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Supra Continuous $\breve{g}$ in Topological Spaces

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Abstract: The aim of this paper is to introduce and investigate a new class of sets and functions between topological space called supra $\breve{g}$-open sets and supra $\breve{g}$-closed functions respectively, furthermore introduced the concept of supra $\breve{g}$-open mapping and supra $\breve{g}$-closed maps and investigated several properties of them. Additionally, we relate and compare these functions with some other functions in topological spaces.

MSC: 54C08, 54C10.

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1. Introduction

In 1983, A.S Mashhour [3] introduced the supra topological spaces and studied S-continuous functions and S^*-continuous functions. In 2008, R.Devi [2] introduced and studied a class of sets and maps between topological space called supra α-open sets and supra α-continuous maps. In 2010, O.R.Sayed [5] introduced and investigated several properties of supra b-open set and supra b-continuity on topological space. In 2011, Arokiarani and Trintia pricilla [1] introduced and investigated several properties of new type of sets called T-closed set and supra T-continuity maps. Now, the concepts of supra $\breve{g}$-open sets were introduced and some basic properties of it were studied. Also, we introduced the concept of supra $\breve{g}$-continuous maps, supra $\breve{g}$-open maps and supra $\breve{g}$-closed maps and investigated several properties for these class of maps. In particular, we study the relationship between supra $\breve{g}$-continuous maps and supra $\breve{g}$-open maps.

2. Preliminaries

Throughout this paper $(X,\tau),(Y,\sigma)$ and (Z,η) or X,Y and Z represent non-empty topological spaces on which no separate axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), $cl(A)$ and $int(A)$ denoted closure and interior of A respectively.

Definition 2.1 ([4]). *A subfamily μ of X is said to be supra topology on X, if*

(1). X, $\phi \in \mu$

(2). if $A_i \in \mu$ for all $i \in J$, then $\bigcup A_i \in \mu$. The pair (X,μ) is called supra topological spaces.

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The element of μ are called supra open sets in (X,μ) and the complement of supra open sets is called supra closed sets and it is denoted by μ^c.

Definition 2.2 ([4]). *The supra closure of a set A is denoted by supra $cl(A)$ and defined as supra $cl(A) = \bigcap\{B : B$ is a supra closed and $A \subseteq B\}$. The supra interior of a set is denoted by supra $int(A)$, and defined as supra int(A),and defined as supra int (A) = $\bigcup\{B$: B is a supra open and $A \supseteq B\}$.*

Definition 2.3 ([5]). *Let (X, τ) be a topological space and μ be a supra topology on X. We call μ a supra topology associated with τ, if $\tau \subset \mu$.*

Definition 2.4. *A subset A of a space X is called*

(1). supra semi open set [5], if $A \subseteq cl^{\mu}[int^{\mu}(A)]$.

(2). supra α-open set [2], if $A \subseteq cl^{\mu}[cl^{\mu}(int^{\mu}(A))$.

(3). supra regular open [2], if $A = cl^{\mu}(int^{\mu}(A))$.

(4). supra g-closed [4], if $cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in X.

(5). supra b-open [5], if $A \subseteq cl^{\mu}[int^{\mu}(A)]U(int^{\mu}(A))cl^{\mu}(A)$.

3. Supra $\breve{g}$-Closed Sets

In this section, I introduce a new class of generalized open sets called supra $\breve{g}$ open sets and discuss some of their properties.

Definition 3.1. *A subset A of (X, μ) is said to be supra $\breve{g}$ closed in (X, μ), if $cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is B-open in (X, μ). The complement of supra $\breve{g}$-closed is called supra $\breve{g}$-open set. we denoted the collection of all $\breve{g}^{\mu}$ closed (respectively supra $\breve{g}$-open) in X is denoted by $\breve{G}^{\mu}C(X)$ (respectively $\breve{G}^{\mu}O(X)$). The intersection of all supra $\breve{g}$-closed sets containing A is called supra $\breve{g}$-closure of A and denoted by $cl^{\mu}_{\breve{g}}(A)$ and the supra $\breve{g}$-interior of A is the union of all supra $\breve{g}$-open sets contained in A and is denoted by $Int^{\mu}_{\breve{g}}(A)$ in X.*

Definition 3.2. *A subset A of a space (x, μ) is called a supra $\breve{g}_{\alpha}$-closed set if $\alpha cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is B-open in (x, μ). The complement of supra $\breve{g}_{\alpha}$-closed set is called supra $\breve{g}_{\alpha}$-open set.*

Proposition 3.3. *Every supra closed set is $\breve{g}^{\mu}$-closed set.*

Proof. If A is any supra closed set in X and G is any supra B-open set containing A, then G $\supseteq A = cl^{\mu}(A)$. Hence A is $\breve{g}^{\mu}$ in X. □

The converse of proposition need not be true.

Proposition 3.4. *Every $\breve{g}^{\mu}$-closed set is $\breve{g}^{\mu}_{\alpha}$-closed.*

Proof. If A is a $\breve{g}^{\mu}$-closed set in X and G is any supra B-open set containing A, the G $\supseteq cl^{\mu}(A) \supseteq \alpha cl^{\mu}(A)$. Hence A is $\breve{g}^{\mu}_{\alpha}$-closed. □

Proposition 3.5. *Every supra $\breve{g}$-closed set is supra sg-closed.*

Proof. If A is a supra $\breve{g}$-closed in X and G is any supra semi open set containing A, since every supra semi open set is supra B-open and A is supra $\breve{g}$-closed set, We have G $\supseteq cl^{\mu}(A) \supseteq scl(A)$. Hence A is supra sg-closed. □

The converse of proposition need not be true.

Proposition 3.6. *Every $\breve{g}^{\mu}$-closed set in g^{μ}-closed.*

Proof. If A is a supra $\breve{g}$-closed set is (x, μ) and G is any open set containing A, since every supra open set in supra B-open, then $G \supseteq cl^{\mu}(A)$. Hence A is supra g-closed. □

The converse of proposition is need not be true.

Proposition 3.7. *Every supra $\breve{g}$-closed set is supra gs-closed.*

Proof. If A is a supra $\breve{g}$-closed set in X and G is any supra open set containing A, since every supra open set is supra B-open, then $G \supseteq cl^{\mu}(A) \supseteq scl^{\mu}(A)$. Hence A is supra gs-closed. □

The converse of proposition is need not to be true.

Proposition 3.8. *Every supra $\breve{g}$-closed set is supra gsp-closed.*

Proof. If A is a supra $\breve{g}$-closed set in X and G is any supra open set containing A, since every supra open set is supra B-open, then $G \supseteq cl^{\mu}(A) \supseteq spcl^{\mu}(A)$. Hence A is supra gsp-closed. □

The converse of proposition is need not be true.

Definition 3.9. *The intersection of all supra B-open subsets in (X, μ) containing A is called supra B-kernel of A and is denoted by $B^{\mu} - ker(A)$.*

Lemma 3.10. *A subset A of (X, μ) is supra $\breve{g}$-closed if and only if $cl^{\mu}(A) \subseteq B^{\mu} - ker(A)$.*

Proof. Suppose that A is $\breve{g}^{\mu}$ -closed, then $cl^{\mu} \subseteq U$ whenever $A \subseteq U$ and U is supra B-open. Let $x \in cl^{\mu}(A)$. If $x \notin B^{\mu} - ker(A)$, then there is B^{μ}-open set U containing A such that $x \notin U$. Since U is an B^{μ}-open set containing A, then $x \notin cl^{\mu}(A)$ and this is a contradiction. Conversely, let $cl^{\mu}(A) \subseteq B^{\mu}$-ker(A), if U is any B^{μ}-open set containing A, then $cl^{\mu}(A) \subseteq B^{\mu} - Ker(A) \subseteq U$. Therefore, A is $\breve{g}^{\mu}$-closed. □

Proposition 3.11. *If A and B are $\breve{g}^{\mu}$-closed set in (X, μ), then $A \bigcup B$ is $\breve{g}^{\mu}$-closed in (X, μ).*

Proof. If $A \bigcup B \subseteq G$ and G is B^{μ}-open, then $A \subseteq G$ and $B \subseteq G$. since A and B are $\breve{g}^{\mu}$-closed, $G \supseteq cl^{\mu}(A)$ and $G \supseteq cl^{\mu}(B)$ and hence $G \supseteq cl^{\mu}(A) \bigcup cl^{\mu}(B) = cl(A \bigcup B)$. Thus $A \bigcup B$ is $\breve{g}^{\mu}$-closed sets in (X, μ). □

4. Supra $\breve{g}$-Continuous

In this section, I introduce a new type of continuous functions called supra $\breve{g}$-continuous function and obtain some of their properties and characterizations.

Definition 4.1. *Let (X, τ) and (X, σ) be two topological spaces and μ be an associated supra topology with τ. A map $f : (X, \tau) \to (Y, \sigma)$ is called supra $\breve{g}$-continuous maps, if the inverse image of each open in Y is supra $\breve{g}$-open set in X.*

Theorem 4.2. *Every continuous map is supra $\breve{g}$-continuous.*

Proof. Let (X, τ) and (Y, σ) be two supra topological spaces. Let $f : (X, \tau) \to (Y, \sigma)$ be continues map and A is open in Y. then $f^{-1}(A)$ is an open set in X. since μ is associated with τ, then $\tau \subset \mu$. Therefore, $f^{-1}(A)$ is supra open set in X. Hence f is supra $\breve{g}$ continuous. □

The converse of the above theorem is not true as shown in the following examples.

Theorem 4.3. *Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ. Let f be a map from X into Y, then the following are equivalent:*

(1). f is a supra $\breve{g}$-continuous map.

(2). The inverse image of a closed set in Y is a supra $\breve{g}$-closed set in X.

(3). The inverse image of a open set in Y is a supra $\breve{g}$-open set in X.

(4). $cl^{\mu}_{\breve{g}}(f-1(A) \subseteq f^{-1}(cl(A)))$ for every set A in Y.

(5). $f(cl^{\mu}_{\breve{g}}(A) \subseteq cl(f(A))$ for every set in X.

(6). $f^{-1}(int(B)) \subseteq int^{\mu}_{\breve{g}}(f^{-1}(B))$ for every B in Y.

Proof. $(i) \Rightarrow (ii)$. Let A be a closed set in Y.Then Y-A is open set in Y then $f^{-1}(Y-A) = X - f^{-1}(A)$ is a supra $\breve{g}$-open set in X. It follows that $f^{-1}(A)$ is a supra $\breve{g}$-closed subset of X.
$(ii) \Rightarrow (iii)$ Let A be any subset of Y. Since cl(A) is closed in Y, Then $f^{-1}(cl(A)$ is supra $\breve{g}$ closed in X. Therefore $cl^{\mu}_{\breve{g}} \subset cl^{\mu}_{\breve{g}}(f^{-1}(cl(A))) = f^{-1}(cl(A))$.
$(iii) \Rightarrow (iv)$ Let A be any subset of X.By (3) we have $f^{-1}(cl(f(A))) \supseteq cl^{\mu}_{\breve{g}}(f^{-1}(f(A)) \supseteq cl^{\mu}_{\breve{g}}(A)$.Therefore $f(cl^{\mu}_{\breve{g}}) \subseteq cl(f(A)$.
$(iv) \Rightarrow (v)$. Let B be any subset of Y. By (iv), $f(cl^{\mu}_{\breve{g}}(X - f^{-1}(B))) \subseteq cl(f(x - f^{-1}(B)))$ and $f(X - int^{\mu}_{\breve{g}}(f^{-1}(B))) \subseteq cl(Y - B) = Y - int(B)$.
Therefore we have $X - int^{\mu}_{\breve{g}} f^{-1}(B) \subseteq f^{-1}(Y - int(B))$ and $f^{-1}(int(B)) \subset int^{\mu}_{\breve{g}}(f^{-1}(B))$. □

Theorem 4.4. *Let $(X,\tau),(Y,\sigma)$ and (Z,v) be three topological spaces. If a map $f:(X,\tau) \rightarrow (Y,\sigma)$ is supra $\breve{g}$-continuous and $g:(Y,\sigma) \rightarrow (Z,v)$ is a continuous map,then $g \circ f:(X,\tau) \rightarrow (z,v)$ is supra $\breve{g}$-continuous.*

Proof. Let F be any closed set in (Z,v). Since $g:(Y,\sigma) \rightarrow (Z,v)$ is continuous. $g^{-1}(F)$ is closed in (Y,σ). Since $f:(X,\tau) \rightarrow (Y,\sigma)$ is supra $\breve{g}$-continuous. $f^{-1}[g^{-1}(F)] = (g \circ f)^{-1}(F)$ is $\breve{g}$-closed in (X,τ) and so $g \circ f$ is supra $\breve{g}$-continuous. □

Theorem 4.5. *Let (X,τ) and (Y,σ) be two topological spaces and μ and v be the associated supra topologies with τ and σ respectively. Then $f:(X,\tau) \rightarrow (Y,\sigma)$ is a supra $\breve{g}$-continuous maps. If one of the following holds.*

(i). $f^{-1}(int^{v}_{b}(B) \subseteq int(f-1(B)$ for every set B in Y.

(ii). $cl(f^{-1}(B)) \subseteq f^{-1}(cl^{v}_{b}(B))$ for every set B in Y

(iii). $f(cl(A)) \subseteq cl^{\mu}_{b}(f(A))$ for every set A in X.

Proof. Let B be any open set of Y. If condition (i) is satisfied, then $f^{-1}(int^{v}_{b}(B)) \subseteq int(f^{-1}(B))$. We get $f^{-1}(B) \subseteq int(f^{-1}(B))$. Therefore, $f^{-1}(B)$ is an open set. Every open set is supra $\breve{g}$-continuous map. If condition (2) is satisfied, then we can easily prove that f is a supra $\breve{g}$-continuous map. Let condition (3) be satisfied, and B be any open set of Y. Then $f^{-1}(B)$ is a set in X and $f(cl(f^{-1}(B))) \subseteq cl^{\mu}_{b}(f(f^{-1}))(B)$. This implies $f(cl(f^{-1}(B))) \subseteq cl^{\mu}_{b}(B)$. This is nothing but condition (2). Hence f is a supra $\breve{g}$-continuous map. □

5. Some Forms of Supra $\breve{g}$-Continuous Functions

Definition 5.1. *A map $f:(X,\tau)\to(Y,\sigma)$ is called strongly supra $\breve{g}$-continuous function if the inverse image of every supra $\breve{g}$-closed set in (Y,σ) is supra closed in (X,τ).*

Definition 5.2. *A map $f:(X,\tau)\to(Y,\sigma)$ is called perfectly supra $\breve{g}$-continuous function if the inverse image of every supra $\breve{g}$-closed set in (Y,σ) is both supra open and supra closed in (X,τ).*

Theorem 5.3. *Every perfectly supra $\breve{g}$ continuous function is strongly supra $\breve{g}$-continuous.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ be a perfectly $\breve{g}$-continuous function. Let V be $\breve{g}$-closed set in (Y,σ). Since f is perfectly $\breve{g}$-continuous function $f^{-1}(V)$ is both supra open and closed in (X,τ). Therefore f is strongly supra $\breve{g}$-continuous function. □

The converse of the above theorem need not be true.

Theorem 5.4. *Let $f:(X,\tau)\to(Y,\sigma)$ be strongly supra $\breve{g}$-continuous function and $g:(Y,\sigma)\to(Z,\upsilon)$ be strongly supra $\breve{g}$-continuous function Then their composition $g\circ f:(X,\tau)\to(Z,\upsilon)$ is a strongly supra $\breve{g}$- continuous function.*

Proof. Let V be supra $\breve{g}$-closed set in (Z,υ). Since g is strongly $\breve{g}$-continuous, $g^{-1}(V)$ is supra closed in (Y,σ). We know that every supra closed set in supra $\breve{g}$-closed set, $g^{-1}(V)$ is supra $\breve{g}$-closed in (Y,σ). Since f is strongly $\breve{g}$-continuous, $f^{-1}(g^{-1}(V))$ is supra closed in (X,τ), implies $(g\circ f)(V)$ is supra closed in (X,τ). Therefore $g\circ f$ is strongly $\breve{g}$-continuous. □

Definition 5.5. *A functions $f:(X,\tau)\to(Y,\sigma)$ is called totally supra $\breve{g}$ continuous functions if the inverse image of every supra open set V of (Y,σ) is both supra $\breve{g}$ open and supra $\breve{g}$ closed subset of (X,τ). (i.e.,) $f^{-1}(V)$ is supra clopen set in X, for every supra open set V of Y.*

Theorem 5.6. *Every totally supra $\breve{g}$ continuous functions is supra $\breve{g}$ continuous.*

Proof. Let O be an supra open set of (Y,σ). Since f is totally supra $\breve{g}$ continuous functions, $f^{-1}(O)$ is both supra $\breve{g}$-open and supra $\breve{g}$-closed in (X,τ). Therefore F is supra $\breve{g}$ continuous. □

Remark 5.7. *The converse of above theorem need not be true.*

Theorem 5.8. *Every totally supra continuous is totally supra $\breve{g}$ continuous.*

Proof. Let O be an supra open set of (Y,σ). Since, f is totally supra continuous functions, $f^{-1}(O)$ is both supra open and supra closed in (X,σ). Since every supra open set is supra $\breve{g}$-open and every supra closed set is supra $\breve{g}$-closed. $f^{-1}(O)$ is both supra $\breve{g}$-open and supra $\breve{g}$-closed in (X,τ). Therefore, f is totally supra $\breve{g}$ continuous. □

Remark 5.9. *The converse of above theorem need not be true.*

Theorem 5.10. *Every perfectly supra $\breve{g}$ continuous map is totally supra $\breve{g}$ continuous.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ be a perfectly supra $\breve{g}$ continuous map. Let O be an supra open set of (Y,σ). Then O is supra $\breve{g}$ open in (Y,σ). Since f is perfectly supra $\breve{g}$ continuous, $f^{-1}(O)$ is both supra open and supra closed in (X,τ), implies $f^{-1}(O)$ is both supra $\breve{g}$-open and supra $\breve{g}$-closed in (X,τ). Therefore, f is totally supra $\breve{g}$ continuous. □

Remark 5.11. *The converse of above theorem need not to be true.*

Theorem 5.12. *If $f:X\to Y$ is totally supra $\breve{g}$ continuous map and X is supra $\breve{g}$ connected, then Y is an indiscrete space.*

Proof. Suppose that Y is not an indiscrete space. Let A be an non-empty supra open subset of Y. Since f is totally supra $\breve{g}$ continuous map, then $f^{-1}(A)$ is a non empty supra $\breve{g}$ clopen subset of X. Then $X = f^{-1}(A) \cup (f^{-1}(A))^c$. Thus, X is union of two non-empty disjoint supra $\breve{g}$ open sets. which is contradiction to the fact, that X is supra $\breve{g}$ connected. Therefore, Y must be an indiscrete space. □

Theorem 5.13. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then $g \circ f : X \to Z$.*

(i). If f is supra $\breve{g}$ irresolute and g is totally supra $\breve{g}$ continuous then $g \circ f$ is totally supra $\breve{g}$ continuous.

(ii). If f is totally supra $\breve{g}$ continuous and g is continuous, then $g \circ f$ is totally supra $\breve{g}$ continuous.

Proof.

(i). Let O be an supra open set in Z. Since g is totally supra $\breve{g}$ continuous, $g^{-1}(O)$ is supra $\breve{g}$ clopen in Y. Since f is supra $\breve{g}$-irresolute, $f^{-1}(g^{-1}(O))$ is supra $\breve{g}$-open and supra $\breve{g}$-closed in X. Since $(g \circ f)^{-1}(O) = f^{-1}(g^{-1}(O))$.

(ii). Let O be an supra open set in Z. Since g is continuous, $g^{-1}(O)$ is open in Y. Since f is totally supra $\breve{g}$ continuous, $f^{-1}(g^{-1}(O))$ is supra $\breve{g}$ clopen in X. Hence, $g \circ f$ is totally supra $\breve{g}$ continuous. □

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