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Some Characterization of Fuzzy Soft Γ-Semigroups

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Abstract: The purpose of this paper is to introduce the notion of fuzzy soft prime ideals of Γ-semigroup and fuzzy soft Γ-left quasi regular and also to obtain some interesting properties of them.

Keywords: Soft set, fuzzy set ,soft prime, fuzzy soft Γ-left quasi regular.

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1. Introduction

The concept of fuzzy set was introduced by Zadeh [20]. He studied their properties on the parellel lines to set theory. In 1971, Rosenfeld [13] defined fuzzy subgroups and gave some of its properties. Kuroki [6] introduced fuzzy semigroups as a generalized of classical semigroups. Mordeson [9] obtained some characterization of fuzzy semigroups. Sen and saha [14, 15] have introduced Γ-semigroups and their properties. Soft set theory was proposed by Molotsov [8] in 1999. Maji [7] worked on soft set theory and fuzzy soft set theory. Ali [1] introduced new operations on soft sets. Mukherjee [11] studied about the fuzzy ideals and fuzzy prime ideals. Sujit kumar sardar [17] worked on characterization of prime ideals of Γ-semigroups in terms of fuzzy subsets. Thawhat [18] introduced soft Γ-semigroups and disscused interesting many results. Chinnadurai [2] studied the fuzzy soft Γ-regular semigroup. In this paper we have discussed some properties of prime ideals of Γ-semigroups and fuzzy soft Γ-left quasi regular semigroups.

2. Preliminaries

Definition 2.1 ([14]). *Let $S = \{a, b, c, ...\}$ and $\Gamma = \{\alpha, \beta, \gamma, ...\}$ be two non-empty sets. Then S is called a Γ-semigroup if it satisfies the conditions*

(1). $a\alpha b \in S$,

(2). $(a\beta b)\gamma c = a\beta(b\gamma c)\ \forall\ a, b, c \in S$ *and* $\alpha, \beta, \gamma \in \Gamma$.

Definition 2.2 ([9]). *An ideal I of a Γ-semigroup S is called prime Γ-ideal if for any ideals A and B of $S, A\Gamma B \subseteq I$.*

Definition 2.3 ([9]). *An ideal I of a Γ-semigroup S is called semiprime Γ-ideal if for any ideals A , if $A\Gamma A \subseteq I$ implies $A \subseteq I$.*

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Definition 2.4 ([18]). *Let S be a Γ- semigroup. A non empty subset A of S is said to be idempotent $A\Gamma A = A$.*

Definition 2.5 ([10]). *A soft semigroup (F, A) over S is called a soft regular semigroup if for each $\alpha \in A, F(\alpha)$ is regular.*

Definition 2.6 ([16]). *A semigroup S is called left quasi regular if every left ideal of S is idempotent. Also S is left quasi regular if and only if $a \in SaSa$, that is there exist element $x, y \in S$, such that $a = xaya$.*

Definition 2.7 ([8]). *Let U be the universel set and E be the set of parameters, $P(U)$ denote the power set of U and A be a non empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.*

Definition 2.8 ([1]). *The extended union of two fuzzy soft sets (F, A) and (G, B) over a common universe U is fuzzy soft set denoted by $(F, A) \cup_\epsilon (G, B)$ defined as $(F, A) \cup_\epsilon (G, B) = (H, C)$ where $C = A \cup B$, $\forall\ c \in C$.*

$$H(c) = \begin{cases} llF(c) & \text{if } c \in A - B \\ G(c) & \text{if } c \in B - A \\ F(c) \cup G(c) & \text{if } c \in A \cap B. \end{cases}$$

Definition 2.9 ([1]). *The extended intersection of two fuzzy soft sets (F, A) and (G, B) over a common universe U is fuzzy soft set denoted by $(F, A) \cap_\epsilon (G, B)$ defined as $(F, A) \cap_\epsilon (G, B) = (H, C)$ where $C = A \cup B$, $\forall\ c \in C$.*

$$H(c) = \begin{cases} llF(c) & \text{if } c \in A - B \\ G(c) & \text{if } c \in B - A \\ F(c) \cap G(c) & \text{if } c \in A \cup B. \end{cases}$$

Definition 2.10 ([18]). *A soft set (F, A) over S is called a soft Γ-semigroup over S if $(F, A)\tilde{\circ}(F, A)\tilde{\subset}(F, A)$.*

Definition 2.11 ([12]). *The restricted product (H, C) of two fuzzy soft sets (F, A) and (G, B) over a semigroup S is defined as $(H, C) = (F, A)\tilde{\circ}(G, B)$ where $C = A \cap B$ by $H(c) = F(c)\tilde{\circ}G(c), \forall\, c \in C$.*

Definition 2.12 ([20]). *Let X be a non-empty set. A fuzzy subset μ of X is a function from X into the closed unit interval $[0, 1]$. The set of all fuzzy subset of X is called the fuzzy power set of X and is denoted by $FP(X)$.*

Definition 2.13 ([9]). *A semigroup S is called a fuzzy left(right)duo if every fuzzy left(right)ideal of S is a fuzzy ideal of S*

Definition 2.14 ([9]). *A fuzzy ideal f of S is called prime if for any two fuzzy ideals g and $h, g \circ h \subseteq f$ implies $g \subseteq f$ or $h \subseteq f$.*

Definition 2.15 ([9]). *A fuzzy subset f of a semigroup S is called a fuzzy semiprime if $f(a) \geq f(a^2)\ \forall\ a \in S$.*

Definition 2.16 ([3]). *Let X be non empty set and A be subset of X. Then the characteristic function $\chi_A : X \to [0, 1]$ is defined by*

$$\chi_A(x) = \begin{cases} ll1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

3. Prime Fuzzy Soft Ideals of Γ-semigroups

In this section, the notion of prime fuzzy soft ideals of Γ-semigroup introduced. We have also obtained equivalent conditions simple soft Γ-semigroups.

Definition 3.1. *Let (F, A) be fuzzy soft Γ-ideal of S is called prime fuzzy soft Γ-ideal if and only if it is prime fuzzy Γ-ideal of S, (i.e) for each $e \in A$ satisfied the condition $F(e)(p) = F(e)(p\gamma q)\ \forall\ p, q \in S$ and $\gamma \in \Gamma$.*

Definition 3.2. *Let (F, A) be fuzzy soft Γ-ideal of S is called semiprime fuzzy soft Γ-ideal if and only if it is semiprime fuzzy Γ-ideal of S, (i.e) for each $e \in A$ satisfied the condition $F(e)(p) = F(e)(p\gamma q)\ \forall\ p \in S$ and $\gamma \in \Gamma$.*

Theorem 3.3. *A soft set (F, A) over S is a prime soft Γ-ideal of S if and only if $(\chi_{F(e)}, A)$ is a prime fuzzy soft Γ- ideal of S.*

Proof. Suppose that (F, A) is a prime soft Γ- ideal of S. Since $F(e)$ is a Γ-prime ideal of S. Let $a \in S$ such that $p\gamma q \in F(e)$, then $\chi_{F(e)}(p\gamma q) = 1\ \forall\ e \in A$. Since $F(e)$ is a Γ-prime ideal of $S, p\gamma q \in F(e) \Rightarrow p \in F(e)$ and $q \in F(e)$ and $\chi_{F(e)}(p) = 1 = \chi_{F(e)}(p\gamma q)$ and $\chi_{F(e)}(q) = 1 = \chi_{F(e)}(p\gamma q)$. If $p\gamma q \neq F(e)$ then we have $\chi_{F(e)}(p) = 0 = \chi_{F(e)}(p\gamma q)$ and $\chi_{F(e)}(q) = 0 = \chi_{F(e)}(p\gamma q)$. Hence $(\chi_{F(e)}, A)$ is a prime fuzzy soft Γ- ideal of S.
Conversely, assume that $(\chi_{F(e)}, A)$ is a prime fuzzy soft Γ- ideal of S. Let $p\gamma q \in F(e)\ \forall\ e \in A$. Then $\chi_{F(e)}(p\gamma q) = 1$ and by hypothesis we have, $\chi_{F(e)}(p) = \chi_{F(e)}(p\gamma q) = 1$ and $\chi_{F(e)}(q) = \chi_{F(e)}(p\gamma q) = 1$. It follows that $\chi_{F(e)}(p) = 1$,$\chi_{F(e)}(q) = 1$ and so $p, q \in F(e)$. Hence $F(e)$ is prime soft Γ-ideal of S. Hence (F, A) is a prime soft Γ-ideal of S. □

Theorem 3.4. *If $(F, A), (G, B)$ and (H, C) be fuzzy soft sets over a Γ-semigroup S such that $(F, A) \subseteq (G, B)$. If $B \cap C$ is non-empty or $C \subseteq A$ then $(F\tilde{\circ}H, A \cup C) \subseteq (G\tilde{\circ}H, B \cup C)$ and $(H\tilde{\circ}F, C \cup A) \subseteq (H\tilde{\circ}G, C \cup B)$.*

Proof. Note that $A \cup C \subseteq B \cup C$. Suppose $B \cap C$ is non-empty, let $e \in A \cup C$. If $e \in A, e \notin C$, we have $e \in B \cup C$, but $e \neq C$, and hence $e \in B$. Therefore $(F\tilde{\circ}H)(e) = F(e) \leq G(e) = (G\tilde{\circ}H)(e)$. Hence $(F\tilde{\circ}H)(e) \leq (G\tilde{\circ}H)(e)$. If $e \in C, e \notin A$ thus $(F\tilde{\circ}H)(e) = H(e)$ and $(G\tilde{\circ}H)(e) = H(e)$. Assume that $C \subseteq A$. If $e \in A \cap C$, then $(F\tilde{\circ}H)(e) = F(e)\tilde{\circ}H(e) \leq G(e)\tilde{\circ}H(e) = (G\tilde{\circ}H)(e)$. Therefore $(F\tilde{\circ}H)(e) \leq (G\tilde{\circ}H)(e)$. If $e \in A, e \notin C$, we have $e \in B \cup C$, but $e \neq C$, hence $e \in B$. Therefore $(F\tilde{\circ}H)(e) = F(e) \leq G(e) = (G\tilde{\circ}H)(e)$. Hence $(F\tilde{\circ}H)(e) \leq (G\tilde{\circ}H)(e)$. Therefore $(F\tilde{\circ}H), A \cap C) \subseteq (G\tilde{\circ}H), B \cap C)$. Similarly $(H\tilde{\circ}F, C \cup A) \subseteq (H\tilde{\circ}G), C \cup B)$. □

Note that, converse of Theorem 3.4 is not true in general, which is shows in the following example.

Example 3.5. *Let $S = \{z_1, z_2, z_3, z_4\}$ and $\Gamma = \{\alpha, \beta\}$, where α, β is defined on S with the following Cayley table:*

α	z_1	z_2	z_3	z_4
z_1	z_1	z_1	z_1	z_1
z_2	z_1	z_1	z_2	z_1
z_3	z_1	z_1	z_3	z_1
z_4	z_1	z_2	z_1	z_4

β	z_1	z_2	z_3	z_4
z_1	z_1	z_1	z_1	z_1
z_2	z_1	z_1	z_1	z_1
z_3	z_1	z_1	z_1	z_1
z_4	z_1	z_2	z_1	z_4

Consider $E = \{e_1, e_2, e_3, e_4\}$,and $A = \{e_1, e_2\}, B = \{e_1, e_2, e_3\}, C = \{e_1, e_3\}$, $B \cap C \neq \phi$ and $C \nsubseteq A$, consider fuzzy soft sets over a soft Γ-semigroup S.
$(F, A) = \{F(e_1) = \{(z_1, 0.8), (z_2, 0.5), (z_3, 0.7), (z_4, 0.5)\}, F(e_2) = \{(z_1, 0.7), (z_2, 0.3), (z_3, 0.5), (z_4, 0.3)\}\}$; $(G, B) = \{G(e_1) = \{(z_1, 0.5), (z_2, 0.2), (z_3, 0.4), (z_4, 0.2)\}, G(e_2) = \{(z_1, 0.6), (z_2, 0.2), (z_3, 0.4), (z_4, 0.2)\}, G(e_3) = \{(z_1, 0.5), (z_2, 0.1), (z_3, 0.3), (z_4, 0.1)\}\}$; $(H, C) = \{H(e_1) = \{(z_1, 0.9), (z_2, 0.6), (z_3, 0.8), (z_4, 0.6)\}, H(e_3) = \{(z_1, 0.6), (z_2, 0.3),$

$(z_3, 0.4), (z_4, 0.3)\}\}$. *Consider* $(F, A)\tilde{\circ}(H, C) = (F\tilde{\circ}H, A \cup C)$.

$(F\tilde{\circ}H, A \cup C) = \{\{(F\tilde{\circ}H)(e_1) = (z_1, 0.8), (z_2, 0.5), (z_3, 0.7), (z_4, 0.5) = (H\tilde{\circ}F)(e_1); \{(F\tilde{\circ}H)(e_2) = (z_1, 0.7), (z_2, 0.3), (z_3, 0.5), (z_4, 0.3) = (H\tilde{\circ}F)(e_2)\}; \{(F\tilde{\circ}H)(e_3) = (z_1, 0.6), (z_2, 0.3), (z_3, 0.4), (z_4, 0.3) = (H\tilde{\circ}F)(e_3)\}\}$ *and consider* $(G\tilde{\circ}H, B \cup C) = \{\{(G\tilde{\circ}H)(e_1) = (z_1, 0.5), (z_2, 0.2), (z_3, 0.4), (z_4, 0.2) = (H\tilde{\circ}G)(e_1), \{(G\tilde{\circ}H)(e_2) = (z_1, 0.6), (z_2, 0.2), (z_3, 0.4), (z_4, 0.2) = (H\tilde{\circ}G)(e_2)\}, \{(G\tilde{\circ}H)(e_3) = (z_1, 0.5), (z_2, 0.1), (z_3, 0.3), (z_4, 0.1) = (H\tilde{\circ}G)(e_3)\}\}$. *Since* $(F\tilde{\circ}H)(e_i) \nleq (G\tilde{\circ}H)(e_i), (H\tilde{\circ}F)(e_i) \nleq (H\tilde{\circ}G)(e_i)\ \forall\ i = 1, 2, 3$. *Hence* $(F\tilde{\circ}H, A \cup C) \nsubseteq (G\tilde{\circ}H, B \cup C), (H\tilde{\circ}F, C \cup A) \nsubseteq (H\tilde{\circ}G, C \cup B)$.

Theorem 3.6. *A soft set* (F, A) *over* S *is a semiprime soft* Γ*-ideal of* S *if and only if* $(\chi_{F(e)}, A)$ *is a semiprime fuzzy soft* Γ*- ideal of* S.

Proof. Suppose that (F, A) is a semiprime soft Γ- ideal of S . Since $F(e)$ is a Γ-semiprime ideal of S.Let $a \in S$ such that $p\gamma p \in F(e)$, then $\chi_{F(e)}(p\gamma p) = 1\ \forall\ e \in A$. Since $F(e)$ is a Γ-semiprime ideal of $S, p\gamma q \in F(e) \Rightarrow p \in F(e)$ and $q \in F(e)$ and $\chi_{F(e)}(p) = 1 = \chi_{F(e)}(p\gamma p)$. If $p\gamma p \neq F(e)$ then $\chi_{F(e)}(p) = 0 = \chi_{F(e)}(p\gamma p)$. Hence $(\chi_{F(e)}, A)$ is a semiprime fuzzy soft Γ-ideal of S.

Conversely assume that $(\chi_{F(e)}, A)$ is a fuzzy soft Γ-semiprime ideal of S. Let $p\gamma p \in F(e),\ \forall\ e \in A$. Then $\chi_{F(e)}(p\gamma p) = 1$ and by hypothesis we have, $\chi_{F(e)}(p) = \chi_{F(e)}(p\gamma p) = 1$, it follows that $\chi_{F(e)}(p) = 1$,which implies $p \in F(e)$. Hence $F(e)$ is semiprime soft Γ-ideal of S,therefore (F, A) is a semiprime Γ-ideal of S. □

Theorem 3.7. *Let* $(F, A), (G, B)$ *and* (H, C) *be fuzzy soft -bi-ideals over a* Γ*-semigroup* S*, suppose that* (F, A) *is a prime fuzzy soft* Γ*-bi-ideal with* $(G\tilde{\circ}H, B \cup C) \subseteq (F, A)$*, then the following conditions are satisfied.*

(i). If $|(B \cap C)|$ *is empty , then* $(G, B) \subseteq (F, A)$ *and* $(H, C) \subseteq (F, A)$.

(ii). If $|(B \cap C)|$ *is non-empty, then* $(G, B) \subseteq (F, A)$ *or* $(H, C) \subseteq (F, A)$.

Proof. Suppose that $(G\tilde{\circ}H, B\cup C) \subseteq (F, A)$, that is $B\cup C\ \subseteq A$ and $(G\tilde{\circ}H)(e) \leq F(e)$ for all $e \in B\cup C$. Since $B\cup C \subseteq A$, implies that $B \subseteq A$ and $C \subseteq A$.

(i). Assume that $|(B \cap C)|$ is empty ,$e \in B$,then $G(e) = (G\tilde{\circ}H)(e) \leq F(e)$. That is $(G, B) \subseteq (F, A)$, and similarly $e^{'} \in C$, then $H(e^{'}) = (G\tilde{\circ}H)(e^{'}) \leq F(e^{'})$. Therefore $(H, C) \subseteq (F, A)$.

(ii). Assume that $|(B \cap C)|$ is not empty , suppose $(G, B) \nsubseteq (F, A)$, then there exists $e^{'} \in B$ such that $G(e^{'}) \nleq F(e^{'})$. If $e^{'} \notin C$ then $G(e^{'}) = (G\tilde{\circ}H)(e^{'}) \leq F(e^{'})$.Therefore $q \in B \cap C$.Let $e^{"} \in C$.

Claim: $H(e^{"}) \leq F(e^{"})$. If $e^{"} \in C$,and $e^{"} \notin B$, then $H(e^{"}) = (G\tilde{\circ}H)(e^{"}) \leq F(e^{"})$. If $e^{"} \in C$,and $e^{"} \in B$, then $e^{"} = e^{'}$. Hence $G(e^{"})\tilde{\circ}H(e^{"}) = (G\tilde{\circ}H)(e^{"}) \leq F(e^{"})$.Since (F, A) is a prime fuzzy soft Γ-bi-ideal over a Γ semigroup S.That is $G(e^{"}) \leq F(e^{"})$ or $H(e^{"}) \leq F(e^{"})$,but $G(e^{"}) \nleq F(e^{"})$,and hence $H(e^{"}) \leq F(e^{"})$. Therefore $(H, C) \subseteq (F, A)$. □

Example 3.8. *From the Example 3.5, consider* $E = \{e_1, e_2, e_3, e_4\}$*, and* $A = \{e_1, e_2, e_3, e_4\}$, $B = \{e_1, e_2\}$, $C = \{e_1, e_2, e_4\}$. *Let* $(F, A) = \{F(e_1) = \{(z_1, 0.8), (z_2, 0.4), (z_3, 0.2), (z_4, 0.5)\}, F(e_2) = \{(z_1, 0.9), (z_2, 0.5), (z_3, 0.1), (z_4, 0.7)\}, F(e_3) = \{(z_1, 1), (z_2, 0.6), (z_3, 0.5), (z_4, 0.8)\}, F(e_4) = \{(z_1, 0.7), (z_2, 0.5), (z_3, 0.4), (z_4, 0.6)\}\}$. *Let* $(G, B) = \{G(e_1) = \{(z_1, 0.9), (z_2, 0.7), (z_3, 0.5), (z_4, 0.8)\}, G(e_2) = \{(z_1, 0.8), (z_2, 0.4), (z_3, 0.1), (z_4, 0.7)\}\}$. *Let* $(H, C) = \{H(e_1) = \{(z_1, 0.7), (z_2, 0.3), (z_3, 0.1), (z_4, 0.4)\}, H(e_2) = \{(z_1, 0.6), (z_2, 0.3), (z_3, 0), (z_4, 0.5)\}, H(e_4) = \{(z_1, 0.6), (z_2, 0.4), (z_3, 0.2), (z_4, 0.5)\}\}$. *Consider* $(G, B)\tilde{\circ}(H, C) = (G\tilde{\circ}H, B \cup C)$.

$(G\tilde{\circ}H, B \cup C) = \{(G\tilde{\circ}H)(e_1) = \{(z_1, 0.7), (z_2, 0.3), (z_3, 0.1), (z_4, 0.4)\}, (G\tilde{\circ}H)(e_2) = \{(z_1, 0.6), (z_2, 0.3), (z_3, 0), (z_4, 0.5)\}, (G\tilde{\circ}H)(e_4) = \{(z_1, 0.6), (z_2, 0.4), (z_3, 0.2), (z_4, 0.5)\}\}$. *Thus* $(G\tilde{\circ}H, B \cup C) \subseteq (F, A)$*, since* $|(B \cap C)|$ *is non-empty,and* $(G, B) \nsubseteq (F, A)$ *hence* $(H, C) \subseteq (F, A)$

4. Fuzzy Soft Γ-left Quasi Regular Semigroups

In this section S denotes fuzzy soft Γ- left quasi regular semigroup.

Definition 4.1. *A soft Γ-semigroup S is called soft left quasi regular if every soft Γ-left ideal of S is idempotent.*

Example 4.2. *Let $S = \{z_1, z_2, z_3, z_4\}$ and $\Gamma = \{\alpha, \beta, \gamma\}$, where α, β, γ is defined on S with the following Cayley table:*

α	z_1	z_2	z_3	z_4
z_1	z_1	z_1	z_1	z_1
z_2	z_1	z_2	z_3	z_4
z_3	z_1	z_3	z_3	z_3
z_4	z_1	z_3	z_3	z_3

β	z_1	z_2	z_3	z_4
z_1	z_1	z_1	z_1	z_1
z_2	z_1	z_2	z_3	z_4
z_3	z_1	z_3	z_3	z_3
z_4	z_1	z_2	z_3	z_4

γ	z_1	z_2	z_3	z_4
z_1	z_1	z_1	z_1	z_1
z_2	z_1	z_3	z_3	z_3
z_3	z_1	z_3	z_3	z_3
z_4	z_1	z_2	z_3	z_4

Consider $E = \{e_1, e_2, e_3, e_4\}$, and $A = \{e_1, e_2, e_3, e_4\}F(e_1) = \{z_1, z_3\}, F(e_2) = \{z_1, z_2, z_3\}$,$F(e_3) = \{z_1, z_3, z_4\}$, $F(e_4) = \{z_1, z_2, z_3, z_4\}$. Here (F, S) is soft Γ-left quasi regular semigroup.

Theorem 4.3. *Let (F, A) be a soft Γ-left quasi regular then every fuzzy soft Γ-left ideal is idempotent.*

Proof. Let (F, A) be a soft Γ-left quasi regular semigroup and $F(e)$ be fuzzy soft Γ-left ideal of S. Let $a \in S$ there exists $p, q \in S$ such that $a = p\alpha a\beta q\gamma a$, and $\alpha, \beta, \gamma \in \Gamma$, $\forall\ e \in A$. Thus

$$\begin{aligned}(F(e)\tilde{\circ}F(e))(a) &= \sup_{a=x\Gamma y} \min\{F(e)(x), F(e)(y)\}\\ &\geq \min\{F(e)(p\alpha a), F(e)(q\gamma a)\}\\ &\geq \min\{F(e)(a), F(e)(a)\}\\ &= F(e)(a)\end{aligned}$$

That is $(F, A)\tilde{\circ}(F, A) \supseteq (F, A)$. Since $F(e)$ is a fuzzy soft Γ- left ideal of S,then $(F, A)\tilde{\circ}(F, A) \subseteq (F, A)$. Hence $(F, A)\tilde{\circ}(F, A) = (F, A)$,this impies that (F, A) is a soft idempotent. □

Theorem 4.4. *If (F, A) is both soft Γ-intra regular and soft Γ-left quasi regular,then $(F, A) \cap_\epsilon (G, A) \cap_\epsilon (H, B) \subseteq (F, A)\tilde{\circ}(G, A)\tilde{\circ}(H, B)$ for every fuzzy soft Γ-generalized bi-ideal, (H, B) every fuzzy soft Γ-left ideal,(F, A) and every fuzzy soft Γ-right ideal of (G, A) of S.*

Proof. Assume that (F, A) is both soft Γ-intra regular and soft Γ-left quasi regular, let (H, B) be any fuzzy soft Γ-generalized bi-ideal, (F, A) and every fuzzy soft Γ-left ideal, (G, A) be fuzzy soft Γ-right ideal of S. Let $a \in S, (F, A) \cap_\epsilon (G, A) \cap_\epsilon (H, B) = K_1, A \cup B$ and $(F, A) \cap_\epsilon (G, A) \cap_\epsilon (H, B) = K_2, A \cup B$ for any $e \in A \cup B$. By using the cases
Case 1: If $e \in A - B$ then $K_1(e) = A \cap B \cap (H, B) = K_2(e), A \cap B\ \forall\ e \in A \cap B$, we have $K_1(e) = K_2(e)$
Case 2: If $e \in B - A$ then $K_1(e) = H(e) = K_1(e)$
Case 3: If $e \in A \cap B$ then $K_1(e) = F(e) \cap G(e) \cap H(e)$ and $K_2(e) = F(e)\tilde{\circ}G(e)\tilde{\circ}H(e)$.
To prove $(F, A) \cap_\epsilon (G, A) \cap_\epsilon (H, B) \subseteq (F, A)\tilde{\circ}(G, A)\tilde{\circ}(H, B)$, since S is soft Γ-intra regular, then there exists $p, q \in S$ such that $a = p\delta a\lambda a\eta q$ and since S is soft Γ-left quasi regular, then there exists $m, n \in S$ such that $a = m\alpha a\beta n\gamma a = m\alpha(p\delta a\lambda a\eta q)\beta n\gamma a = ((m\alpha p)\delta a)(\lambda a\eta(q\beta n)\gamma a)$. Consider

$$\begin{aligned}(F(e)\tilde{\circ}G(e)\tilde{\circ}H(e))(x) &= \sup_{a=x\Gamma y} \min\{F(e)(x), G(e)\tilde{\circ}H(e))(y)\}\\ &\geq \min\{F(e)((m\alpha p)\delta a), (G(e)\tilde{\circ}H(e))((\lambda a\eta(q\beta n)\gamma a)\}\end{aligned}$$

$$\begin{aligned} &\geq \min\left\{F(e)(a), \sup_{a=x\Gamma y} \min\{G(e)(x), H(e)(y)\}\right\} \\ &\geq \min\{F(e)(a), G(e)(\lambda a\eta q\beta n), H(e)(a)\}\} \\ &\geq \min\{F(e)(a), \min\{G(e), H(e)(a)\}\} \\ &= \min\{F(e), G(e), H(e)\}(a) \\ &= (F(e) \cap G(e) \cap H(e))(a) \end{aligned}$$

Therefore $(F, A) \cap_\epsilon (G, A) \cap_\epsilon (H, B) \subseteq (F, A)\tilde{\circ}(G, A)\tilde{\circ}(H, B)$. □

Definition 4.5. *A soft Γ-regular semigroup (F, A) is called fuzzy soft duo if it is fuzzy soft Γ- left duo and fuzzy soft Γ-right duo.*

Example 4.6. *From the Example 4.2 (β and γ). Consider $E = \{e_1, e_2, e_3, e_4\}$ and $A = \{e_1, e_2, e_3, e_4\}$, $F(e_1) = F(e_3) = \{z_1, z_3\}$, $F(e_2) = F(e_4) = \{z_1, z_2, z_3, z_4\}$. Hence (F, S) is soft regular duo Γ- semigroup.*

Theorem 4.7. *In a soft regular right duo(left duo,duo) Γ-semigroup S, the following conditions are equivalent*

(i). (F, A) is a fuzzy soft Γ-left ideal(right) of S.

(ii). (F, A) is a fuzzy soft Γ- bi-ideal of S.

Proof. Let $F(e)$ be a fuzzy soft left Γ-ideal of S, and let $p, q, r \in S, \alpha, \beta \in \Gamma$ and $e \in A$. Consider

$$\begin{aligned} F(e)(p\alpha q\beta r) &= F(e)((p\alpha q)\beta r) \\ &\geq F(e)(r) \\ &\geq \min\{F(e)(p), F(e)(r)\} \end{aligned}$$

Hence $F(e)$ is a fuzzy soft bi-ideal of S.

Conversely, let $F(e)$ be a fuzzy soft bi-ideal of S, and $p, q \in S, \gamma \in \Gamma$ and $e \in A$. Then $p\gamma q \in S$, since S is soft Γ-regular and right duo, then $p\gamma q \in S\Gamma(q\Gamma S\Gamma q) \subseteq q\Gamma S\Gamma q$, consequently then there exist $r \in S, \alpha, \beta \in \Gamma$ and $e \in A$ such that $p\gamma q = q\alpha r\beta q$. Consider

$$\begin{aligned} F(e)(p\gamma q) &= F(e)(q\alpha r\beta q) \\ &\geq \min\{F(e)(q), F(e)(q)\} \\ &= F(e)(q) \end{aligned}$$

Hence $F(e)$ is a fuzzy soft Γ-left ideal of S. Similarly we can prove that (ii). □

Example 4.8. *From example 4.6, $F(e_1) = \{(z_1, 0.4), (z_2, 0.1), (z_3, 0.2), (z_4, 0.1)\}$; $F(e_3) = \{(z_1, 0.5), (z_2, 0.2), (z_3, 0.3), (z_4, 0.2)\}$. Thus (F, A) is a fuzzy soft Γ-ideal and bi-ideal of regular duo Γ-semigroup S.*

Theorem 4.9. *In a soft regular left duo Γ-semigroup S, the following conditions are equivalent*

(i). (F, A) is a fuzzy soft Γ- bi-ideal of S.

(ii). (F, A) is a fuzzy soft Γ-(1, 2)-ideal of S.

Proof. Let $F(e)$ be a fuzzy soft Γ-bi-ideal of S, and let $p, s, q, r \in S, \alpha, \beta \in \Gamma$ and $a \in A$. Consider

$$\begin{aligned} F(e)(p\alpha s\beta(q\gamma r)) &= F(e)((p\alpha s\beta q)\gamma r) \\ &\geq \min\{F(e)(p\alpha s\beta q), F(e)(r)\} \\ &\geq \min\big\{\min\{F(e)(p), F(e)(q)\}, F(e)(r)\big\} \\ &= \min\{F(e)(p), F(e)(q), F(e)(r)\} \end{aligned}$$

Hence $F(e)$ is a fuzzy soft Γ-$(1,2)$-ideal of S.

Conversely, assume that $F(e)$ is a fuzzy soft Γ-$(1,2)$-ideal of S, and S be a soft regular left duo Γ-semigroup S. Let $p, s, q \in S, \alpha, \eta \in \Gamma$ and $e \in A$. Then $p\gamma q \in S$ since S is soft Γ-regular and left duo, then $p\gamma q \in S\Gamma(p\alpha s) \in (p\Gamma S\Gamma p)\Gamma S \subseteq (p\Gamma S\Gamma p)$, this equivalent to $p\alpha s = p\beta s\gamma p, s \in S$ and $\beta, \gamma \in \Gamma, e \in A$. Consider

$$\begin{aligned} F(e)(p\alpha s\eta q) &= F(e)((p\beta s\gamma p)\eta q) \\ &= F(e)(p\beta s\gamma(p\eta q)) \\ &\geq \min\{F(e)(p), F(e)(p), F(e)(q)\} \\ &= \min\{F(e)(p), F(e)(q)\} \end{aligned}$$

Therefore $F(e)$ is a fuzzy soft Γ-bi- ideal of S. □

Example 4.10. *Let $S = \{z_1, z_2, z_3, z_4\}$ and $\Gamma = \{\alpha, \beta\,\gamma\}$, where α, β, γ is defined on S with the following Cayley table:*

α	z_1	z_2	z_3	z_4
z_1	z_1	z_1	z_1	z_1
z_2	z_1	z_1	z_2	z_1
z_3	z_1	z_1	z_3	z_1
z_4	z_1	z_2	z_1	z_4

β	z_1	z_2	z_3	z_4
z_1	z_1	z_1	z_1	z_1
z_2	z_1	z_1	z_1	z_1
z_3	z_1	z_1	z_1	z_1
z_4	z_1	z_2	z_1	z_4

γ	z_1	z_2	z_3	z_4
z_1	z_1	z_1	z_1	z_1
z_2	z_1	z_1	z_1	z_1
z_3	z_1	z_1	z_1	z_1
z_4	z_1	z_1	z_2	z_1

Consider $E = \{e_1, e_2, e_3, e_4\}$ and $A = \{e_1, e_2, e_3\}$, $F(e_1) = \{z_1, z_2\}$, $F(e_2) = F(e_3) = \{z_1, z_2, z_3\}$. Hence (F, S) is soft regular duo Γ- semigroup. Consider $F(e_1) = \{(z_1, 0.6), (z_2, 0.2), (z_3, 0.1), (z_4, 0.4)\}$; $F(e_2) = \{(z_1, 0.7), (z_2, 0.3), (z_3, 0.2), (z_4, 0.5)\}$. Thus (F, A) is a fuzzy soft Γ-$(1,2)$-ideal and Γ-bi-ideal of regular duo Γ-semigroup S.

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References

[1] M. I. Ali,F. Feng, X. Y.Liu and M. Shabir, *On some new operations in soft set theory*, Comput Math. Appl., 57(2009), 1547-1553.

[2] V. Chinnadurai and K. Arulmozhi, *Fuzzy soft Γ- regular semigroups*, International Journal of Multidisciplinary Research and Modern Education, 2(3)(2016), 238-250.

[3] V. Chinnadurai, *Fuzzy ideals in algebraic structures*, LAP LAMBERT Academic publishing, (2013).

[4] Chinram and Jirojkul, *On bi-Γ-ideals in Γ-semigroups*, Songklanakarin Sci. Technol Journal., 29(2007), 231-234.

[5] T. K. Dutta and N.C. Adhikari, *On Prime Radical of Ã-Semigroup*, Bull. Cal. Math. Soc., 86(5)(1994), 437-444.

[6] N. Kuroki, *On fuzzy semigroups*, Inform. Sci., 53(1991), 203236.

[7] P. K. Maji, R. Biswas and R. Roy, *Soft set theory*, Comput. Math. Appl., 45(2003), 555-562.

[8] D. Molodtsov, *Soft set theory first results*, Comput. Math. Appl., 37(1999), 19-31.

[9] J. N. Mordeson, D. S. Malik and N. Kuroki, *Fuzzy semigroups*, Springer-Verlag Berlin Heidelberg GmbH, (2003).

[10] Muhammad Ifran Ali, Muhammad Shabir and K. P. Shum, *On soft ideals over semigroups*, Southeast Asian Bulletion of Mathematics, 34(2010), 595-610.

[11] T. K. Mukherjee and M. K. Sen, *On fuzzy ideals of a ring*, Proceedings, Seminar on Fuzzy Systems and Non-standard Logic, Calcutta, May, (1984).

[12] Munazza Naz, *On fuzzy soft semigroup*, World Applied Science Journal, 22(special issue of applied Math)(2013), 62-83.

[13] Rosenfeld, *Fuzzu groups*, J.Math. Anal. Appl., 35(1971), 512-517.

[14] M. K. Sen, *On Γ-semigroups*, Proceedings of International conference on Algebra and its Application Decker publication, New yark, 301(1981).

[15] M. K. Sen and N. K Saha, *On Γ- semigroup I*, Bull. Calcutta Math. Soc., 78(1986), 180-186.

[16] A. Sezgin Sezer, N. C Atgman, A. O. Atagun, M. I. Ali and E. Turkmen, *Title?*, Filomat, 29(5)(2015), 917946.

[17] Sujit Kumar Sardar, *On Fuzzy Ideals in Γ-Semigroups*, International Journal of Algebra, 3(16)(2009), 775-784.

[18] Thawhat Changphas and Boonyen Thongkam, *On soft Γ-semigroups*, Annals of Fuzzy Mathematics and Informatics, 4(2)(2012), 217-223.

[19] M. Uckun, M. A. Ozturk and Y. B. Jun, *Intuitionistic fuzzy sets in Γ-semigroups*, Bull. Korean Math. Soc., 44(2)(2007), 359367.

[20] L. A Zadeh, *Fuzzy sets*, Inform and Control, 8(1965), 338-353.