Int. J. Math. And Appl., 6(2–B)(2018), 75–79
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

CR-Submanifolds of P-Sasakian manifold Endowed with a Semi-Symmetric Non-metric Connection

N. V. C. Shukla[1] and Ram Asray Verma[1,*]

1 Department of Mathematics & Astronomy, University of Lucknow, Lucknow, Uttar Pradesh, India.

Abstract: We define a semi-symmetric non-metric connection in a para-Sasakian manifold and study CR-submanifolds of a para-Sasakian manifold endowed with a semi-symmetric non-metric connection. Moreover, we also obtain integrability conditions of the distributions on CR-submanifolds.Parallel horizontal distributions of CR−submanifolds.

MSC: 53C07, 53C25.

Keywords: p-sasakian manifold, CR-Submanifolds, a Semi Symmetric Non metric Connection, parallel horizontal distribution.

 Accepted on: 21.03.2018

1. Introduction

A. Bejancu [5] studied CR-submanifolds of a Kahler manifold. Later, CR-submanifolds of Sasakian manifold were studied by M. Kobayashi [9]. K. Matsumotu introduced the idea of LP-Sasakian structure and studied its several properties [6]. B. Prasad [11] and S. Prasad, R. H. Ojha [12] studied submanifolds of a LP-Sasakian manifold. U. C. De and Anup Kumar Sengupta studied CR-submanifolds of a LP-Sasakian manifold in [7]. In this paper, we have studied and obtained some results related to CR-submanifolds of a P-Sasakian manifold endowed with a semi-symmetric non-metric connection. On the other-hand, A. Friedmannand, J. A. Schouten ([8, 14]) introduced the idea of a semi-symmetric linear connection. A linear connection ∇ is said to be semi-symmetric connection if its torsion tensor T is of the form

$$T(X,Y) = \eta(Y)X - \eta(X)Y,$$

where η is a 1-form, K. Yano [16] studied some properties of semi−symmetric metric connection. N. S. Agashe and M. R. Chaffle [3] studied some properties of semi-symmetric non-metric connection. The first author and C. Ozgur [4], defined a semi-symmetric non-metric connection and studied some properties of hypersurfaces of almost−r−paracontact Riemannian manifold with semi-symmetric non-metric connection. In this paper, we study CR-submanifolds of a P-Sasakian manifold endowed with a semi-symmetric non-metric connection.

* *E-mail: ramasray1986@gmail.com*

2. Preliminaries

An n-dimensional differentiable manifold $\overline{M}$ is said to admit an almost para- contact Riemannian structure (ϕ, ξ, η, g) where ϕ is a $(1,1)$ tensor field, ξ is a vector field, η is a $1-$form and g is the Riemannian metric on $\overline{M}$ such that

$$\phi^2(X) = X - \eta(X)\xi, \tag{1}$$

$$\phi\xi = 0, \quad \eta(\phi X) = 0, \tag{2}$$

$$\eta(\xi) = 1, \tag{3}$$

$$g(X, \xi) = \eta(X) \tag{4}$$

$$\phi^2(X) = X - \eta(X)\xi, \tag{5}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{6}$$

$$g(\phi X, Y) = g(X, \phi Y) = \psi(X, Y) \tag{7}$$

for any vector fields X, Y tangent to $\overline{M}$. In addition, if (ϕ, ξ, η, g), satisfy the equations

$$d\eta = 0, \tag{8}$$

$$\left(\overline{\overline{\nabla}}_X \phi\right) Y = -g(X, Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{9}$$

$$\nabla_X \xi = \phi X, \tag{10}$$

Then $\overline{M}$ is called para-Sasakian manifold or briefly a $P-$Sasakian manifold [1]. For any vector fields X, Y tangent to $\overline{M}$. where $\overline{\overline{\nabla}}$ is the Riemannian connection with respect to g.

2.1. Semi-symmetric Non-metric Connection

Let $\overline{M}$ be an n-dimensional Riemannian manifold with Riemannian metric g. If $\overline{\nabla}$ is the semi-symmetric non-metric connection of a Riemannian manifold M, a linear connection $\overline{\nabla}$ is given by

$$\overline{\nabla}_X Y = \overline{\overline{\nabla}}_X Y + \eta(Y)X, \tag{11}$$

Then $\overline{R}$ and R are related by

$$\overline{R}(X, Y)Z = R(X, Y)Z + \alpha(X, Z)Y - \alpha(Y, Z)X, \tag{12}$$

for all vector fields X, Y, Z on M , where α is a $(0,2)$ tensor field denoted by

$$\alpha(X, Z) = (\nabla_X \eta)(Z) - \eta(X)\eta(Z), \tag{13}$$

From (11)

$$\left(\overline{\nabla}_W g\right)(X, Y) = -\eta(X)g(Y, W) - \eta(Y)g(X, W). \tag{14}$$

Combining (11) and (10), we get

$$\left(\overline{\nabla}_X \phi\right) Y = -g(Y, X)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi - \eta(Y)\phi X, \tag{15}$$

Combining (11) and (9)

$$\overline{\nabla}_X \xi = \phi X + X, \tag{16}$$

We denote by g the metric tensor of $\overline{M}$ as well as that include on M. Let $\overline{\nabla}$ be the semi- symmetric non-metric connection on $\overline{M}$ and ∇ be the semi-symmetric non-metric connection on M with respect to unit normal N, Gauss equation and Weingarten formula for CR-submanifolds of P-Sasakian manifold with respect to the semi-symmetric non-metric connection are given Respectively by

$$\overline{\nabla}_X Y = \nabla_X Y + h\left(X, Y\right) \tag{17}$$

$$\overline{\nabla}_X N = -A_N X + \nabla^{\perp} N \tag{18}$$

for any vector fields $X, Y \in TM$, $N \in T^{\perp}M$, $h(resp.A_N)$ is the second fundamental form (resp.tensor) of M in $\overline{M}$ and $\nabla^{\perp}$ denotes the operator of the normal connection. Moreover, we have [5]

$$g\left(h\left(X, Y\right), N\right) = g\left(A_N X, Y\right) \tag{19}$$

any vector X tangent to M is given as

$$X = PX + QX \tag{20}$$

where PX and QX belongs to distribution D and $D^{\perp}$ respectively. For any vector field N normal to M, we put

$$\phi N = BN + CN, \tag{21}$$

where BN denote the tangential component of ϕN and CN denote the normal component of ϕN.

3. Integrabilities of Horizontal Distribution D and Vertical Distribution $D^{\perp}$

Lemma 3.1. *Let M be a CR-submanifolds of an $P-$Sasakian manifold $\overline{M}$ with semi-symmetric non-metric connection. Then*

$$P\nabla_X \phi PY - PA_{\phi QY}Y = \phi P\nabla_X Y - g(X,Y)P\xi - \eta(X)PY + \eta\left(X\right)\phi PY + 2\eta\left(X\right)\eta\left(Y\right)P\xi \tag{22}$$

$$Q\nabla_X \phi PY - QA_{\phi QY}Y = -g(X,Y)Q\xi - \eta(X)QY + \eta\left(X\right)\phi QY + 2\eta\left(X\right)\eta\left(Y\right)Q\xi + Bh\left(X,Y\right) \tag{23}$$

$$h\left(X, \phi PY\right) + \nabla^{\perp}\phi QY = \phi Q\nabla_X Y + Ch\left(X, Y\right) \tag{24}$$

for $X, Y \in TM$.

Proof. By virtue of (15), (17), (18), (20) and (21), we can easily get

$$\begin{aligned} P\nabla_X \phi PY - Q\nabla_X \phi PY + h(X, \phi PY) - PA_{\phi QY}X - QA_{\phi QY}X + \nabla^{\perp}\phi QY = &-g(X,Y)P\xi - g(X,Y)Q\xi - \eta(X)PY \\ &- \eta(X)QY + \eta(X)\phi PY + \eta(X)\phi QY + 2\eta(X)\eta(Y)P\xi + 2\eta(X)\eta(Y)Q\xi \\ &+ \phi P\nabla_X Y + \phi Q\nabla_X Y + Bh(X,Y) + Ch(X,Y). \end{aligned}$$

Equations (22)-(24) follow by equating horizontal, vertical and normal components. □

Lemma 3.2. *Let M be a CR-submanifolds of an P-Sasakian manifold $\overline{M}$ with semi-symmetric non-metric connection. Then*

$$\phi P[X,Y] = A_{\phi Y}Z - A_{\phi Z}Y - \eta(Z)Y + \eta(Y)Z + \eta(Z)\phi Y - \eta(Y)\phi Z \ \ \textit{for all } Y, Z \in D^{\perp}.$$

Proof. By virtue of (15) , (17) and (18) we have

$$-A_{\phi Z}Y + \nabla^{\perp}\phi Z = -g(Y,Z)\xi - \eta(Y)Z + \eta(Y)\phi Z + 2\eta(Y)\eta(Z)\xi + \phi(\nabla_Y Z + h(Y,Z))$$

for any $Y, Z \in D^{\perp}$. Using (24), we get

$$\phi P\nabla_Y Z = -A_{\phi Z}Y - g(Z,Y)\xi - \eta(Z)Y + \eta(Z)\phi Y - 2\eta(Y)\eta(Z)\xi - Bh(Y,Z)$$

Interchanging Y and Z, we find

$$\phi P\nabla_Z Y = -A_{\phi Y}Z - g(Y,Z)\xi - \eta(Y)Z + \eta(Y)\phi Z - 2\eta(Z)\eta(Y)\xi - Bh(Z,Y)$$

On subtracting above two equations, we obtain

$$\phi P[Y,Z] = A_{\phi Y}Z - A_{\phi Z}Y - \eta(Z)Y + \eta(Y)Z + \eta(Z)\phi Y - \eta(Y)\phi Z$$

for $Y, Z \in D^{\perp}$. Thus we have □

Theorem 3.3. *Let M be a CR-submanifolds of an P-Sasakian manifold $\overline{M}$ with semi-symmetric non-metric connection. Then the distribution $D^{\perp}$ is integrable if and only if*

$$A_{\phi Y}Z - A_{\phi Z}Y = \eta(Z)Y - \eta(Y)Z + \eta(Y)\phi Z - \eta(Z)\phi Y, \ \ \textit{for all } Y, Z \in D^{\perp}.$$

4. Parallel Horizontal Distributions of CR-submanifolds

Definition 4.1. *The horizontal distribution D is said to be parallel with respect to the connection ∇ on M if $\nabla_X Y \in D$ for all vector fields $X, Y \in D$.*

Proposition 4.2. *Let M be a $\xi-$vertical CR-submanifolds of a P-Sasakian manifold $\overline{M}$ with semi-symmetric non-metric connection. Then the distribution $D^{\perp}$ is parallel with respect to the connection ∇ on M, if and only if, $AX \in D^{\perp}$ for each $X \in D^{\perp}$ and $N \in TM^{\perp}$.*

Proof. Let $X, Y \in D^{\perp}$. Then using (17) and (18), we have

$$-A_{\phi Y}X + \nabla^{\perp}\phi Y = \phi\nabla_X Y + \phi h(Y,X) - \eta(X)Y - g(Y,X)\xi + \eta(X)\phi Y + 2\eta(X)\eta(Y)\xi.$$

Taking inner product with $Z \in D$, we get

$$-g(A_{\phi Y}X, Z) = g(\nabla_X Y, \phi Z).$$

Therefore,

$$\nabla_X Y = 0$$

if and only if $A_{\phi Y}X \in D^{\perp}$ for all $X \in D^{\perp}$. From which our assertion follows. □

Definition 4.3. *A CR-submanifolds M of an $P-$ Sasakian manifold $\overline{M}$ with semi-symmetric non-metric connection is said to be totally geodesic if and only if $h(X,Y)=0$ for $X \in D$ and $Y \in D^{\perp}$.*

Let M be a mixed totally geodesic $\xi-$vertical CR-submanifolds of an P$-$ Sasakian manifold $\overline{M}$ admitting a semi-symmetric non-metric connection. From (15), we have

$$(\overline{\nabla}_X \phi)N = 0$$

for $X \in D$ and $Y \in \phi D^{\perp}$. Since

$$\overline{\nabla}_X \phi N = \left(\overline{\nabla}_X \phi\right) N + \phi\left(\overline{\nabla}_X N\right)$$

so that

$$\overline{\nabla}_X \phi N = \phi\left(\overline{\nabla}_X N\right)$$

Using (17) and (18) in above equation, we get

$$\nabla_X(\phi N) = \phi A_N X + \phi \nabla^{\perp} N$$

as $\phi A_N X \in D$, so that $\nabla_X \phi N \in D$ if and only if $\phi \nabla^{\perp} N = 0$. Thus we have the following theorem.

Theorem 4.4. *Let M be a mixed totally ξ -vertical CR-submanifolds of an P-Sasakian manifold $\overline{M}$ with semi-symmetric non-metric connection. Then the normal section $N \in \phi D^{\perp}$ is $D-$parallel if and only if $\nabla_X \phi N \in D$ for $X \in D$.*

References

[1] T. Adati and K. Motsumoto, *On conformally recurrent and conformally symmetric $P-$Sasakian manifolds*, TRU Math., 13(1977), 25-32.

[2] T. Adati and T. Miyazama, *Some properties of $P-$Sasakian manifold*, TRU. Math., ,13(1)(1977), 33-42.

[3] N. S. Agashe and M. R. Chaffle, *A semi-symmetric non-metric connection of a Riemannian manifold*, Indian J. Pure and Appl. Math., 23(1992), 399-409.

[4] M. Ahmad and C. Ozgur, *Hypersurfaces of an almost r-paracontact Riemannian manifold endowed with a semi-symmetric non-metric connection*, Result. Math., 55(2009), 1-10.

[5] A. Bejancu, *CR-submanifolds of Kaehler manifold*, Proc. Amer. Math. Soc., 69(1)(1978), 134-142.

[6] A. Barman, *Semi-symmetric non-metric connection in a p-sasakian manifold*, Novi Sad J. Math., 43(2)(2013), 117-124.

[7] U. C. De and A. K. Sengupta, *CR-submanifolds of a Lorentzian para-Sasakian manifold*, Bull. Malaysian Math. Sci. Soc., 23(2000), 99-106.

[8] A. Friedmann and J. A. Schouten, *Uber die Geometric der halbsymmetrischen Ubertragung*, Math. Z., 21(1924), 211-223.

[9] M. Kobayashi, *CR-submanifolds of Sasakian manifold*, Tensor (N.S.), 35(1981), 297-307.

[10] C. Özgür, *On a class of Para-Sasakian manifolds*, Turk. J. Math., 29(2005), 249-257.

[11] B. Prasad, *Semi-invariant submanifolds of a Lorentzian para-Sasakian manifold*, Bull. Malaysian Math. Soc.(second series), 21(1998), 21-26.

[12] S. Prasad and R. H. Ojha, *Lorentzian para-contact submanifolds*, Pub. Math. Debrecen, 44(1994), 215-223.

[13] B. B. Sinha and Kalpana, *Semi-symmetric connection in an $SP-$Sasakian manifold*, Progress of Mathematics, 14(1 & 2)(1980), 77-83.

[14] J. A. Schouten, *Ricci calculus*, Springer, (1954).

[15] D. Tarafdar and U. C. De, *On a type of P-Sasakian manifold*, Extracta Mathematicae, 8(1993), 31-36.

[16] K. Yano, *On semi-symmetric metric connction*, Rev. Roumaine Math. Pures Appl., 15(1970), 1579-1586.