Int. J. Math. And Appl., 6(2–B)(2018), 81–89
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Some Decompositions of Weaker Form of Continuity

P. Sekar[1] and K. Vidhyalakshmi[2,*]

1 Principal, C. Kandaswami Naidu College For Men, Chennai, Tamil Nadu, India.

2 Research Scholar, University of Madras, Chennai, Tamil Nadu, India.

Abstract: In this paper we introduce the notions of αgs-$\mathcal{I}$-open sets, pgs-$\mathcal{I}$-open sets, sg-$\mathcal{I}$-open sets, ω_t-$\mathcal{I}$-sets, ω_{α^*}-$\mathcal{I}$-sets and ω_S-$\mathcal{I}$-sets in ideal topological spaces and investigate some of their properties. Using these notions we obtain decompositions of ω-continuity.

MSC: 8308.

Keywords: ω-open set, $\mathcal{I}$-open set, ω-continuity set, αg-$\mathcal{I}$-open set.

 Accepted on: 26.02.2018

1. Introduction

In 1961, Levine [10] obtained a decomposition of continuity which was later improved by Rose [15]. Tong [18] decomposed continuity into A-continuity and showed that his decomposition is independent of Levine's. The concept of ω-continuity was introduced and studied by Sheik John [16]. In 2000, Sundaram and Rajamani [17] obtained two different decompositions of g-continuity by introducing the notions of C(S)-sets and C*-sets in topological spaces. Recently, Noiri [13] introduced αg-$\mathcal{I}$-open sets, gp-$\mathcal{I}$-open sets, gs-$\mathcal{I}$-open sets, C(S)-$\mathcal{I}$-sets, C*-$\mathcal{I}$-sets and S*-$\mathcal{I}$-sets to obtain decompositions of g-continuity. In this paper we introduce αgs-$\mathcal{I}$-open sets, pgs-$\mathcal{I}$-open sets, sg-$\mathcal{I}$-open sets, ω_t-$\mathcal{I}$-sets, ω_{α^*}-$\mathcal{I}$-sets and ω_S-$\mathcal{I}$-sets to obtain decompositions of ω-continuity.

1.1. Preliminaries

Throughout this paper, (X,τ) and (Y,σ) (or X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), $Cl(A)$, $Int(A)$ and A^c denote the closure of A, the interior of A and the complement of A respectively.

Definition 1.1. *A subset A of a topological space (X,τ) is called semi-open [9] (respectively preopen [11], α-open [12]) if $A \subset Cl(Int(A))$ (respectively $A \subset Int(Cl(A))$, $A \subset Int(Cl(Int(A)))$). The complement of semi-open (respectively preopen, α-open) set is called semi-closed (respectively preclosed, α-closed).*

Definition 1.2 ([13])**.** *The largest semi-open (respectively preopen, α-open) set contained in A is called the semi-interior (respectively preinterior, α-interior) of A and is denoted by s-Int(A) (respectively p-Int(A), α-Int(A)). The smallest semi-closed (respectively preclosed, α-closed) set containing A is called the semi-closure (respectively preclosure, α-closure) of A and is denoted by s-Cl(A) (respectively p-Cl(A), α-Cl(A)).*

* *E-mail: vidhusat22@yahoo.co.in*

Definition 1.3. *An ideal $\mathcal{I}$ on a topological space (X,τ) is a non-empty collection of subsets of X which satisfies the following two conditions.*

(1). $A \in \mathcal{I}$ and $B \subset A$ imply $B \in \mathcal{I}$ and

(2). $A \in \mathcal{I}$ and $B \in \mathcal{I}$ imply $A \cup B \in \mathcal{I}$.

Given a topological space (X,τ) with an ideal $\mathcal{I}$ on X if $P(X)$ is the set of all subsets of X, a set operator $(.)* : P(X) \to P(X)$, called a local function [8] of A with respect to τ and $\mathcal{I}$ is defined as follows: for $A \subset X$, $A^*(\mathcal{I},\tau) = \{x \in X | U \cap A \notin \mathcal{I}$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau | x \in U\}$. A Kuratowski closure operator $Cl^*(.)$ for a topology $\tau^*(\mathcal{I},\tau)$, called the $*$-topology, finer than τ is defined by $Cl^*(A) = A \cup A^*(\mathcal{I},\tau)$ [19]. We will simply write A^* for $A^*(\mathcal{I},\tau)$ and τ^* for $\tau^*(\mathcal{I},\tau)$. If $\mathcal{I}$ is an ideal on X, then $(X,\tau,\mathcal{I})$ is called an ideal topological space.

Definition 1.4. *A subset A of a topological space (X,τ) is called:*

(1). ω-open if $F \subset Int(A)$ whenever $F \subset A$ and F is semi-closed in (X,τ) [16].

(2). αgs-open if $F \subset \alpha$-$Int(A)$ whenever $F \subset A$ and F is semi-closed in (X,τ) [6].

(3). pgs-open if $F \subset p$-$Int(A)$ whenever $F \subset A$ and F is semi-closed in (X,τ) [6].

(4). sg-open if $F \subset s$-$Int(A)$ whenever $F \subset A$ and F is semi-closed in (X,τ) [1].

(5). a t-set if $Int(A) = Int(Cl(A))$ [3].

(6). an α^*-set if $Int(A) = Int(Cl(Int(A)))$ [4].

(7). ω_t-set if $A = U \cap V$, where U is ω-open and V is a t-set in (X,τ) [6].

(8). ω_{α^*}-set if $A = U \cap V$, where U is ω-open and V is an α^*-set in (X,τ) [6].

The collection of all ω_t-sets (respectively ω_{α^*}-sets) of X is denoted by $\omega_t(X,\tau)$ (respectively $\omega_{\alpha^*}(X,\tau)$).

Theorem 1.5. *[7] Let (X,τ) be a topological space with ideals $\mathcal{I}$, $\mathcal{J}$ on X and A, B be subsets of X. Then*

(1). $A \subset B \Rightarrow A^* \subset B^*$,

(2). $A^* = Cl(A^*) \subset Cl(A)$,

(3). $A^* \cup B^* = (A \cup B)^*$,

(4). $(A^*)^* \subset A^*$,

(5). $\mathcal{I} \subset \mathcal{J} \Rightarrow A^*(\mathcal{J}) \subset A^*(\mathcal{I})$.

Definition 1.6. *[13] A subset A of an ideal topological space $(X,\tau,\mathcal{I})$ is called:*

(1). pre-$\mathcal{I}$-open if $A \subset Int(Cl*(A))$.

(2). semi-$\mathcal{I}$-open if $A \subset Cl*(Int(A))$.

(3). α-$\mathcal{I}$-open if $A \subset Int(Cl^*(Int(A)))$.

(4). t-$\mathcal{I}$-set if $Int(Cl^*(A)) = Int(A)$.

(5). α^*-$\mathcal{I}$-*set if* $Int(Cl^*(Int(A))) = Int(A)$.

(6). *S*-$\mathcal{I}$-*set if* $Cl^*(Int(A)) = Int(A)$.

In the light of these definitions, we have α-$\mathcal{I}$-$Int(A) = A \cap Int(Cl*(Int(A)))$, *p*-$\mathcal{I}$-$Int(A) = A \cap Int(Cl*(A))$ *and s*-$\mathcal{I}$-$Int(A) = A \cap Cl*(Int(A))$, *where* α-$\mathcal{I}$-$Int(A)$ *denotes* α-$\mathcal{I}$-*interior of A in* $(X, \tau, \mathcal{I})$ *which is the union of all* α-$\mathcal{I}$-*open sets of* $(X, \tau, \mathcal{I})$ *contained in A. p*-$\mathcal{I}$-$Int(A)$ *and s*-$\mathcal{I}$-$Int(A)$ *have similar meanings.*

Proposition 1.7 ([3])**.** *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space and A a subset of X. Then the following hold:*

(1). If A is a t-set, then A is a t-$\mathcal{I}$-*set.*

(2). If A is a t-$\mathcal{I}$-*set, then A is an* α^*-$\mathcal{I}$-*set.*

(3). If A is an α^*-*set, then A is an* α^*-$\mathcal{I}$-*set.*

Proposition 1.8. *In a topological space* (X, τ), *the following hold:*

(1). Every αgs-open set is pgs-open but not conversely [6].

(2). Every αgs-open set is sg-open but not conversely [14].

Proposition 1.9 ([6])**.** *Let S be a subset of* (X, τ). *If S is an* ω-*open set in X, then* $S \in \omega_t(X, \tau)$ *and* $S \in \omega_\alpha^*(X, \tau)$.

Remark 1.10 ([6])**.** *The converse of Proposition 1.9 need not be true.*

Example 1.11 ([6])**.** *Let* $X = \{a, b, c\}$ *and* $\tau = \{\emptyset, \{b\}, \{a, b\}, X\}$. *Then* $\{a, c\}$ *is both* ω_t-*set and* ω_α^*-*set, but it is not* ω-*open set.*

2. αgs-$\mathcal{I}$-open Sets, pgs-$\mathcal{I}$-open Sets and sg-$\mathcal{I}$-open Sets

Definition 2.1. *A subset A of an ideal topological space* $(X, \tau, \mathcal{I})$ *is called:*

(1). α*gs*-$\mathcal{I}$-*open if* $F \subset \alpha$-$\mathcal{I}$-$Int(A)$ *whenever* $F \subset A$ *and F is semi-closed in X.*

(2). pgs-$\mathcal{I}$-*open if* $F \subset p$-$\mathcal{I}$-$Int(A)$ *whenever* $F \subset A$ *and F is semi-closed in X.*

(3). sg-$\mathcal{I}$-*open if* $F \subset s$-$\mathcal{I}$-$Int(A)$ *whenever* $F \subset A$ *and F is semi-closed in X.*

Proposition 2.2. *For a subset of an ideal topological space, the following hold:*

(1). Every α*gs*-$\mathcal{I}$-*open set is* α*gs-open.*

(2). Every pgs-$\mathcal{I}$-*open set is pgs-open.*

(3). Every sg-$\mathcal{I}$-*open set is sg-open.*

Proof.

(1). Let A be an αgs-$\mathcal{I}$-open. Then we have, $F \subset \alpha$-$\mathcal{I}$-$Int(A)$ whenever $F \subset A$ and F is semi-closed in X. Now, $F \subset A \cap Int(Cl*(Int(A))) \subset A \cap Int(Cl(Int(A))) = \alpha - Int(A)$. This shows that A is αgs-open.

(2). Let A be a pgs-$\mathcal{I}$-open set. Then we have, $F \subset p - \mathcal{I}$-$Int(A)$ whenever $F \subset A$ and F is semi-closed in X. Now, $F \subset A \cap Int(Cl*(A)) \subset A \cap Int(Cl(A)) = p - Int(A)$. This shows that A is pgs-open.

(3). Let A be an sg-$\mathcal{I}$-open set. Then we have, $F \subset s\text{-}\mathcal{I}\text{-}Int(A)$ whenever $F \subset A$ and F is semi-closed in X. Now, $F \subset A \cap Cl^*(Int(A)) \subset A \cap Cl(Int(A)) = s - Int(A)$. This shows that A is sg-open. □

Proposition 2.3. *For a subset of an ideal topological space, the following hold:*

(1). Every ω-open set is αgs-$\mathcal{I}$-open.

(2). Every αgs-$\mathcal{I}$-open set is pgs-$\mathcal{I}$-open.

(3). Every αgs-$\mathcal{I}$-open set is sg-$\mathcal{I}$-open.

Proof.

(1). Let A be a ω-open set. Then we have, $F \subset Int(A)$ whenever $F \subset A$ and F is semi-closed in X. Now, $F \subset Int((Int(A))^*) \cup Int(A) = Int((Int(A))^*) \cup Int(Int(A)) \subset Int[(Int(A))^* \cup Int(A)] = Int(Cl^*(Int(A)))$. That is, $F \subset A \cap Int(Cl^*(Int(A))) = \alpha\text{-}\mathcal{I}\text{-}Int(A)$. Hence A is αgs-$\mathcal{I}$-open.

(2). Let A be an αgs-$\mathcal{I}$-open set. Then we have, $F \subset \alpha\text{-}\mathcal{I}\text{-}Int(A)$ whenever $F \subset A$ and F is semi-closed in X. Now, $F \subset A \cap Int(Cl^*(Int(A))) \subset A \cap Int(Cl^*(A)) = p\text{-}\mathcal{I}\text{-}Int(A)$. Hence A is pgs-$\mathcal{I}$-open.

(3). Let A be an αgs-$\mathcal{I}$-open set. Then we have, $F \subset \alpha\text{-}\mathcal{I}\text{-}Int(A)$ whenever $F \subset A$ and F is semi-closed in X. Now, $F \subset A \cap Int(Cl^*(Int(A))) \subset A \cap Cl^*(Int(A)) = s\text{-}\mathcal{I}\text{-}Int(A)$. Hence A is sg-$\mathcal{I}$-open. □

Remark 2.4. *The converses of Propositions 2.2 and 2.3 need not be true as seen from the next six Examples.*

Example 2.5. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $\{a, c\}$ is an αgs-open set, but it is not an αgs-$\mathcal{I}$-open set.*

Example 2.6. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. Then $\{b, d\}$ is a pgs-open set, but it is not a pgs-$\mathcal{I}$-open set.*

Example 2.7. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{a, b, c\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $\{a, c, d\}$ is a sg-open set, but it is not an sg-$\mathcal{I}$-open set.*

Example 2.8. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then $\{a, c, d\}$ is an αgs-$\mathcal{I}$-open set, but it is not a ω-open set.*

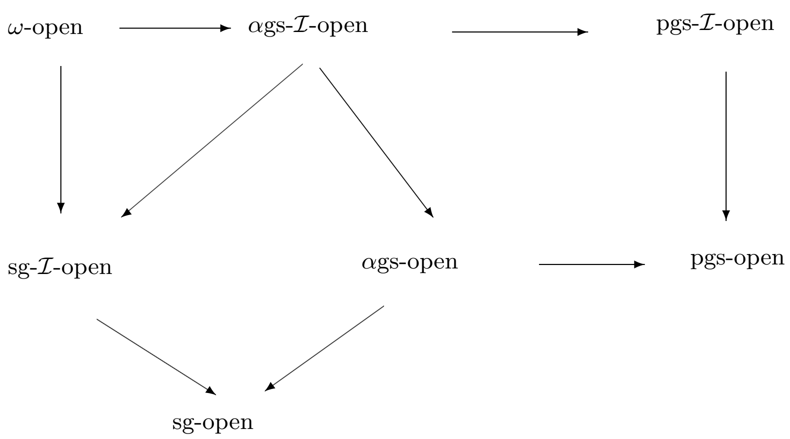
Example 2.9. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then $\{b, c\}$ is a pgs-$\mathcal{I}$-open set, but it is not an αgs-$\mathcal{I}$-open set.*

Example 2.10. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{d\}, \{a, d\}, X\}$ and $\mathcal{I} = \{\emptyset, \{d\}\}$. Then $\{a, c\}$ is an sg-$\mathcal{I}$-open set, but it is not an αgs-$\mathcal{I}$-open set.*

Example 2.11. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $\{b, c\}$ is an sg-$\mathcal{I}$-open set, but it is not a pgs-$\mathcal{I}$-open set.*

Example 2.12. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then $\{b, c\}$ is a pgs-$\mathcal{I}$-open set, but it is not an sg-$\mathcal{I}$-open set.*

From Propositions 1.8, 2.2, 2.3 and Remark 2.4, we have the following diagram.



However, none of the above implications is reversible and that the notions of pgs-$\mathcal{I}$-open sets and sg-$\mathcal{I}$-open sets are independent.

3. ω_t-$\mathcal{I}$-sets, ω_{α^*}-$\mathcal{I}$-sets and ω_S-$\mathcal{I}$-sets

Definition 3.1. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called:*

(1). ω_t-$\mathcal{I}$-set if $A = U \cap V$, where U is ω-open and V is a t-$\mathcal{I}$-set.

(2). ω_{α^*}-$\mathcal{I}$-set if $A = U \cap V$, where U is ω-open and V is an α^*-$\mathcal{I}$-set.

(3). ω_S-$\mathcal{I}$-set if $A = U \cap V$, where U is ω-open and V is an S-$\mathcal{I}$-set.

Proposition 3.2. *For a subset of an ideal topological space, the following hold:*

(1). Every t-$\mathcal{I}$-set is ω_t-$\mathcal{I}$-set.

(2). Every α^*-$\mathcal{I}$-set is ω_{α^*}-$\mathcal{I}$-set.

(3). Every S-$\mathcal{I}$-set is ω_S-$\mathcal{I}$-set.

(4). Every ω-open set is ω_t-$\mathcal{I}$-set.

(5). Every ω-open set is ω_{α^*}-$\mathcal{I}$-set.

(6). Every ω-open set is ω_S-$\mathcal{I}$-set.

Remark 3.3. *The converses of Proposition 3.2 need not be true as seen from the following Examples.*

Example 3.4. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{d\}, \{a, c\}, \{a, c, d\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{d\}, \{a, d\}\}$. Then $\{c, d\}$ is a ω_t-$\mathcal{I}$-set, but it is not a t-$\mathcal{I}$-set.*

Example 3.5. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. Then $\{a\}$ is a ω_{α^*}-$\mathcal{I}$-set, but it is not an α^*-$\mathcal{I}$-set.*

Example 3.6. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{d\}, \{a, d\}, X\}$ and $\mathcal{I} = \{\emptyset, \{d\}\}$. Then $\{a\}$ is a ω_S-$\mathcal{I}$-set, but it is not an S-$\mathcal{I}$-set.*

Example 3.7. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. Then $\{b, c\}$ is both ω_t-$\mathcal{I}$-set and ω_{α^*}-$\mathcal{I}$-set, but it is not a ω-open set.*

Example 3.8. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then $\{a, c\}$ is a ω_S-$\mathcal{I}$-set, but it is not a ω-open set.*

Proposition 3.9. *For a subset of an ideal topological space, the following hold:*

(1). Every ω_t-set is ω_t-$\mathcal{I}$-set.

(2). Every ω_t-set is ω_{α^*}-set.

(3). Every ω_{α^*}-set is ω_{α^*}-$\mathcal{I}$-set.

(4). Every ω_t-$\mathcal{I}$-set is ω_{α^*}-$\mathcal{I}$-set.

(5). Every ω_S-$\mathcal{I}$-set is ω_{α^*}-$\mathcal{I}$-set.

Proof. (1), (2), (3) and (4), the proof follows from Proposition 1.7. (5) If A is a S-$\mathcal{I}$-set, then $Cl^*(Int(A)) = Int(A) \Rightarrow Int(Cl^*(Int(A))) = Int(A)$. Hence it is an α^*-$\mathcal{I}$-set. □

Remark 3.10. *The converses of Proposition 3.9 need not be true as seen from the next four Examples.*

Example 3.11. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{c\}, \{b, c\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, X\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then $\{c, d\}$ is a ω_t-$\mathcal{I}$-set, but it is not a ω_t-set.*

Example 3.12. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $\{b, c\}$ is a ω_{α^*}-set, but it is not a ω_t-set.*

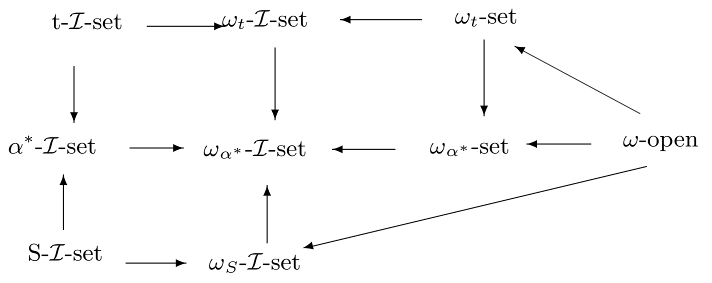
Example 3.13. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $\{a, c\}$ is a ω_{α^*}-$\mathcal{I}$-set, but it is neither ω_{α^*}-set nor ω_t-$\mathcal{I}$-set.*

Example 3.14. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. Then $\{a, b, d\}$ is a ω_{α^*}-$\mathcal{I}$-set, but it is not a ω_S-$\mathcal{I}$-set.*

Example 3.15. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then $\{a, c\}$ is α^*-$\mathcal{I}$-set, but it is not t-$\mathcal{I}$-set.*

Example 3.16. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $\{b, c\}$ is α^*-$\mathcal{I}$-set, but it is not S-$\mathcal{I}$-set.*

By Propositions 1.7, 1.9, 3.2 and 3.9, we have the following diagram.



None of the above implications is reversible.

Example 3.17. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then $\{b,\ c\}$ is a ω_{α^*}-set, but it is not a ω_t-$\mathcal{I}$-set.*

Example 3.18. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{c\}, \{b, c\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, X\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then $\{c,\ d\}$ is a ω_t-$\mathcal{I}$-set, but it is not a ω_{α^*}-set.*

Example 3.19. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $\{b,\ c\}$ is a ω_S-$\mathcal{I}$-set, but it is not a ω_t-$\mathcal{I}$-set.*

Example 3.20. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{d\}, \{a, d\}, X\}$ and $\mathcal{I} = \{\emptyset, \{d\}\}$. Then $\{a,\ b,\ c\}$ is a ω_t-$\mathcal{I}$-set, but it is not a ω_S-$\mathcal{I}$-set.*

Example 3.21. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. Then $\{a,\ b\}$ is pgs-$\mathcal{I}$-open set, but it is not ω_t-$\mathcal{I}$-set.*

Example 3.22. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $\{a,\ c\}$ is ω_t-$\mathcal{I}$-set, but it is not pgs-$\mathcal{I}$-open set .*

Example 3.23. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. Then $\{a,\ b\}$ is αgs-$\mathcal{I}$-open set, but it is not $\omega_\alpha *$-$\mathcal{I}$-set.*

Example 3.24. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $\{b,\ c\}$ is $\omega_\alpha *$-$\mathcal{I}$-set, but it is not αgs-$\mathcal{I}$-open set.*

Example 3.25. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $\{b,\ c\}$ is sg-$\mathcal{I}$-open set, but it is not ω_S-$\mathcal{I}$-set.*

Example 3.26. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $\{b,\ c,\ d\}$ is ω_S-$\mathcal{I}$-set, but it is not sg-$\mathcal{I}$-open set.*

Remark 3.27. *From the above ten examples, we have*

(1). The notions of ω_t-$\mathcal{I}$-sets and ω_{α^*}-sets are independent.

(2). The notions of ω_t-$\mathcal{I}$-sets and ω_S-$\mathcal{I}$-sets are independent.

(3). The notions of pgs-$\mathcal{I}$-open sets and ω_t-$\mathcal{I}$-sets are independent.

(4). The notions of αgs-$\mathcal{I}$-open sets and ω_{α^*}-$\mathcal{I}$-sets are independent.

(5). The notions of sg-$\mathcal{I}$-open sets and ω_S-$\mathcal{I}$-sets are independent.

Proposition 3.28. *A subset A of (X, τ, $\mathcal{I}$) is ω-open if and only if it is both pgs-$\mathcal{I}$-open and ω_t-$\mathcal{I}$-set.*

Proof. Necessity is trivial. We prove the sufficiency. Assume that A is pgs-$\mathcal{I}$-open and ω_t-$\mathcal{I}$-set in X. Let F⊂A and F be semi-closed in X. Since A is a ω_t-$\mathcal{I}$-set in X, $A = U \cap V$, where U is ω-open and V is a t-$\mathcal{I}$-set. Now F is semi-closed and U is ω-open implies $F \subset Int(U)$. Since A is pgs-$\mathcal{I}$-open, $F \subset p\text{-}\mathcal{I}\text{-}Int(A) = A \cap Int(Cl^*(A)) = (U \cap V) \cap Int(Cl^*(U \cap V)) \subset (U \cap V) \cap Int(Cl^*(U) \cap Cl^*(V)) = U \cap V \cap Int(Cl^*(U)) \cap Int(Cl^*(V))$. Hence $F \subset Int(Cl^*(V))$. But V is a t-$\mathcal{I}$-set, therefore $Int(V) = Int(Cl^*(V))$, which implies $F \subset Int(V)$. Therefore $F \subset Int(U) \cap Int(V) = Int(U \cap V) = Int(A)$. Hence A is ω-open in X. □

Proposition 3.29. *A subset A of $(X, \tau, \mathcal{I})$ is ω-open if and only if it is both αgs-$\mathcal{I}$-open and ω_{α^*}-$\mathcal{I}$-set.*

Proof. Necessity is trivial. We prove the sufficiency. Assume that A is αgs-$\mathcal{I}$-open and ω_{α^*}-$\mathcal{I}$-set in X. Let $F \subset A$ and F be semi-closed in X. Since A is a ω_{α^*}-$\mathcal{I}$-set in X, $A = U \cap V$, where U is ω-open and V is an α*-$\mathcal{I}$-set. Now F is semi-closed and U is ω-open implies $F \subset Int(U)$. Since A is αgs-$\mathcal{I}$-open, $F \subset \alpha\text{-}\mathcal{I}\text{-}Int(A) = A \cap Int(Cl^*(Int(A))) = (U \cap V) \cap Int(Cl^*(Int(U \cap V))) = (U \cap V) \cap Int(Cl^*(Int(U) \cap Int(V))) \subset (U \cap V) \cap Int(Cl^*(Int(U)) \cap Cl^*(Int(V))) = U \cap V \cap IntCl^*(Int(U))) \cap Int(Cl^*(Int(V)))$. Hence $F \subset Int(Cl^*(Int(V)))$. But V is an α*-$\mathcal{I}$-set, therefore $Int(V) = Int(Cl^*(Int(V)))$, which implies $F \subset Int(V)$. Therefore $F \subset Int(U) \cap Int(V) = Int(U \cap V) = Int(A)$. Hence A is ω-open in X. □

Proposition 3.30. *A subset A of $(X, \tau, \mathcal{I})$ is ω-open if and only if it is both sg-$\mathcal{I}$-open and ω_S-$\mathcal{I}$-set.*

Proof. Necessity is trivial. We prove the sufficiency. Assume that A is sg-$\mathcal{I}$-open and ω_S-$\mathcal{I}$-set in X. Let $F \subset A$ and F be semi-closed in X. Since A is a ω_S-$\mathcal{I}$-set in X, $A = U \cap V$, where U is ω-open and V is an S-$\mathcal{I}$-set. Now F is semi-closed and U is ω-open implies $F \subset Int(U)$. Since A is sg-$\mathcal{I}$-open, $F \subset s\text{-}\mathcal{I}\text{-}int(A) = A \cap Cl^*(Int(A)) = (U \cap V) \cap Cl^*(Int(U \cap V)) \subset Cl^*(Int(U \cap V)) = Cl^*(Int(U) \cap Int(V)) \subset Cl^*(Int(U)) \cap Cl^*(Int(V))$. Hence $F \subset Cl^*(Int(V))$. But V is an S-$\mathcal{I}$-set, therefore $Int(V) = Cl^*(Int(V))$, which implies $F \subset Int(V)$. Therefore $F \subset Int(U) \cap Int(V) = Int(U \cap V) = Int(A)$. Hence A is ω-open in X. □

4. Decompositions of ω-continuity

Definition 4.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is called ω-continuous [16] if for every $V \in \sigma$, $f^{-1}(V)$ is ω-open in (X, τ).*

Definition 4.2 ([6]). *A function $f : (X, \tau) \to (Y, \sigma)$ is called αgs-continuous (respectively pgs-continuous) if for every $V \in \sigma$, $f^{-1}(V)$ is αgs-open (respectively pgs-open) in (X, τ).*

Definition 4.3 ([6]). *A function $f : (X, \tau) \to (Y, \sigma)$ is called*

(1). ω_t-continuous if for every $V \in \sigma$, $f^{-1}(V) \in \omega_t(X, \tau)$.

(2). ω_{α^*}-continuous if for every $V \in \sigma$, $f^{-1}(V) \in \omega_{\alpha^*}(X, \tau)$.

Definition 4.4. *A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is called αgs-$\mathcal{I}$-continuous (respectively pgs-$\mathcal{I}$-continuous, sg-$\mathcal{I}$-continuous, ω_t-$\mathcal{I}$-continuous, ω_{α^*}-$\mathcal{I}$-continuous and ω_S-$\mathcal{I}$-continuous) if for every $V \in \sigma$, $f^{-1}(V)$ is αgs-$\mathcal{I}$-open (respectively pgs-$\mathcal{I}$-open, sg-$\mathcal{I}$-open, ω_t-$\mathcal{I}$-set, ω_{α^*}-$\mathcal{I}$-set and ω_S-$\mathcal{I}$-set) in $(X, \tau, \mathcal{I})$. From Propositions 3.28, 3.29 and 3.30, we have the following decompositions of ω-continuity.*

Theorem 4.5. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. For a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, the following properties are equivalent.*

(1). f is ω-continuous.

(2). f is pgs-$\mathcal{I}$-continuous and ω_t-$\mathcal{I}$-continuous.

(3). f is αgs-$\mathcal{I}$-continuous and ω_{α^*}-$\mathcal{I}$-continuous.

(4). f is sg-$\mathcal{I}$-continuous and ω_S-$\mathcal{I}$-continuous.

Corollary 4.6 ([6]). *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $\mathcal{I} = \{\emptyset\}$. For a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, the following properties are equivalent.*

(1). f is ω-continuous.

(2). f is pgs-continuous and ω_t-continuous.

(3). f is αgs-continuous and ω_{α^*}-continuous.

Proof. Since $\mathcal{I} = \{\emptyset\}$, we have $A^* = Cl(A)$ and $Cl^*(A) = A^* \cup A = Cl(A)$ for any subset A of X. Therefore, we obtain

(1). A is αgs-$\mathcal{I}$-open (respectively pgs-$\mathcal{I}$-open) if and only if it is αgs-open (respectively pgs-open) and

(2). A is ω_t-$\mathcal{I}$-set (respectively ω_{α^*}-$\mathcal{I}$-set) if and only if it is ω_t-set (respectively ω_{α^*}-set). The proof follows immediately from Theorem 4.5. □

References

[1] P. Bhattacharyya and B. K. Lahiri, *Semi-generalized closed sets in topology*, Indian J. Math., 29(1987), 375-382.

[2] J. Dontchev, *On pre-I-open sets and a decomposition of $\mathcal{I}$-continuity*, Banyan Math. J., 2(1996).

[3] E. Hatir and T. Noiri, *On decompositions of continuity via idealization*, Acta Math. Hungar., 96(4)(2002), 341-349.

[4] E. Hatir, T. Noiri and S. Yuksel, *A decomposition of continuity*, Acta Math. Hungar., 70(1996), 145-150.

[5] S. Jafari, K. Viswanathan and M. Rajamani, *A decomposition of α-continuity and αgs-continuity in topological spaces*, Fasciculi Math., 41(2009), 73-79.

[6] S. Jafari, T. Noiri, K. Viswanathan and M. Rajamani, *Decompositions of ω-continuity in topological spaces*, J. Adv. Math. Studies, 3(2)(2010), 49-55.

[7] D. Jankovic and T. R. Hamlett, *New topologies from old via ideals*, Amer. Math. Monthly, 97(4)(1990), 295-310.

[8] K. Kuratowski, *Topology*, Vol. 1, Academic Press, New York, (1966).

[9] N. Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly, 70(1963), 36-41.

[10] N. Levine, *A decomposition of continuity in topological spaces*, Amer. Math. Monthly, 68(1961), 44-46.

[11] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, *On precontinuous and weak precontinuous mappings*, Proc. Math. Phys. Soc. Egypt, 53(1982), 47-53.

[12] O. Njastad, *On some classes of nearly open sets*, Pacific J. Math., 15(1965), 961-970.

[13] T. Noiri, M. Rajamani and V. Inthumathi, *On decompositions of g-continuity via idealization*, Bull. Cal. Math. Soc., 99(4)(2007), 415-424.

[14] M. Rajamani and K. Viswanathan, *On αgs-closed sets in topological spaces*, Acta Ciencia Indica, 31(1)(2005), 293-303.

[15] D. Rose, *On Levine's decomposition of continuity*, Canad. Math. Bull., 21(1978), 477-481.

[16] M. Sheik John, *A study on generalizations of closed sets and continuous maps in topological and bitopological spaces*, Ph.D Thesis, Bharathiar University, Coimbatore, India, (2002).

[17] P. Sundaram and M. Rajamani, *On decompositions of generalized continuous maps in topological spaces*, Acta Ciencia Indica, 26(2)(2000), 101-104.

[18] J. Tong, *A decomposition of continuity*, Acta Math. Hungar., 48(1986), 11-15.

[19] R. Vaidyanathaswamy, *The localization theory in set topology*, Proc. Indian Acad. Sci., Sect A, 20(1944), 51-61.