Int. J. Math. And Appl., 6(2–B)(2018), 377–384
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Solving Fuzzy Fractional Differential Equation with Fuzzy Laplace Transform Involving Trigonometric Function

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Abstract: This paper deals with fuzzy Laplace transform to obtain the solution of fuzzy fractional differential equation (FFDEs) under Caputo's H-differentiability. In order to solve FFDEs, first we have to understand about fuzzy Laplace transform of Caputo's H-differentiability with fractional order $(0 < \beta < 1)$. An analytical solution is presented to confirm the capability of proposed method.

MSC: 34A07, 34A08.

Keywords: Fuzzy fractional differential equation, Fuzzy Laplace transforms, trigonometric function, hyper geometric function.

 Accepted on: 05.02.2018

1. Introduction

Fractional calculus is a mathematical branch investigating the properties of derivatives and integrals of non-integer orders. It has been applied in modelling physical and chemical processes and in engineering [4, 6, 18] Podlubny and kilbas[10,12,] gave the idea of fractional calculus and consider Caputo's differentiability to solve FFDEs. Agarwal [2] proposed the concept of solutions for fractional differential equations with uncertainty. Laplace transform is used for solving differential equations. To solve fuzzy fractional differential equation, fuzzy initial and boundary value problems, we use fuzzy Laplace transform. The advantage of fuzzy Laplace transform is that it solves the problem directly without determining a general solution. Here we have seen some basic definitions section 2. In section 3, Caputo H-differentiability is introduced and some of the properties are considered. Fuzzy Laplace transforms are introduced and we discuss the properties in section4. The solutions of FFDEs are determined by fuzzy Laplace transform under Caputo H-differentiability and solve the example in section 5. In section 6, a conclusion is drawn.

2. Preliminaries

Definition 2.1 ([8]). *Fuzzy number is a mapping $u : R \to [0, 1]$ with the following properties:*

(1). u is upper semi continuous.

(2). u is fuzzy convex. i.e., $u(\lambda x + (1 - \lambda) y) \geq \min\{u(x), u(y)\}$ for all $x, y \in R$, $\lambda \in [0, 1]$.

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(3). u is normal. i.e., $\exists\ x_0 \in R$ *for which* $u(x_0) = 1$.

(4). Supp $u = \{x \in R/u(x) > 0\}$ *is the support of the u, and its closure cl (supp u) is compact.*

Definition 2.2 ([13, 14]). *A fuzzy number u in parametric form is a pair* $(\underline{u}, \overline{u})$ *of functions* $\underline{u}(r), \overline{u}(r)$, $0 \leq r \leq 1$, *which satisfy the following requirements:*

(1). $\underline{u}(r)$ *is a bounded non-decreasing left continuous function in (0,1], and right continuous at 0,*

(2). $\overline{u}(r)$ *is a bounded non-increasing left continuous function in (0,1], and right continuous at 0,*

(3). $\underline{u}(r) \leq \overline{u}(r)$, $0 \leq r \leq 1$.

Theorem 2.3 ([15]). *Let f be fuzzy valued function on* $[a, \infty)$ *represented by* $\left(\underline{f}(x;r), \overline{f}(x;r)\right)$. *For any fixed* $r \in [0,1]$, *assume* $\underline{f}(x;r)$ *and* $\overline{f}(x;r)$ *are Riemann-integrable on* $[a,b]$ *for every* $b \geq a$, *and assume there are two positive functions* $\underline{M}(r)$, $\overline{M}(r)$ *such that* $\int_a^b \left|\underline{f}(x;r)\right| dx \leq \underline{M}(r)$ *and* $\int_a^b \left|\overline{f}(x;r)\right| dx \leq \overline{M}(r)$ *for every* $b \geq a$. *Then* $f(x)$ *is improper fuzzy Riemann integrable on* $[a, \infty)$ *and the improper fuzzy Riemann integral is a fuzzy number. Furthermore, we have* $\int_a^\infty f(x;r)dx = \left[\int_a^\infty \underline{f}(x;r)\, dx, \int_a^\infty \overline{f}(x;r)\, dx\right]$.

Definition 2.4. *Let* $x, y \in E$. *If there exists* $z \in E$ *such that* $x = y + z$, *then z is called the H-difference of x and y and it is denoted by* $x \ominus y$.

3. Caputo's H-differentiability [7]

$C^F[a,b]$ is the space of all continuous fuzzy valued functions on $[a,b]$. Also we denote the space of all Lebesgue integrable fuzzy valued functions on $[a,b]$ by $L^F[a,b]$. In this section, concept of the fuzzy Caputo derivatives as in [7] is revisited using hukuhara difference $\left({}^C D_{a+}^{\beta} f\right)(x)$ are as defined as:

Let $f \in C^F[a,b] \cap L^F[a,b]$ be a fuzzy set value fnction; then f is Caputo fuzzy H-differentiable at x when

$$\left({}^C D_{a+}^{\beta} f\right)(x) = \left[\frac{1}{G(1-\beta)} \int_a^x \frac{f'(t)\, dt}{(x-t)^{\beta}}\right].$$

Definition 3.1. *Let* $f \in C^F[a,b] \cap L^F[a,b]$, x_0 *in* (a,b) *and* $F(x) = \frac{1}{G(1-\beta)} \int_a^x \frac{f(t)dt}{(x-t)^{\beta}}$. *We say that f is Caputo's H-differentiable about order* $0 < \beta < 1$ *at* x_0, *if there exists an element* $({}^C D_{a+}^{\beta} f)(x_0) \in E$ *such that for* $h > 0$ *sufficiently small*

(1). $({}^C D_{a+}^{\beta} f)(x_0) = \lim\limits_{h \to 0^+} \frac{F(x_0+h) \ominus F(x_0)}{h} = \lim\limits_{h \to 0^+} \frac{F(x_0) \ominus F(x_0-h)}{h}$ *(or)*

(2). $({}^C D_{a+}^{\beta} f)(x_0) = \lim\limits_{h \to 0^+} \frac{F(x_0) \ominus F(x_0+h)}{-h} = \lim\limits_{h \to 0^+} \frac{F(x_0-h) \ominus F(x_0)}{-h}$ *(or)*

(3). $({}^C D_{a+}^{\beta} f)(x_0) = \lim\limits_{h \to 0^+} \frac{F(x_0+h) \ominus F(x_0)}{h} = \lim\limits_{h \to 0^+} \frac{F(x_0-h) \ominus F(x_0)}{-h}$ *(or)*

(4). $({}^C D_{a+}^{\beta} f)(x_0) = \lim\limits_{h \to 0^+} \frac{F(x_0) \ominus F(x_0+h)}{-h} = \lim\limits_{h \to 0^+} \frac{F(x_0) \ominus F(x_0-h)}{h}$.

We say that the fuzzy valued function f is $({}^C(i) - \beta)$ *differentiable if it is differentiable as in the Definition 2.5 Case (i), and f is* $({}^C(ii) - \beta)$ *differentiable if it is differentiable as in the Definition 2.5 of Case (2) and so on for other cases.*

Theorem 3.2 ([17]). *Let* $f \in C^F[a,b] \cap L^F[a,b]$, x_0 *in* (a,b) *and* $0 < \beta < 1$. *Then*

(1). Let us consider f is $({}^C(i) - \beta)$ *differentiable fuzzy valued function, then*

$$\left({}^C D_{a+}^{\beta} f\right)(x_0;r) = \left[\left({}^C D_{a+}^{\beta} \underline{f}\right)(x_0;r), \left({}^C D_{a+}^{\beta} \overline{f}\right)(x_0;r)\right], \quad 0 \leq r \leq 1$$

(2). Let us consider f is $({}^{C}(ii)-\beta)$ differentiable fuzzy valued function, then

$$\left({}^{C}D^{\beta}_{a+}f\right)(x_0;r)=\left[\left({}^{C}D^{\beta}_{a+}\underline{f}\right)(x_0;r),\left({}^{C}D^{\beta}_{a+}\overline{f}\right)(x_0;r)\right],\quad 0\le r\le 1$$

Where

$$\left({}^{C}D^{\beta}_{a+}\underline{f}\right)(x_0;r)=\left[\frac{1}{G(1-\beta)}\frac{d}{dx}\int_a^x\frac{\underline{f}(t;r)\,dt}{(x-t)^{\beta}}\right]_{x=x_0} \tag{1}$$

$$\left({}^{C}D^{\beta}_{a+}\overline{f}\right)(x_0;r)=\left[\frac{1}{G(1-\beta)}\frac{d}{dx}\int_a^x\frac{\overline{f}(t;r)\,dt}{(x-t)^{\beta}}\right]_{x=x_0} \tag{2}$$

4. Fuzzy Laplace Transforms

Definition 4.1 ([16]). *Let f be continuous fuzzy valued function. Suppose that $f(x)?e^{-px}$ is improper fuzzy Riemann integrable on $[0,\infty)$, then $\int_0^\infty f(x)\odot e^{-px}dx$ is called fuzzy Laplace transforms and denoted by*

$$L[f(x)]=\int_0^\infty f(x)\odot e^{-px}dx\quad (p>0 \text{ and integer}) \tag{3}$$

Using Theorem 2.1 we have $0\le r\le 1$;

$$\int_0^\infty f(x;r)\odot e^{-px}dx=\left[\int_0^\infty \underline{f}(x;r)\odot e^{-px}dx,\ \int_0^\infty \overline{f}(x;r)\odot e^{-px}dx\right]$$

Using the classical Laplace transforms,

$$l\left[\underline{f}(x;r)\right]=\int_0^\infty \underline{f}(x;r)e^{-px}dx\ \text{ and }\ l\left[\overline{f}(x;r)\right]=\int_0^\infty \overline{f}(x;r)e^{-px}dx$$

Then we get

$$L[f(x;r)]=\left[l\left[\underline{f}(x;r)\right],\ l\left[\overline{f}(x;r)\right]\right]$$

Definition 4.2. *Hypergeom (n,d,z) is the generalized hyper geometric function $F(n,d,z)$, also known as Barnes extended hyper geometric function. For scalars a, b and c, hypergeom([a,b], c ,z) is a Gauss hyper geometric function $2F_1(a,b;c;z)$. The Gauss hypergeometric function $2F_1(a,b;c;z)$ is defined in the unit disc as the sum of the hypergeometric series*

$$2F_1(a,b;c;z)=\sum_{k=0}^{\infty}\frac{(a)_k(b)_k}{(c)_k}\frac{z^k}{k!},|z|<1$$

Definition 4.3. *The pochhammer symbol $(a)_k$ is defined by*

$$(a)_0=1,$$

$$(a)_n=a(a+1)\dots(a+n-1),\quad n\in N$$

Definition 4.4. *A two parameters function of Mittag -Leffler type is defined by the series expansion*

$$E_{\alpha,\beta}(z)=\sum_{r=0}^{\infty}\frac{z^r}{G(\alpha r+\beta)};\quad (\alpha,\beta>0)$$

An error function is defined by $er\,fc(x)=\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt$.

Theorem 4.5 ([16]). *Let f and g are continuous fuzzy valued functions. Suppose that c_1 and c_2 are constants.*

$$L\left[(c_1 \odot f(x)) \oplus (c_2 \odot g(x))\right] = (c_1 \odot L\left[f(x)\right]) \oplus (c_2 \odot L\left[g(x)\right])$$

Lemma 4.6 ([16]). *Let f be continuous fuzzy valued function on $[0,\infty)$ and $\lambda \in R$ then $L[\lambda \odot f(x)] = \lambda \odot L[f(x)]$.*

Theorem 4.7. *Suppose that $f \in C^F[0,\infty) \cap L^F[0,\infty)$. Then*

$$L\left[({}^C D^{\beta}_{a+} f)(x)\right] = s^{\beta} L\left[f(x)\right] \ominus s^{\beta-1} f(0), \tag{4}$$

if f is $({}^C(i)-\beta)$ differentiable, and

$$L\left[({}^C D^{\beta}_{a+} f)(x)\right] = -s^{\beta-1} f(0) \ominus -\left(s^{\beta} L\left[f(x)\right]\right) \tag{5}$$

if f is $({}^C(ii)-\beta)$ differentiable.

5. Fuzzy Fractional Differential Equations Under Caputo's H-differentiability

Let $f \in C^F[a,b] \cap L^F[a,b]$ and consider the fuzzy fractional differential equation of order $0 < \beta < 1$ with the initial condition and $x \in (a,b)$.

$$\begin{cases} ({}^C D^{\beta}_{a+} y)(x) = f\left[x, y(x)\right], \\ y(0) \in E \end{cases} \tag{6}$$

Determining the solutions:

Here we use fuzzy Laplace transform and its inverse to derive the solution. By taking Laplace transform on both sides, we get

$$L[({}^C D^{\beta}_{a+} y)(x)] = L[f(x, y(x))], \tag{7}$$

Based on the caputo's H-differentiability, we have the following cases:

Case (1): Let us consider $y(x)$ is a $({}^C(i)-\beta)$ differentiable function then the equation (7) is extended based on the it's lower and upper functions as follows

$$\begin{aligned} s^{\beta} l\left[\underline{y}(x;r)\right] - s^{\beta-1}\underline{y}(0;r) &= l\left[\underline{f}(x, y(x); r)\right]; \quad 0 \le r \le 1 \\ s^{\beta} l\left[\overline{y}(x;r)\right] - s^{\beta-1}\overline{y}(0;r) &= l\left[\overline{f}(x, y(x); r)\right]; \quad 0 \le r \le 1 \end{aligned} \tag{8}$$

Where

$$\underline{f}(x, y(x); r) = \min\{f(x,u)/u \in \left[\underline{y}(x;r), \overline{y}(x;r)\right]\}$$

$$\overline{f}(x, y(x); r) = \max\{f(x,u)/u \in \left[\underline{y}(x;r), \overline{y}(x;r)\right]\}$$

To solve the linear system (8), we assume that $H_1(p;r)$, $k_1(p;r)$ are the solutions

$$l\left[\underline{y}(x;r)\right] = H_1(p;r)$$

$$l\left[\overline{y}(x;r)\right] = k_1(p;r)$$

By using inverse Laplace transform $\underline{y}(x;r)$ and $\overline{y}(x;r)$ are computed as follows,

$$\begin{aligned}\underline{y}(x;r) &= l^{-1}[H_1(p;r)]\\ \overline{y}(x;r) &= l^{-1}[k_1(p;r)]\end{aligned} \tag{9}$$

Case (2): Let us consider $y(x)$ is a $({}^C(ii)-\beta)$ differentiable function then the equation (7) can be written as follows

$$\begin{cases} -s^{\beta-1}\underline{y}(0;r) \ominus (-s^{\beta} l[\underline{y}(x;r)]) = l\left[\underline{f}(x, y(x); r)\right] \\ -s^{\beta-1}\overline{y}(0;r) \ominus (-s^{\beta} l[\overline{y}(x;r)]) = l\left[\overline{f}(x, y(x); r)\right] \end{cases} \quad 0 \le r \le 1 \tag{10}$$

Where

$$\underline{f}(x, y(x); r) = \min\{f(x,u)/u \in \left[\underline{y}(x;r), \overline{y}(x;r)\right]\}$$
$$\overline{f}(x, y(x); r) = \max\{f(x,u)/u \in \left[\underline{y}(x;r), \overline{y}(x;r)\right]\}$$

To solve the linear system (10), we assume that $H_2(p;r)$, $k_2(p;r)$ are the solutions

$$l\left[\underline{y}(x;r)\right] = H_2(p;r)$$
$$l\left[\overline{y}(x;r)\right] = k_2(p;r)$$

By using inverse Laplace transform $\underline{y}(x;r)$ and $\overline{y}(x;r)$ are computed as follows,

$$\begin{aligned}\underline{y}(x;r) &= l^{-1}[\, H_2(p;r)]\\ \overline{y}(x;r) &= l^{-1}[\, k_2(p;r)]\end{aligned} \tag{11}$$

Example 5.1. *Let us consider the following fuzzy fractional differential equation*

$$\begin{cases} ({}^C D_{0+}^{\beta} y)(x) = \lambda \odot y(x) + cosx, \quad 0 < \beta, \quad x < 1 \\ ({}^C D_{0+}^{\beta-1} y)(0) = ({}^C {y_0}^{(\beta-1)}) \in E \end{cases} \tag{12}$$

Solution.

Case (1): Suppose $\lambda \in R^{+} = (0, +\infty)$, then applying Laplace transform on both sides

$$\begin{aligned} L[({}^C D_{0+}^{\beta} y)(x)] &= L[\lambda \odot y(x) + cosx], \\ L[({}^C D_{0+}^{\beta} y)(x)] &= L[\lambda \odot y(x)] + L[cosx], \end{aligned} \tag{13}$$

Using $({}^C(i)-\beta)$ differentiability, we get

$$\begin{cases} s^{\beta} l\left[\underline{y}(x;r)\right] - s^{\beta-1}\underline{y}(0;r) = \lambda l\left[\underline{y}(x;r)\right] + \frac{S}{s^2+1} \\ s^{\beta} l\left[\overline{y}(x;r)\right] - s^{\beta-1}\overline{y}(0;r) = \lambda l\left[\overline{y}(x;r)\right] + \frac{S}{s^2+1} \end{cases} \tag{14}$$

$$\Rightarrow (s^{\beta} - \lambda) l\left[\underline{y}(x;r)\right] = s^{\beta-1}\underline{y}(0;r) + \frac{s}{s^2+1}$$

$$(s^{\beta}-\lambda)l\left[\overline{y}\,(x;r)\right]=s^{\beta-1}\overline{y}(0;r)+\frac{s}{s^2+1}$$
$$l\left[\underline{y}\,(x;r)\right]=s^{\beta-1}\underline{y}(0;r)\frac{1}{(s^{\beta}-\lambda)}+\frac{s}{(s^2+1)(s^{\beta}-\lambda)}$$
$$l\left[\overline{y}\,(x;r)\right]=s^{\beta-1}\overline{y}(0;r)\frac{1}{(s^{\beta}-\lambda)}+\frac{s}{(s^2+1)(s^{\beta}-\lambda)} \tag{15}$$

Applying inverse transform on both sides,

$$\underline{y}\,(x;r)=\underline{y}\,(0;r)\,l^{-1}\left[\frac{s^{\beta-1}}{(s^{\beta}-\lambda)}\right]+l^{-1}\left[\frac{s}{(s^2+1)(s^{\beta}-\lambda)}\right]$$
$$\overline{y}\,(x;r)=\overline{y}\,(0;r)\,l^{-1}\left[\frac{s^{\beta-1}}{(s^{\beta}-\lambda)}\right]+l^{-1}\left[\frac{s}{(s^2+1)(s^{\beta}-\lambda)}\right] \tag{16}$$

1^{st} term in equation (16), consider,

$$l^{-1}\left[\frac{s^{\beta-1}}{(s^{\beta}-\lambda)}\right]=\sum_{r=0}^{\infty}\frac{(\lambda x^{\beta})^r}{\Gamma\,(\beta r+1)}=E_{\beta,1}(\lambda x^{\beta})$$

Convolution theorem in Laplace transform we have 2^{nd} term in equation (16)

$$l^{-1}\left[\frac{s}{(s^2+1)(s^{\beta}-\lambda)}\right]=\int_0^x(x-t)^{(\beta-1)}E_{\beta,\beta}(\lambda(x-t)^{\beta})\cos t\;dt$$

(16) $\Rightarrow$

$$\underline{y}\,(x;r)=\underline{y}\,(0;r)\,E_{\beta,1}(\lambda x^{\beta})+\int_0^x(x-t)^{(\beta-1)}E_{\beta,\beta}(\lambda(x-t)^{\beta})\cos t\;dt$$
$$\overline{y}\,(x;r)=\overline{y}\,(0;r)\,E_{\beta,1}(\lambda x^{\beta})+\int_0^x(x-t)^{(\beta-1)}E_{\beta,\beta}(\lambda(x-t)^{\beta})\cos t\;dt \tag{17}$$

Case (2): Suppose $\lambda\in R^{-}=(-\infty,0)$, then using $(^{C}(ii)-\beta)$ differentiability the solution will obtain similar to equation (17). For the special case, let us consider $\beta=0.5$, $\lambda=1$ and $y\,(0;r)=[1+r,3-r]$ in Case (1)

$$\underline{y}\,(x;r)=[1+r,3-r]\odot E_{\frac{1}{2},1}(x^{\frac{1}{2}})+\int_0^x(x-t)^{(-\frac{1}{2})}E_{\frac{1}{2},\frac{1}{2}}(x-t)^{\frac{1}{2}}\cos t\;dt$$
$$\overline{y}\,(x;r)=[1+r,3-r]\odot E_{\frac{1}{2},1}(x^{\frac{1}{2}})+\int_0^x(x-t)^{(-\frac{1}{2})}E_{\frac{1}{2},\frac{1}{2}}(x-t)^{\frac{1}{2}}\cos t\;dt \tag{18}$$

Now consider 1^{st} term in equation (18)

$$E_{\frac{1}{2},1}(x^{\frac{1}{2}})=e^{(x^{\frac{1}{2}})^2}erfc(-x^{\frac{1}{2}})$$
$$=e^{x}\;erfc(-\sqrt{x})\quad\{E_{\frac{1}{2},1}(z)=e^{z^2}erfc(-z)\}$$

2^{nd} term in equation (18)

$$\int_0^x(x-t)^{(-\frac{1}{2})}E_{\frac{1}{2},\frac{1}{2}}(x-t)^{\frac{1}{2}})\cos t\;dt=\int_0^x(x-t)^{(-\frac{1}{2})}\sum_{k=0}^{\infty}\frac{(x-t)^{\frac{k}{2}}}{\Gamma(\frac{k+1}{2})}\cos t\;dt$$
$$=\int_0^x\sum_{k=0}^{\infty}\frac{(x-t)^{\frac{k-1}{2}}}{\Gamma(\frac{k+1}{2})}\cos t\;dt$$
$$=\int_0^x\frac{(x-t)^{-\frac{1}{2}}}{\Gamma(\frac{1}{2})}\cos t\;dt+\int_0^x\frac{(x-t)^{0}}{\Gamma(1)}\cos t dt+\int_0^x\frac{(x-t)^{\frac{1}{2}}}{\Gamma(\frac{3}{2})}\cos t\;dt$$

$$+\int_0^x \frac{(x-t)^1}{\Gamma(2)}\cos t\ dt + \int_0^x \frac{(x-t)^{\frac{3}{2}}}{\Gamma(\frac{5}{2})}\cos t\ dt + \int_0^x \frac{(x-t)^2}{\Gamma(3)}\cos t\ dt + \dots$$

Rearrange the terms split series

$$= \frac{x^{\frac{1}{2}}}{(\frac{1}{2})!}\left[{}^{hypergeom}_{even}\left(1,\frac{3}{2},ix\right)\right] + \frac{x^{\frac{3}{2}}}{(\frac{3}{2})!}\left[{}^{hypergeom}_{even}\left(1,\frac{5}{2},ix\right)\right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!}\left[{}^{hypergeom}_{even}\left(1,\frac{7}{2},ix\right)\right] + \dots$$
$$+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^5}{5!}+\frac{x^6}{6!}+\frac{x^9}{9!}+\frac{x^{10}}{10!}+\dots$$
$$= \sum_{n=0}^{\infty}\frac{x^{n+\frac{1}{2}}}{(n+\frac{1}{2})!}\left\{{}^{hypergeom}_{even}\left(1,n+\frac{3}{2},ix\right)\right\} + \sum_{n=0}^{\infty}\frac{x^{4n+1}}{(4n+1)!} + \sum_{n=0}^{\infty}\frac{x^{4n+2}}{(4n+2)!}$$

(18) $\Rightarrow$

$$\underline{y}(x;r) = [1+r]\,e^x erfc(-\sqrt{x}) + \sum_{n=0}^{\infty}\frac{x^{n+\frac{1}{2}}}{(n+\frac{1}{2})!}\left\{{}^{hypergeom}_{even}\left(1,n+\frac{3}{2},ix\right)\right\} + \sum_{n=0}^{\infty}\frac{x^{4n+1}}{(4n+1)!} + \sum_{n=0}^{\infty}\frac{x^{4n+2}}{(4n+2)!}$$
$$\overline{y}(x;r) = [3-r]\,e^x erfc(-\sqrt{x}) + \sum_{n=0}^{\infty}\frac{x^{n+\frac{1}{2}}}{(n+\frac{1}{2})!}\left\{{}^{hypergeom}_{even}\left(1,n+\frac{3}{2},ix\right)\right\} + \sum_{n=0}^{\infty}\frac{x^{4n+1}}{(4n+1)!} + \sum_{n=0}^{\infty}\frac{x^{4n+2}}{(4n+2)!}$$

6. Conclusion

In this paper, solving FFDEs of order $0 < \beta < 1$ using fuzzy Laplace transforms under Caputo's-H differentiability was discussed. As an example, we solved a problem involving cosine term.

References

[1] S. Abbasbandy and A. Shirzadi, *Homotopy analysis method for multiple solutions of the fractional Sturm-Liouville problems*, Numer. Algor., 54(2010), 521-32.

[2] R. P. Agarwal, V. Lakshmikantham and J. J. Nieto, *On the concept of solution for fractional differential equations with uncertainty*, Nonlinear Anal., 72(2010), 2859-2862.

[3] T. Allahviranloo and S. Salahshour, *A new approach for solving first order fuzzy differential equations*, Commun Comput Inform Sci., 81(2010), 522-531.

[4] A. Arara, M. Benchohra, N. Hamidi and J. J. Nieto, *Fractional order differential equations on an unbounded domain*, Nonlinear Anal., 72(2010), 580-586.

[5] V. Lakshmikantham and A. S. Vatsala, *Basic theory of fractional differential equations*, Nonlinear Anal., 69(2008), 2677-2682.

[6] Y. I. Babenko, *Heat and Mass Transfer*, Chemia, Leningrad, (1986).

[7] B. Bede and S. G. Gal, *Generalizations of the differentiability of fuzzy-number-valued functions with applications to fuzzy differential equations*, Fuzzy Sets Syst., 151(2005), 581-599.

[8] H. J. Zimmermann, *Fuzzy set theory and its applications*, Dordrecht: Kluwer Academi Publishers, (1991).

[9] S. Rubanraj and J. Sangeetha, *Fuzzy Laplace Transform With Fuzzy Fractional DifferentialEquation*, IJMCR, 4(year?), 1276-1282

[10] A. A. Kilbas, H.M. Srivastava and J. J. Trujillo, *Theory and applications of fractional differential equations*, Amesterdam: Elsevier Science B.V, (2006).

[11] H. Beyer and S. Kempfle, *Definition of physically consistent damping laws with fractional derivatives*, ZAMM, 75(1995), 623-635.

[12] I. Podlubny, *Fractional differential equation*, San Diego: Academic Press, (1999).

[13] M. Friedman, M. Ma and A. Kandel, *Numerical solution of fuzzy differential and integral equations*, Fuzzy Sets Syst., 106(1999), 35-48.

[14] M. Ma, M. Friedman and A. Kandel, *Numerical solution of fuzzy differential equations*, Fuzzy Sets Syst., 105(1999), 133-138.

[15] H. C. Wu, *The improper fuzzy Riemann integral and its numerical integration*, Inform Sci., 111(1999), 109-137.

[16] T. Allahviranloo and M. B. Ahmadi, *Fuzzy Laplace transforms*, Soft Comput., 14(2010), 235-243.

[17] T. Allahviranloo, S. Salahshour and S. Abbasbandy, *Explicit solutions of fractional differential equations with uncertainty*, Soft Comput., doi:10.1007/s00500-011-0743-y.

[18] R. L. Bagley, *On the fractional order initial value problem and its engineering applications*, In: Nishimoto K, editor. Fractional calculus and its applications. Tokyo: College of Engineering, Nihon University, (1990), 12-20.