Int. J. Math. And Appl., 6(3)(2018), 137–144
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

Centered Polygonal Sum Labeling of Graphs

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Abstract: Let G be a (p, q) graph. A graph G admits centered polygonal sum labeling if a one to one function $f : V(G) \to N$ (where N is a set of all non-negative integers) that induces a bijection $f^+ : E(G) \to \{{}_{\mathbf{C}}\mathbf{P_k(1)}, {}_{\mathbf{C}}\mathbf{P_k(2)}, \cdots, {}_{\mathbf{C}}\mathbf{P_k(n)}\}$ of the edges of G defined by $f^+(uv) = f(u) + f(v)$ for every $e = uv\epsilon E(G)$, where ${}_{\mathbf{C}}\mathbf{P_k(1)}, {}_{\mathbf{C}}\mathbf{P_k(2)}, \cdots, {}_{\mathbf{C}}\mathbf{P_k(q)}$, $k \geq 3$ are the first q centered polygonal numbers. A graph which admits such labeling is called centered polygonal sum graph. In this paper we characterize some families of graphs such as Path, Comb, Star graph, Subdivision of star, Bistar, $S_{m,n,r}$, Coconut tree admit centered polygonal sum labeling.

MSC: 05C78.

Keywords: Centered polygonal numbers, Centered polygonal sum labeling, Centered polygonal sum graph.

Accepted on: 16.05.2018

1. Introduction

Through out this paper, we consider non-trivial, finite, simple undirected graph. The set of vertices and edges of a graph $G(p, q)$ will be denoted by $V(G)$ and $E(G)$ respectively. The various graph theoretic notations and terminology we follow Frank Harary [2] and for number theory we follow Burton [1]. Many kind of labeling have been defined and studied by many authors and an excellent survey of graph labelings can be found in [3]. In 2013, S. Murugesan [4] introduced a labeling called centered triangular sum labeling. An injection $f : V(G) \to N$ that induces a bijection $f^+ : E(G) \to \{{}_{\mathbf{C}}\mathbf{P_3(1)}, {}_{\mathbf{C}}\mathbf{P_3(2)}, \cdots, {}_{\mathbf{C}}\mathbf{P_3(q)}\}$ of the edges of G defined by $f^+(uv) = f(u)+f(v)$ for every $e = uv \in E(G)$. A graph which admits such labeling is called centered triangular sum graph. In this paper, we define generalized centered polygonal sum labeling and prove some families of graphs such as Path, Comb, Star graph, Subdivision of star, Bistar, $S_{m,n,r}$, Coconut tree admit centered polygonal sum labeling.

Definition 1.1. *A centered polygonal number is a centered figurative number that represents a polygon with dot in the center and all other dots surrounding the center in successive polygon layers. If the n^{th} centered polygonal number is denoted by ${}_{\mathbf{C}}\mathbf{P_k(n)}$, then ${}_{\mathbf{C}}\mathbf{P_k(n)} = \frac{k}{2}[n(n-1)] + 1$, where $k \geq 3$ is the number of sides of the polygon. For $k = 3$ it gives centered triangular numbers, For $k = 4$ it gives centered tetragonal numbers and so on.*

Definition 1.2. *A centered polygonal sum labeling of a graph G is a one to one function $f : V(G) \to N$ (where N is set of all non-negative integers) that induces a bijection $f^+ : E(G) \to \{{}_{\mathbf{C}}\mathbf{P_k(1)}, {}_{\mathbf{C}}\mathbf{P_k(2)}, \cdots, {}_{\mathbf{C}}\mathbf{P_k(q)}\}$ of the edges of G defined by $f^+(uv) = f(u) + f(v)$ for every $e = uv \in E(G)$. The graph which admits such labeling is called centered polygonal sum*

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graph. For $k = 3$ the above labeling gives centered triangular sum labeling, For $k = 4$ it gives centered tetragonal sum labeling and so on.

2. Main Results

Theorem 2.1. *The path P_n admits centered polygonal sum labeling.*

Proof. Let $P_n : u_1u_2u_3 \cdots u_n$ be a path and $v_i = u_iu_{i+1}$ $(1 \leq i \leq n-1)$ be the edges. For $i = 1, 2, 3, \ldots, n$, define

$$f(u_i) = \begin{cases} \frac{k}{4}(i-1)^2 & \text{if } i \text{ is odd} \\ \frac{1}{4}[ki^2 - 2ki + 4] & \text{if } i \text{ is even} \end{cases}$$

We will prove that the induced edge labels obtained by the sum of the labels of end vertices are the first $n-1$ centered polygonal numbers.

Case (1): i is odd. For $i = 1, 2, 3, \ldots, n-1$,

$$\begin{aligned} f(u_i) + f(u_{i+1}) &= \frac{k}{4}(i-1)^2 + \frac{1}{4}[k(i+1)^2 - 2k(i+1) + 4] \\ &= \frac{k}{2}[i(i-1)] + 1 \\ &= {}_{\mathbf{C}}\mathbf{P_k(i)} \\ &= f^+(v_i). \end{aligned}$$

Case (2): i is even. For $i = 1, 2, 3, \ldots, n-1$,

$$\begin{aligned} f(u_i) + f(u_{i+1}) &= \frac{1}{4}[ki^2 - 2ki + 4] + \frac{k}{4}((i+1)-1)^2 \\ &= \frac{k}{2}[i(i-1)] + 1 \\ &= {}_{\mathbf{C}}\mathbf{P_k(i)} \\ &= f^+(v_i). \end{aligned}$$

Thus the induced edge labels are the first $n-1$ centered polygonal numbers. Hence P_n admits centered polygonal sum labeling. $\square$

Example 2.2. *The centered tetragonal sum labeling of P_9 is shown in the following figure.*

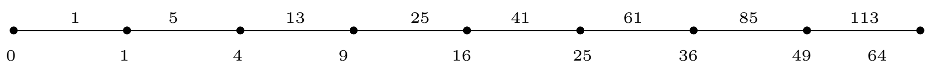

Figure 1

Theorem 2.3. *The comb $P_n \odot K_1$ admits centered polygonal sum labeling.*

Proof. Let $P_n : u_1u_2u_3 \cdots u_n$ be a path and $v_i = u_iu_{i+1}$ $(1 \leq i \leq n-1)$ be the edges. Let $w_1, w_2, \cdots, w_n$ be the pendant vertices adjacent to $u_1, u_2, \cdots, u_n$ respectively and $t_i = u_iw_i (1 \leq i \leq n)$ be the edges. For $i = 1, 2, 3, \ldots, n$, define

$$f(u_i) = \begin{cases} \frac{k}{4}(i-1)^2 & \text{if } i \text{ is odd} \\ \frac{1}{4}[ki^2 - 2ki + 4] & \text{if } i \text{ is even} \end{cases}$$

and

$$f(w_i) = \begin{cases} \frac{1}{4}[ki^2 + 4k(n-1)i + (2kn^2 - 6kn + 3k + 4)] & \text{if } i \text{ is odd} \\ \frac{1}{4}[ki^2 + 4k(n-1)i + (2kn^2 - 6kn + 4k)] & \text{if } i \text{ is even} \end{cases}$$

We will prove that the induced edge labels obtained by the sum of the labels of end vertices are the first $2n-1$ centered polygonal numbers.

Case (1): i is odd. For $i = 1, 2, 3, \ldots, n-1$,

$$\begin{aligned} f(u_i) + f(u_{i+1}) &= \frac{k}{4}(i-1)^2 + \frac{1}{4}[k(i+1)^2 - 2k(i+1) + 4] \\ &= \frac{k}{2}[i(i-1)] + 1 \\ &= {}_{\mathbf{C}}\mathbf{P_k(i)} \\ &= f^+(v_i). \end{aligned}$$

and for i = 1,2,3,. . . ,n,

$$\begin{aligned} f(u_i) + f(w_i) &= \frac{k}{4}(i-1)^2 + \frac{1}{4}[ki^2 + 4k(n-1)i + (2kn^2 - 6kn + 3k + 4)] \\ &= \frac{k}{2}[(i+n-1)(i+n-2)] + 1 \\ &= {}_{\mathbf{C}}\mathbf{P_k(n+i-1)} \\ &= f^+(t_i). \end{aligned}$$

Case (2): i is even. For $i = 1, 2, 3, \ldots, n-1$,

$$\begin{aligned} f(u_i) + f(u_{i+1}) &= \frac{1}{4}[ki^2 - 2ki + 4] + \frac{k}{4}((i+1)-1)^2 \\ &= \frac{k}{2}[i(i-1)] + 1 \\ &= {}_{\mathbf{C}}\mathbf{P_k(i)} \\ &= f^+(v_i). \end{aligned}$$

and for i = 1,2,3,. . . ,n,

$$\begin{aligned} f(u_i) + f(w_i) &= \frac{1}{4}[ki^2 - 2ki + 4] + \frac{1}{4}[(ki^2 + 4k(n-1)i + (2kn^2 - 6kn + 4k)] \\ &= \frac{k}{2}[(i+n-1)(i+n-2)] + 1 \\ &= {}_{\mathbf{C}}\mathbf{P_k(n+i-1)} \\ &= f^+(t_i). \end{aligned}$$

Thus the induced edge labels are the first $2n-1$ centered polygonal numbers. Hence $P_n \odot K_1$ admits centered polygonal sum labeling. ☐

Example 2.4. *The centered pentagonal sum labeling of $P_7 \odot K_1$ is shown in the following figure.*

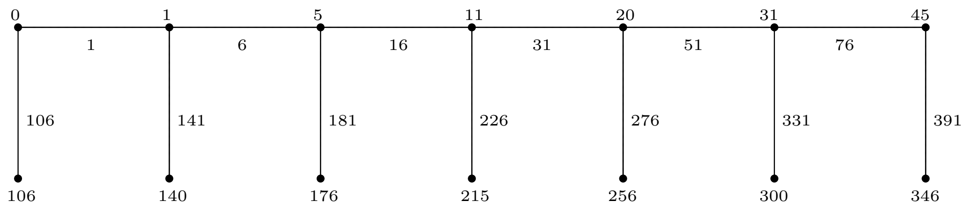

Figure 2

Theorem 2.5. *The star graph $K_{1,n}$ admits centered polygonal sum labeling.*

Proof. Let v be the apex vertex and let $v_1, v_2, \cdots, v_n$ be the pendant vertices of the star $K_{1,n}$. Define f by

$$\begin{aligned} f(v) &= 0. \\ f(v_i) &= \frac{k}{2}[i(i-1)] + 1, \quad 1 \leq i \leq n. \end{aligned}$$

We see that the induced edge labels are the first n centered polygonal numbers. Hence the star graph $K_{1,n}$ admits centered polygonal sum labeling. □

Example 2.6. *The centered heptagonal sum labeling of $K_{1,13}$ is shown in the following figure.*

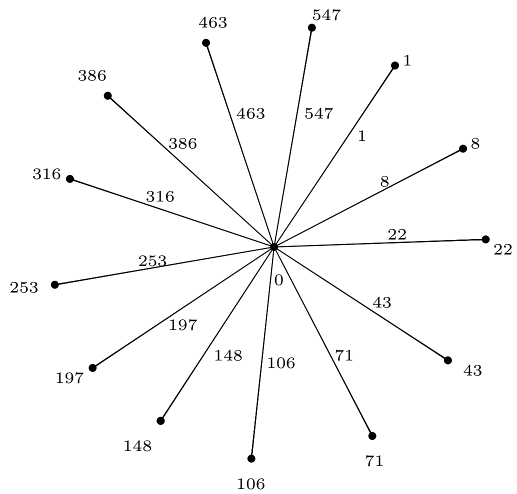

*Figure*3

Theorem 2.7. *$S(K_{1,n})$, the subdivision of the star $K_{1,n}$ admits centered polygonal sum labeling.*

Proof. Let $V(S(K_{1,n})) = \{v, v_i, u_i : 1 \leq i \leq n\}$ and $E(S(K_{1,n})) = \{vv_i, v_iu_i : 1 \leq i \leq n\}$. Define f by

$$\begin{aligned} f(v) &= 0. \\ f(v_i) &= \frac{k}{2}[i(i-1)] + 1, \quad 1 \leq i \leq n. \\ f(u_i) &= \frac{k}{2}[n(n-1) + 2in], \quad 1 \leq i \leq n. \end{aligned}$$

We will prove that the induced edge labels obtained by the sum of the labels of end vertices are the first 2n centered polygonal numbers.

$$\begin{aligned}
f(v)+f(v_i) &= 0+\frac{k}{2}[i(i-1)]+1\\
&= \frac{k}{2}[i(i-1)]+1\\
&= {}_{\mathbf{C}}\mathbf{P_k(i)}.\\
f(v_i)+f(u_i) &= \left\{\frac{k}{2}[i(i-1)]+1\right\}+\frac{k}{2}\left\{n(n-1)+2in\right\}\\
&= \frac{k}{2}[(i+n)^2-(i+n)]+1\\
&= {}_{\mathbf{C}}\mathbf{P_k(n+i)}.
\end{aligned}$$

Thus the induced edge labels are the first 2n centered polygonal numbers. Hence $S(K_{1,n})$ admits centered polygonal sum labeling. $\square$

Example 2.8. *The centered tetrogonal sum labeling of $S(K_{1,5})$ is shown in the following figure.*

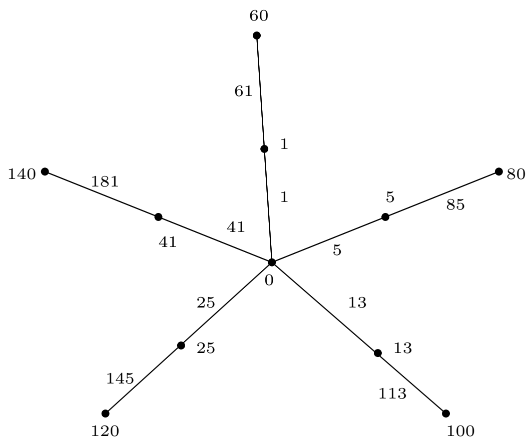

Figure 4

Theorem 2.9. *The bistar $B_{m,n}$ admits centered polygonal sum labeling.*

Proof. Let $B_{m,n}=\{u,v,u_i,v_j:1\leq i\leq m,1\leq j\leq n\}$ and $E(B_{m,n})=\{uv,uu_i,vv_j:1\leq i\leq m,1\leq i\leq n\}$. Define f by

$$\begin{aligned}
f(u) &= 0.\\
f(v) &= 1.\\
f(u_i) &= \frac{k}{2}[i(i+1)]+1, \quad 1\leq i\leq m.\\
f(v_j) &= \frac{k}{2}[j^2+(2m+1)j+(m^2+m)], \quad 1\leq j\leq n.
\end{aligned}$$

We see that the induced edge labels are the first m+n+1 centered polygonal numbers. Hence $B_{m,n}$ admits centered polygonal sum labeling. $\square$

Example 2.10. *The centered hexagonal sum labeling of $B_{6,5}$ is shown in the following figure.*

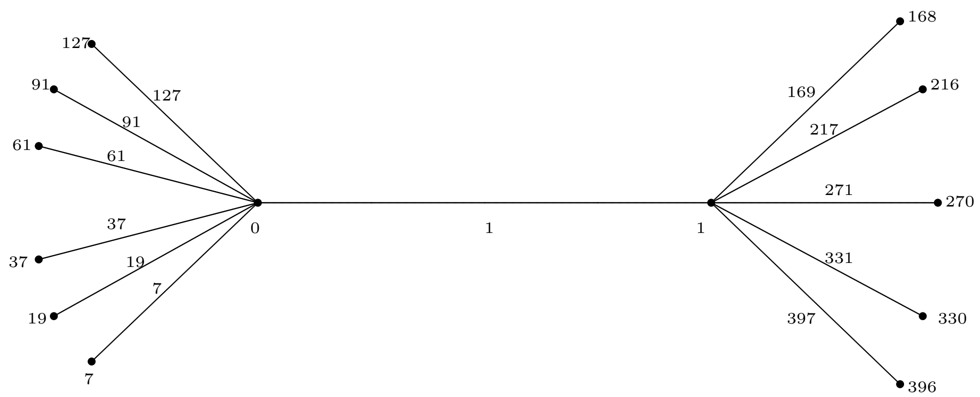

Figure 5

Theorem 2.11. *The graph $S_{m,n,r}$ admits centered polygonal sum labeling.*

Proof. Let P_n : $u_1u_2u_3\cdots u_{r+1}$ be a path of length r $(r \geq 1)$. Let $v_1, v_2, \ldots, v_m$ be the vertices adjacent to u_1 and $w_1, w_2, \ldots w_n$ be the vertices adjacent to u_r. For $i = 1, 2, 3, \ldots, r+1$, define

$$f(u_i) = \begin{cases} \frac{k}{4}(i-1)^2 & \text{if } i \text{ is odd} \\ \frac{1}{4}[ki^2 - 2ki + 4] & \text{if } i \text{ is even} \end{cases}$$

$$f(v_j) = \frac{k}{2}[(j+r)(j+r-1)] + 1, \quad 1 \leq j \leq m.$$

$$f(w_l) = \{\frac{k}{2}[(r+m+l)(r+m+l-1)] + 1\} - f(u_r), \quad 1 \leq l \leq n.$$

We see that the induced edge labels are the first m+n+r centered polygonal numbers. Hence $S_{m,n,r}$ admits centered polygonal sum labeling. □

Example 2.12. *The centered nonagonal sum labeling of $S_{6,5,5}$ is shown in the following figure.*

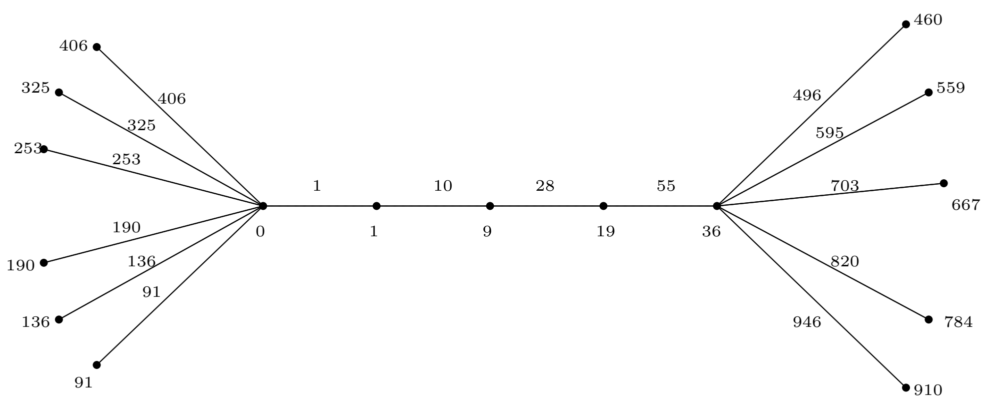

Figure 6

Theorem 2.13. *Coconut tree admits centered polygonal sum labeling.*

Proof. Let P_n : $u_0u_1u_2\cdots u_i$ be a path of length i $(i \geq 1)$ and $u_{i+1}, u_{i+2}, \cdots, u_n$ be the pendant vertices, being adjacent with u_0. For $0 \leq j \leq i$, define

$$f(u_j) = \begin{cases} \frac{k}{4}j^2 & \text{if } j \text{ is even} \\ \frac{1}{4}(kj^2 - k + 4) & \text{if } j \text{ is odd} \end{cases}$$

and for $i+1 \leq m \leq n$, define

$$f(u_m) = \tfrac{k}{2}[m(m-1)] + 1.$$

Case (1): j is odd. For $j = 0, 1, 2, \ldots, i-1$,

$$\begin{aligned}
f(u_j) + f(u_{j+1}) &= \frac{1}{4}[kj^2 - k + 4] + \frac{k}{4}(j+1)^2 \\
&= \frac{1}{4}[kj^2 - k + 4 + kj^2 + 2kj + k] \\
&= \frac{1}{4}[2kj^2 + 2kj + 4] \\
&= \frac{k}{2}[j(j+1)] + 1 \\
&= {}_{\mathbf{C}}\mathbf{P_k}(\mathbf{j+1}).
\end{aligned}$$

Case (2): j is even. For $j = 0, 1, 2, \ldots, i-1$,

$$\begin{aligned}
f(u_j) + f(u_{j+1}) &= \frac{k}{4}j^2 + \frac{1}{4}[k(j+1)^2 - k + 4] \\
&= \frac{1}{4}[2kj^2 + 2kj + 4] \\
&= \frac{k}{2}[j(j+1)] + 1 \\
&= {}_{\mathbf{C}}\mathbf{P_k}(\mathbf{j+1}).
\end{aligned}$$

and for $j = 0$, $m = i+1, i+2, \ldots, n$,

$$\begin{aligned}
f(u_j) + f(u_m) &= 0 + \frac{k}{4}[m(m-1)] + 1 \\
&= \frac{k}{4}[m(m-1)] + 1 \\
&= {}_{\mathbf{C}}\mathbf{P_k}(\mathbf{m}).
\end{aligned}$$

We see that the induced edge labels are the first n centered polygonal numbers. Hence coconut tree admits centered polygonal sum labeling. □

Example 2.14. *The centered octagonal sum labeling of a coconut tree is shown in the following figure.*

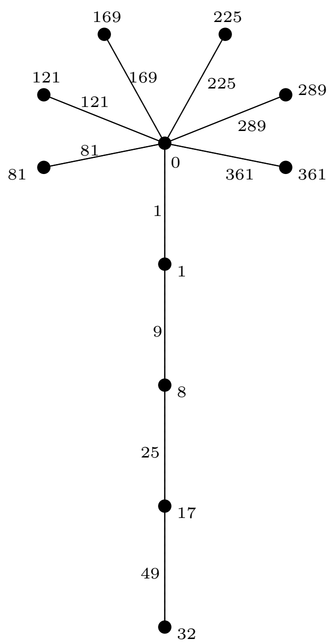

Figure 7

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