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A Study on Multi Server Queuing Model to Optimize the Performance of a Toll Plaza

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Abstract: In this paper we studied multi server queuing model and analyze the performance of toll plaza which is located at Khalghat, Agra-Bombay road (on NH 3). The results of the analysis showed that average queue length, waiting time of vehicle at toll plaza. In particular, we present the optimal number of toll booths to reduce the queue length and waiting time of vehicles.

Keywords: Toll plaza, queue length, waiting time, Queuing model.

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1. Introduction

Increasing traffic volume causes congestions commonly around the toll gate of highways [1]. When the first road covered with a layer of crushed stone was built in 1792 in Pennsylvania, the boom in road construction began. Over the years, the roads were built all over the country, and because of the decreasing federal support of existing and new freeways toll roads now begin to play an important role in the traffic system. The US transportation trust fund is rapidly shrinking and state departments of transportation around the US are facing budget shortages. In the last two rounds of federal highway program reauthorization, the use of toll roads have expanded and now is becoming more popular. Toll roads, in general, can generate funds for repayment of toll revenue bonds, thus the state can collect enough money to finance the operation, maintenance, improvement and construction of new facilities. By the end of year 2006, there were a total of 4917 miles of toll roads built in the United States, including 223 miles of urban toll roads and 2695 miles of rural (Toll Facilities in the United States). A toll plaza is the essential part of toll roads where the toll is collected. There are three difference basic options for tolling: Manual Toll Collection, which has been the most common approach for collecting tolls. In this option, drivers are required to stop and pay a toll collector sitting or standing in a tollbooth [2]. Toll plaza system increasing traffic volume makes congestion commonly around the tollgates of Highway. So, reform measure of congestion around the tollgates is urgently required. The current system for collecting toll is on the basis of manual transaction. In this each vehicle has to stop at the toll plaza for payment. It causes traffic congestion, increase in pollution, and wasting time of people. The goal is to implement the reliable system that leads to:

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- Saving the time at toll plaza for toll collection.
- Reducing traffic congestion and increases security concerns [3].

National Highway 3 (NH 3) commonly referred to as the Mumbai-Agra Highway is a major Indian National Highway that connects the states of Uttar Pradesh, Rajasthan, Madhya Pradesh and Maharashtra in India. The highway originates in Agra in Uttar Pradesh, generally travels southwest through Dhaulpur in Rajasthan, Morena, Gwalior, Shivpuri, Guna, Biaora, Maksi, Dewas and Indore in Madhya Pradesh, and Dhule, Nashik, Thane and terminates at Mumbai in Maharashtra. NH 3 runs for a distance of 1,190 km. The aim of this paper is to study multi server queuing model and analyze the performance of toll plaza which is located at Khalghat, Agra- Bombay road (on NH 3). The results of the analysis showed that average queue length, waiting time of vehicle at toll plaza. In particular, we present the optimal number of toll booths to reduce the queue length and waiting time of vehicles.

2. Queuing System and Mathematical Model Analysis

2.1. The basic indexes of the queuing systems

n = Number of customers in the system

λ = Mean arrival rate

μ = Mean service rate per busy server

ρ = Expected fraction of time for which server is busy

P_n = Steady state probability of exactly n customers in the system

L_q = Expected number of customers waiting in the queue (i.e. queue length)

L_s = Expected number of customers in the system (waiting + being served)

w_q = Expected waiting time for a customer in the queue

W_s = Expected waiting time for a customer in the system (waiting + being served)

2.2. $M/M/S$ Model (Multi server queuing system)

For this queuing system, it is assumed that arrivals follow a Poisson probability distribution at an average rate of λ customers per unit of time and are served on a first come first served basis by any of the servers. The service times are distributed exponentially with an average of μ customers per unit of time. It is further assumed that only one queue is formed. If there are n customers in the queuing system at any point in time, then following two cases may arise:

(1). If n < s (number of customers in the system is less than the number of servers), then there will be no queue. However, $(s-n)$ numbers of servers are not busy. The combined service rate will then be: $\mu_n = n\mu$; $n < s$.

(2). If $n \geq s$ (number of customers in the system is more than or equal to the number of servers) then all servers will be busy and the maximum number of customers in the queue will be $(n-s)$. The combined service rate will be $\mu_n = s\mu$; $n \geq s$.

Thus to derive the result for this model, we have

$$\lambda_n = \lambda \quad forall \quad n \geq 0$$

$$\mu_n = \begin{cases} n\mu; & n < s \\ \mu_n = s\mu; & n \geq s \end{cases}$$

The probability of$n$ customers in the queuing system is given by

$$P_n = \begin{cases} \frac{\rho^n}{n!} P_0; & n \le s \\ \frac{\rho^n}{s! s^{(n-s)}} P_0; & n > s \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\lambda/\mu \right)^n + \frac{1}{s!} \left(\lambda/\mu \right)^s \left(\frac{s\mu}{s\mu - \lambda} \right) \right]^{-1}$$

Expected number of customers waiting in the queue (i.e. queue length)

$$L_q = \left[\frac{1}{(s-1)!} \left(\lambda/\mu \right)^s \left(\frac{\lambda\mu}{(s\mu - \lambda)^2} \right) \right] P_0$$

Expected number of customers in the system

$$L_s = L_q + \frac{\lambda}{\mu}$$

Expected waiting time of a customer in the queue

$$W_q = \frac{L_q}{\lambda}$$

Expected waiting time that a customer spends in the system

$$W_s = W_q + \frac{1}{\mu}$$

3. Analysis of Data

The data were obtained from Khalghat toll plaza, A. B road (on NH 3) Khalghat, M.P., India through Personal Observation on toll plaza. We use TORA software to compute the performance measures of the multi- server queuing model system at Khalghat toll plaza using data

Indore to Bombay Saturday										
Time	Server 1		Server 2		Server 3		Server 4		Server 5	
	Arrival rate	Service rate	Arrival rate	Service rate	Arrival rate	Service rate	Arrival rate	Service rate	Arrival rate	Service rate
9-10 AM	62	40	78	52	50	42	38	26	82	72
10-11 AM	58	40	70	54	60	41	44	28	86	79
11-12 Noon	58	40	70	44	48	40	34	26	66	56
12-1 PM	64	40	66	44	58	40	46	26	62	50
1-2 PM	96	56	90	59	78	40	64	34	44	40
2-3 PM	142	92	132	99	110	71	130	109	98	80
	480	308	506	352	404	274	356	249	438	377

Table 1. **Summary of the data server 1 to server 5**

Total number of vehicles arrived in six hours is 1746 (291 per hour)

Total number of vehicles served in six hours is 1560 (260 per hour)

Average arrival rate $\lambda = 291$ per hour

Average service rate $\mu = 260/5 = 52$ per hour

$$\rho < 1 \quad i.e., \quad \frac{\lambda}{S\mu} < 1, \quad \frac{291}{S(52)} < 1, \quad S > 5.59 \cong 6$$

Here the minimum no. of toll booths are required more than five ($S > 5$)

QUEUEING OUTPUT ANALYSIS

Title: hgg
Comparative Analysis

Scenario	c	Lambda	Mu	L'da eff	p0	Ls	Lq	Ws	Wq
1	6	291.00000	52.00000	291.00000	0.00130	16.97459	11.37843	0.05833	0.03910
2	7	291.00000	52.00000	291.00000	0.00285	7.52865	1.93250	0.02587	0.00664
3	8	291.00000	52.00000	291.00000	0.00340	6.22434	0.62819	0.02139	0.00216

Table 2. **Summary of the performance measures of queuing model**

Bombay to Indore Saturday										
Time	Server 6		Server 7		Server 8		Server 9		Server 10	
	Arrival rate	Service rate	Arrival rate	Service rate	Arrival rate	Service rate	Arrival rate	Service rate	Arrival rate	Service rate
9-10 AM	106	96	62	58	72	62	106	90	76	62
10-11 AM	98	90	64	60	76	66	120	102	70	58
11-12 Noon	84	76	68	64	76	64	98	82	72	58
12-1 PM	78	72	76	68	84	78	102	90	82	70
1-2 PM	58	56	52	28	68	40	88	60	102	76
2-3 PM	78	70	92	76	124	106	92	76	120	106
	502	460	414	354	500	416	606	500	522	430

Table 3. **Summary of the data server 6 to server 10**

The average waiting time of the vehicle in the system is 0.05833 hrs (3.5 min.) and the average waiting time of the vehicle in the queue is 0.0391 hrs (2.34 min.) when six tool booths are available.

Total number of vehicles arrived in six hours is 2544 (424 per hour).

Total number of vehicles served in six hours is 2160 (360 per hour).

Average arrival rate $\lambda = 424$ per hour.

Average service rate $\mu = 360/5 = 72$ per hour.

$$\rho < 1 \;\; i.e., \;\; \frac{\lambda}{S\mu} < 1, \;\; \frac{424}{S(72)} < 1, \;\; S > 5.88 \cong 6$$

Here the minimum no. of toll booths are required more than five ($S > 5$).

QUEUEING OUTPUT ANALYSIS

Title: Bombay to Indore
Comparative Analysis

Scenario	c	Lambda	Mu	L'da eff	p0	Ls	Lq	Ws	Wq
1	6	424.00000	72.00000	424.00000	0.00030	56.19666	50.30778	0.13254	0.11865
2	7	424.00000	72.00000	424.00000	0.00188	8.94553	3.05665	0.02110	0.00721
3	8	424.00000	72.00000	424.00000	0.00244	6.81376	0.92487	0.01607	0.00218

Table 4. **Summary of the performance measures of queuing model**

The average waiting time of the vehicle in the system is 0.13254 hrs (7.9 min.) and the average waiting time of the vehicle in the queue is 0.11865 hrs (7.12 min.) when six tool booths are available.

4. Conclusion

From the Table 2 the average waiting time of the vehicle in the system is 0.05833 hrs (3.5 min.) and the average waiting time of the vehicle in the queue is 0.0391 hrs (2.34 min.) when six tool booths are available. From the table 4 the average waiting time of the vehicle in the system is 0.13254 hrs (7.9 min.) and the average waiting time of the vehicle in the queue is 0.11865 hrs (7.12 min.) when six tool booths are available. After studying and analyzing the data the average waiting time of the vehicle in system is exceeds three minutes this is leads to inconvenience and dissatisfaction to the customers. This paper strongly recommends that to increase the number of toll booths instead of five to at least six on the both sides to avoid inconvenience and dissatisfaction to the customers.

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