - a_2) - a_3 + 2a_2, -2\alpha(a_5 - a_4) + 2a_5 - a_4] * [2\alpha(b_3 - b_2) - b_3 + 2b_2, -2\alpha(b_5 - b_4) + 2b_5 - b_4] & \alpha \in [0.5, 1] \end{cases}$$

Let the Example 3 such that, $\tilde{A_H} = (1, 2, 3, 6, 7, 8)$ and $\tilde{B_H} = (1, 2, 3, 5, 7, 9)$. For $\alpha \in [0, 0.5)$, $A_\alpha * B_\alpha = [(2\alpha + 1)(2\alpha + 1), (-2\alpha + 8)(-4\alpha + 9)]$. For $\alpha \in [0.5, 1]$, $A_\alpha * B_\alpha = [(2\alpha + 1)(2\alpha + 1), (-2\alpha + 8)(-4\alpha + 9)]$. Since the both $\alpha \in [0, 0.5)$ and For $\alpha \in [0.5, 1]$ arithmetic intervals are same. Where $\alpha = 0$ $\quad A_0 * B_0 = [1, 72]$. Like wise $\alpha = 0.5$ $A_{0.5} * B_{0.5} = [4, 49]$. And for $\alpha = 1$ $\quad A_1 * B_1 = [9, 30]$. Hence $A_\alpha * B_\alpha = [1, 4, 9, 30, 49, 72]$ all the points coincides within the approximate value of the two hexagonal fuzzy number. Hence the addition of two α-cuts lies within the interval.



Definition 3.1. *A fuzzy matrix $A = (a_{ij})_{mn}$ of order mxn is called a Hexagonal fuzzy matrix if the element of the matrix are Hexagonal fuzzy number i.e., of the form $(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}, a_{5ij}, a_{6ij})$. Through classical matrix algebra, then some arithmetic operation of Hexagonal fuzzy number of the same order ; then the following results:*

(1). $A + B = (a_{ij} + b_{ij})$

(2). $A - B = (a_{ij} - b_{ij})$

(3). For $A = (a_{ij})_{m\times r}$ *and* $B = (b_{ij})_{r\times n}$, *we have* $A.B = (c_{ij})_{m\times n}$, *where* $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{jk}$ *for* $i = 1, 2, 3, \ldots, m$; $j = 1, 2, 3, \ldots, n$.

(4). $A^T = n(a_{ij})$, *the transpose of A.*

(5). $kA = (ka_{ij})$, *where k is any scalar.*

Some of the special type of Hexagonal fuzzy matrices corresponding to classical matrices are now introduction in this section. However, in fuzzy matrices algebra, we define some other types of Hexagonal fuzzy matrices and their algebraic properties.

4. Properties of Hexagonal Fuzzy Matrices

Property 4.1. *Let $U = (u_{ij})$ and $V = (v_{ij})$ be two square HFMs of the same order m; then, the following holds well.*

(1). $tr(U + V) = tr(U) + tr(V)$.

(2). $tr(U) = tr(U^T)$.

(3). $tr(U.V) = tr(U)tr(V)$.

Proof.

(1). Let U and V be two HFMs order m, where $U = (u_{1ij}, u_{2ij}, u_{3ij}, u_{4ij}, u_{5ij}, u_{6ij})$ and $V = (v_{1ij}, v_{2ij}, v_{3ij}, v_{4ij}, v_{5ij}, v_{6ij})$. Now, $tr(U) = \sum_{i=1}^{m}(U_{ii}) = \sum_{i=1}^{m}(u_{1ii}, u_{2ii}, u_{3ii}, u_{4ii}, u_{5ii}, u_{6ii})$ and $tr(V) = \sum_{i=1}^{m}(V_{ii}) = \sum_{i=1}^{m}(v_{1ii}, v_{2ii}, v_{3ii}, v_{4ii}, v_{5ii}, v_{6ii})$ Then,

$$tr(U + V) = \sum_{i=1}^{m}(u_{ii} + v_{ii}) = \sum_{i=1}^{m}(u_{ii}) + \sum_{i=1}^{m}(v_{ii})$$

$$= \sum_{i=1}^{m}(u_{1ii}, u_{2ii}, u_{3ii}, u_{4ii}, u_{5ii}, u_{6ii}) + \sum_{i=1}^{m}(v_{1ii}, v_{2ii}, v_{3ii}, v_{4ii}, v_{5ii}, v_{6ii})$$

$$tr(U+V) = tr(U) + tr(V).$$

(2). We know that the principle diagonal of a HFM remains invariant under transposition. Hence the proof obvious.

(3). We know that for any two HFM of the same order, their multiplication is well defined. If $U = u_{ij}$ and $V = v_{ij}$ then Again , let $R = r_{ij}$, where $R_{ij} = \sum\limits_{r=1}^{m} u_{ir}v_{rj}$, for $i, j = 1, 2, ..., m$. Now, $tr(R) = \sum\limits_{i=1}^{m}(r_{ii}) = \sum\limits_{r=1}^{m} u_{ir}v_{rj}$. Again let, $S = s_{ij} = V.U$, where $s_{ij} = \sum\limits_{r=1}^{m} v_{ir}u_{rj}$ for $i, j1, 2, ..., m$. Therefore,

$$\begin{aligned} tr(S) = tr(V.U) &= \sum_{r=1}^{m} S_{ii} \\ &= \sum_{r=1}^{m}\left(\sum_{i=1}^{m}(v_{ir}u_{ri})\right) \\ &= \sum_{r=1}^{m}\left(\sum_{i=1}^{m}(u_{ir}v_{ri})\right) \text{(interchanging the dummy i and r)} \\ tr(V.U) &= tr(U.V) = tr(R). \end{aligned}$$

□

Definition 4.2. *The Hexagonal fuzzy Determination of a Hexagonal fuzzy matrix A of order $n \times n$ is denoted by $det(A)$ or $|A| = \sum\limits_{\sigma \in s_n} (sgn\sigma. \prod_{i=1}^{n} a_{i\sigma i})$, where $a_{ii} = ((a_{1i\sigma i}, a_{2i\sigma i}, a_{3i\sigma i}, a_{4i\sigma i}, a_{5i\sigma i}, a_{6i\sigma i}))$ are HFNs and S_n denotes the symmetric group of all permutation, define of indices $1, 2, ..., n$. Additionally, sgn is the signature of the permutation, defined as $sgn\sigma = 1 or - 1$ if the permutation is even or odd, respectively. There are several products and addition of HFMs generates another HFM. Thus, the determinant value of a HFM yields a pentagonal fuzzy number.*

Property 4.3. *If A is square HFM, then the determinant value of A equals to that of its transpose, i.e., $|A| = \left|A^T\right|$.*

Proof. Let $A = (a_{ij})$ be a square HFM of order n and let $R = A^T$ be the transpose of A. Then, by the definition of a hexagonal fuzzy determinant, we have $|A| = \sum\limits_{\sigma \in s_n} (sgn\sigma. \prod_{i=1}^{n} r_{i\sigma i}) = \sum\limits_{\sigma \in s_n} (sgn\sigma. \prod_{i=1}^{n} a_{i\sigma i})$. Let Φ be a permutation on $1, 2, n$ such that $\Phi\sigma = 1$. I being identity permutation. Thus $\Phi = \sigma^{-1}$. Let $\sigma(i) = j$; then $i = \sigma(j)^{-1}$ and $a_{\sigma(i)i} = a_{j\sigma j}, \forall ij$. Therefore,

$$\begin{aligned} |R| &= \sum_{\sigma \in s_n} (sgn\sigma. \prod_{i=1}^{n} r_{1\sigma(i)i}, r_{2\sigma(i)i}, r_{3\sigma(i)i}, r_{4\sigma(i)i}, r_{5\sigma(i)i}), r_{6\sigma(i)i}) \\ &= \sum_{\sigma \in s_n} (sgn\sigma. \prod_{i=1}^{n} a_{1j\sigma(j)}, a_{2j\sigma(j)}, a_{3j\sigma(j)}, a_{4j\sigma(j)}, a_{5j\sigma(j)}), a_{6j\sigma(j)}) \\ &= \sum_{\sigma \in s_n} (sgn\sigma. \prod_{i=1}^{n} a_{1i\sigma(i)}, a_{2i\sigma(i)}, a_{3i\sigma(i)}, a_{4i\sigma(i)}, a_{5i\sigma(i)}), a_{6i\sigma(i)}) \quad \text{(interchanging indices)} \\ &= |R|. \end{aligned}$$

□

Property 4.4.

(1). If A and B are both strictly fuzzy triangular HFMs, then the block matrix $\begin{bmatrix} A & C \\ 0 & B \end{bmatrix}$ *is nilpotent.*

(2). Generally $\begin{bmatrix} A_{11} & & \\ & A_{22}\cdots & \\ & & A_{nn} \end{bmatrix}$ *is nilpotent whenever A_{ii}s are strictly fuzzy triangular HFMs.*

Proof.

(1). Let us consider the concept of block matrix p of the form block matrix $\begin{bmatrix} A & C \\ 0 & B \end{bmatrix}$, where A, B are Strictly fuzzy triangular HFMs of order m, n respectively. If based on the property, A and B are both nilpotent for index m and n, respectively. Now,

$$P^2 = \begin{bmatrix} A & C \\ 0 & B \end{bmatrix} \begin{bmatrix} A & C \\ 0 & B \end{bmatrix} = \begin{bmatrix} A^2 & AC+BC \\ 0 & B^2 \end{bmatrix}$$
$$P^3 = \begin{bmatrix} A^2 & AC+BC \\ 0 & B^2 \end{bmatrix} \begin{bmatrix} A & C \\ 0 & B \end{bmatrix}$$
$$P^3 = \begin{bmatrix} A^3 & A^2C+B^2C+ABC \\ 0 & B^2 \end{bmatrix}$$
$$P^4 = \begin{bmatrix} A^4 & A^3C+A^2CB+AB^2C+B^3C \\ 0 & B^3 \end{bmatrix}$$

Thus, note that when we raise the power to P, the element P_{11} i.e., in general, we and P_{22} increase their power that of element P_{12} i.e., in general, we have $p^k = \begin{bmatrix} A^k & a \\ 0 & B^k \end{bmatrix}$, with k being a positive integer and assuming a as value of the element P_{21} in P^k. If A and B are nilpotent for index m, n respectively. Therefore $A^m = 0, B^n = 0$. Taking $\lambda = lcm(m, n)$ (say)

$$(p^k)^\lambda = \begin{bmatrix} A^{k_\lambda} & a \\ 0 & B^{k_\lambda} \end{bmatrix} = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix},$$

which is a strictly fuzzy triangular HFM. Hence, P is a nilpotent HFM.

(2). It following from the previous properties. □

Definition 4.5. *A square HFM is $A = (a_{ij})$ of order $n \times n$ is called a constant HFM if all the rows are equal to each other, i.e., $(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij}, a_{5ij}, a_{6ij}) = (a_{1rj}, a_{2rj}, a_{3rj}, a_{4rj}, a_{5rj}, a_{6rj})\ \forall\ i, r, j$. For example,*

$$A = \begin{bmatrix} (-1,0,1,2,4,5) & (0,1,2,4,5,6) & (1,2,3,4,5,6) \\ (-1,0,1,2,4,5) & (0,1,2,4,5,6) & (1,2,3,4,5,6) \\ (-1,0,1,2,4,5) & (0,1,2,4,5,6) & (1,2,3,4,5,6) \end{bmatrix}$$

Property 4.6. *Let U and V be two constant HFMs of the same order. Then,the following holds.*

(1). $U + V$ is a constant HFM.

(2). $U.V$ is also a constant HFM.

Proof.

(1). $U = (u_{ij})$ and $V = (v_{ij})$, where $(u_{ij}) = (u_{1ij}, u_{2ij}, u_{3ij}, u_{4ij}, u_{5ij}, u_{6ij})$ and $V_{ij} = (v_{1ij}, v_{2ij}, v_{3ij}, v_{4ij}, v_{5ij}, v_{6ij})$ are two constant HFMs of order n. Then, $(u_{1ij}, u_{2ij}, u_{3ij}, u_{4ij}, u_{5ij}, u_{6ij}) = (u_{1rj}, u_{2rj}, u_{3rj}, u_{4rj}, u_{5rj}, u_{6rj})$ and $(v_{1ij}, v_{2ij}, v_{3ij}, v_{4ij}, v_{5ij}, v_{6ij}) = (v_{1rj}, v_{2rj}, v_{3rj}, v_{4rj}, v_{5rj}, v_{6rj})$. Let,

$$R = (r_{ij}) = (u_{ij} + v_{ij}) = (u_{1ij}, u_{2ij}, u_{3ij}, u_{4ij}, u_{5ij}, u_{6ij}) + (v_{1ij}, v_{2ij}, v_{3ij}, v_{4ij}, v_{5ij}, v_{6ij}); i, r, j = 1, 2, ..., n$$

$$= (u_{1rj}, u_{2rj}, u_{3rj}, u_{4rj}, u_{5rj}, u_{6rj}) + (v_{1rj}, v_{2rj}, v_{3rj}, v_{4rj}, v_{5rj}, v_{6rj}) \text{ [because A, B are constant]}$$

$$= r_{1rj}, r_{2rj}, r_{3rj}, r_{4rj}, r_{5rj}, r_{6rj} = c_{rj} \quad \forall \quad i, r, j.$$

If thus, the rows of $(U + V)$ are similar to each other.

(2). Let,

$$R = (r_{ij}) = (u_{ij} - v_{ij}) = (u_{1ij}, u_{2ij}, u_{3ij}, u_{4ij}, u_{5ij}, u_{6ij}) - (v_{1ij}, v_{2ij}, v_{3ij}, v_{4ij}, v_{5ij}, v_{6ij}); i, r, j = 1, 2, ..., n$$

$$= (u_{1rj}, u_{2rj}, u_{3rj}, u_{4rj}, u_{5rj}, u_{6rj}) - (v_{1rj}, v_{2rj}, v_{3rj}, v_{4rj}, v_{5rj}, v_{6rj}) \quad \text{[because A, B are constant]}$$

$$= r_{1rj}, r_{2rj}, r_{3rj}, r_{4rj}, r_{5rj}, r_{6rj} = c_{rj} \quad \forall \ i, r, j.$$

If thus, the rows $(U - V)$ are similar to each other. □

5. Conclusion

In this paper, special attention is paid to the hexagonal fuzzy number(HFN) and the corresponding hexagonal fuzzy matrix (HFM), along with the related mathematical expression. we studied various types of HFM and their properties (determinate, trace, etc.). second , this paper address the nature of comparable HFM, with some interesting properties. There are several opportunities to develop the applications of such Hexagonal fuzzy number. We are trying to investigate such application.

References

[1] A.K.Adak, M.Bhowmik and M.Pal, *Intuitionistic fuzzy block matrix ans its some properties*, Ann. Pure Appl. Math., 1(1)(2012), 13-31.

[2] A.Bansal, *Some ono linear arithmetic operations on triangular fuzzy number*, Adv. Fuzzy Math., 5(2010), 147-156.

[3] M.Bhowmik and M.Pal, *Some results on generalized interval-valued Intuitionistic fuzzy set*, Int. J. Fuzzy Syst., 14(2)(2012), 193-203.

[4] M.Bhowmik and M.Pal, *Criculant triangular fuzzy number matrices*, V.U.J. Phys Sci., 29(3)(2008), 533-554.

[5] D.Dubois and H.Prade, *Theory and Application*, Fuzzy Set and System, Academic press, London, (1980).

[6] D.Dubois and H.Prade, *Operations on fuzzy numbers*, Internationl Journal of Systems Science, 9(6)(1978), 613-626.

[7] H.Hashimoto, *Convergence of powers of fuzzy transitive matrix*, Fuzzy Sets Syst., 9(1983), 153-160.

[8] H.Hashimoto, *Canonical form of a transitive fuzzy matrix*, Fuzzy Set Syst., 11(1983), 151-156.

[9] K.H.Kim and F.W.Roush, *Generalized fuzzy matrices*, Fuzzy Set Syst., 04(1980), 293-315.

[10] A.Kauffmann, *Introduction to Fuzzy Arithmetic:Theory and Applicatons*, Van Nostrand Reinhold, New York, (1980).

[11] A.K.Shayamal and M.Pal, *Triangular fuzzy matrtices*, Iran. J. Fuzzy Syst., 4(1)(2007), 75-87.

[12] Thomason, *Convergence of powers of fuzzy matrix*, J. Math. Anal. Appl., 57(1977), 476-480.