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On the Hosoya Polynomial and Wiener Index of Semitotal Point Graph and its Complement

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Abstract: The Wiener index $W(G)$ of a graph G is the sum of distances between all (unordered) pairs of vertices of G. In this paper, we obtain the Hosoya polynomial and Wiener index of semitotal point graph and its complement of certain graph families. Further, we obtained sharp upper and lower bounds for Wiener index of semitotal point graph and its complement.

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1. Introduction

The graphs considered in this paper are simple, connected, nontrivial, undirected finite graphs with n vertices and m edges. Let G be such a graph with the vertex set $V(G) = \{v_1, v_2, ..., v_n\}$ and the edge set $E(G) = \{e_1, e_2, ..., e_m\}$. The distance between two vertices v_i and v_j, denoted by $d_G(v_i, v_j)$ is the length of the shortest path between the vertices v_i and v_j in G. The shortest v_i - v_j path is called geodesic. The diameter of G, denoted by diam(G), is the length of any longest geodesic. The degree of a vertex v_i in G is the number of edges incident with v_i and is denoted by $d_G(v_i) = deg(v_i)$. The complement $\overline{G}$ of a graph G is a graph whose vertex set is $V(G)$ in which two vertices are adjacent if and only if they are not adjacent in G. Therefore $\overline{G}$ has n vertices and $\frac{n(n-1)}{2} - m$ edges. In this paper, we denote C_n, P_n, S_n, T, K_n and $K_{r,s}$ for cycle, path, star, tree, complete graph and complete bipartite graph, of order n respectively and a wheel of order $n+1$ by W_n. For undefined terms and notations refer [1]. In [6], Wiener used a linear formula of $W(G)$ and $W_p(G)$ to obtain the boiling points t_B of the paraffins, that is $t_B = aW(G) + bW_p(G) + c$, where a, b and c are constants for a given isomeric group. The Wiener index (or Wiener number) [6] $W(G)$ of a graph G, is the sum of distances between all (unordered) pairs of vertices of G.

$$W(G) = \sum_{i<j} d_G(v_i, v_j).$$

The Wiener index of a graph belongs to the molecular structure-descriptors called topological indices, which are used for the design of molecules with desired properties [3]. Its mathematical properties are reasonably well understood. The Wiener polarity index [6] $W_p(G)$ of a graph G, is equal to the number of unordered pairs of vertices of distance three in G.

$$W_p(G) = |\{(u, v)|d_G(u, v) = 3\}|.$$

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The Wiener Finer index $W_F(G)$ of a graph G, is equal to the number of unordered pairs of vertices of distance four in G.

$$W_F(G) = |\{(u,v)|d_G(u,v) = 4\}|.$$

The Hosoya polynomial was initially defined by Haruo Hosoya [2] and termed in honour of Harold Wiener who coined the Wiener index. The Hosoya polynomial [4] of a connected graph G is denoted by $W(G;q)$ and is defined by,

$$W(G;q) = \sum_{i<j} q^{d_G(v_i,v_j)}$$

where q is a parameter. The relation between Hosoya polynomial and Wiener index is,

$$W(G) = \frac{d}{dq}(W(G;q))\Big|_{q=1}. \tag{1}$$

Hence, we can derive the expression for the Wiener index of G from that of the Hosoya polynomial of G. The following theorems are useful for proving our results.

Theorem 1.1 ([7]). *If G is a connected graph with the connected complement and $diam(G) > 3$, $diam(\overline{G}) = 2$.*

Theorem 1.2 ([4]). *The Hosoya polynomial satisfies the following conditions:*

(1). $deg(W(G;q))$ equals the diameter of G.

(2). $[q^o]W(G;q)$ =0.

(3). $[q^1]W(G;q) = |E(G)|$, where $E(G)$ is an edge set of G.

(4). $W(G;1) = \binom{|V(G)|}{2}$, where $V(G)$ is the vertex set of G.

(5). $W'(G;1) = W(G)$.

Definition 1.3 (Semitotal point graph and its complement). *Let G be a simple (n,m)-graph. The semitotal point graph $T_2(G)$ of a graph G is the graph with vertex set $V(G) \cup E(G)$ and two vertices in $T_2(G)$ are adjacent if and only if they are either adjacent vertices of G or one is a vertex and the other is an edge incident with it in G. The order and size of $T_2(G)$ are respectively $n+m$ and $3m$. Also those of the complement of semitotal point graph $\overline{T_2(G)}$ are $n+m$ and $\binom{n+m}{2} - 3m$ respectively.*

We refer [5] for details of semitotal point graph and its related concepts.

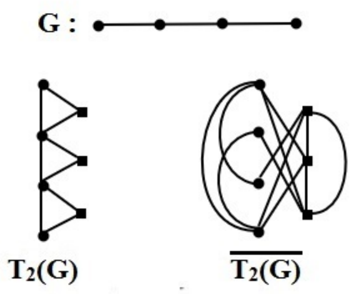

Figure 1. self-explanatory examples of the parent graph G and semitotal point graph with its complement.

2. Hosoya Polynomial of $T_2(G)$

Theorem 2.1. *For any graph G, the semitotal point graph $T_2(G)$ is connected if and only if G is connected.*

Proof. Suppose G is connected. Then $T_2(G)$ is connected as $S(G)$ is a connected spanning subgraph of $T_2(G)$. Conversely, assume G has atleast two components say G_1 and G_2. Then $T_2(G) = T_2(G_1) \cup T_2(G_2)$. Clearly $T_2(G)$ is disconnected, a contradiction. □

The Wiener polarity index $W_p(G)$ and Wiener Finer Index $W_F(G)$ of the semitotal point graph of some graph families are given in the following observation:

Observation 2.2. *Let G be a connected graph with n vertices and m edges. Then*

(1). $W_p(T_2(K_{r,s})) = \frac{rs}{2}(rs+1-r-s)$ *and* $W_F(T_2(K_{r,s})) = 0$.

(2). $W_p(T_2(K_n)) = \frac{1}{8}(n^4 - 6n^3 + 11n^2 - 6n)$ *and* $W_F(T_2(K_n)) = 0$.

(3). $W_p(T_2(W_n)) = n(n-2)$ *and* $W_F(T_2(W_n)) = \begin{cases} m & \text{if } n \geq 7 \\ 3 & \text{if } n = 6 \\ 0 & \text{otherwise} \end{cases}$

Theorem 2.3. *Let G be a graph of order n and size m with $diam(G) \leq 4$. Then*

$$W(G;q) = W_F(G)q^4 + W_p(G)q^3 + \left(\binom{n}{2} - W_F(G) - W_p(G) - m\right)q^2 + mq$$

and

$$W(G) = 2W_F(G) + W_p(G) + n(n-1) - m. \tag{2}$$

Proof. Let G be a graph of order n and size m with $diam(G) \leq 3$. Then by definition of Hosoya polynomial, we have

$$W(G;q) = \sum_{u,v \in V(G)} q^{d_G(u,v)}$$

and by Theorem 1.2, the highest power of polynomial is equal to the diameter of G. Let $A_i(G) = |\{(u,v)/d_G(u,v) = i\}|$. Therefore the expected Hosoya polynomial for G is

$$W(G;q) = \sum_{i=1}^{4} A_i(G)q^i.$$

By definition of $A_i(G)$, we have $A_1(G) = m$, $A_4(G) = W_F(G)$, $A_3(G) = W_p(G)$ and $A_2(G) = \binom{n}{2} - m - W_p(G)$. Therefore,

$$W(G;q) = W_F(G)q^4 + W_p(G)q^3 + \left(\binom{n}{2} - W_F(G) - W_p(G) - m\right)q^2 + mq.$$

From Equation (1), the Wiener index for G is

$$\begin{aligned} W(G) &= \frac{d}{dq}(W(G;q))\Big|_{q=1} \\ &= 4W_F(G) + 3W_p(G) + 2\left(\binom{n}{2} - W_F(G) - W_p(G) - m\right) + m \\ &= 2W_F(G) + W_p(G) + n(n-1) - m. \end{aligned}$$

□

Corollary 2.4. *Let $K_{r,s}$ be a complete bipartite graph. Then*

$$W(T_2(K_{r,s});q) = \frac{1}{2}rs(rs+1-r-s)q^3 + \left(\binom{rs+r+s}{2} - \frac{1}{2}rs(rs-r-s-5)\right)q^2 + 3rsq$$

and

$$W(T_2(K_{r,s})) = \frac{3}{2}rs(rs+r+s-2) + r^2 + s^2 - r - s.$$

Corollary 2.5. *Let K_n be a complete graph of order n. Then*

$$W(T_2(K_n);q) = \frac{1}{8}(n^4 - 6n^3 + 11n^2 - 6n)q^3 + \frac{1}{2}(2n^3 - 6n^2 + 4n)q^2 + \frac{3n(n-1)}{2}q$$

and

$$W(T_2(K_n)) = \frac{1}{8}(3n^4 - 2n^3 - 3n^2 + 2n)$$

Corollary 2.6. *Let W_n be a wheel graph. Then*

$$W(T_2(W_n);q) = W_p(C_n)q^4 + n(n-2)q^3 + \left(\frac{1}{2}(7n^2 - 23n + 18) - W_p(C_n)\right)q^2 + 6(n-1)q$$

and

$$W(T_2(W_n)) = 2W_p(C_n) + (10n^2 - 23n + 12).$$

The diameter of the semitotal point graph of a graph increases as the order of the graph increases. Therefore, we have the following three corollaries.

Corollary 2.7. *Let S_n be a star of order n. Then $W(T_2(S_n)) = (4n^2 - 9n + 5)$.*

Corollary 2.8. *Let P_n be a path of order n. Then $W(T_2(P_n) = \frac{2n^3-5n+3}{3}$.*

Corollary 2.9. *For a nontrivial tree T of order n, $4n^2 - 9n + 5 \leq W(T_2(T)) \leq \frac{2n^3-5n+3}{3}$.*

Theorem 2.10. *If C_n is a cycle, then $W(T_2(C_n)) = \frac{n^3+2n^2-n}{2}$.*

Proof. Let $V(G) = \{v_1, v_2, ..., v_n\}$ be the vertex set and $E = \{v'_1, v'_2, ..., v'_n\}$ be the edge set of C_n. Then $T_2(C_n)$ is the semitotal point graph of cycle C_n with vertex set $V' = V \cup \{v'_1, v'_2, ..., v'_n\}$.

Splitting the summation of Wiener index of $T_2(C_n)$ into four parts,

$$\begin{aligned}
W(T_2(C_n)) &= \text{half of the shortest distance between the vertices } v_i \text{ and } v_j \\
&\quad +\text{half of the shortest distance between the vertices } v'_i \text{ and } v'_j \\
&\quad +\text{half of the shortest distance between the vertices } v_i \text{ and } v'_j \\
&\quad +\text{half of the shortest distance between the vertices } v'_i \text{ and } v_j \\
&= \frac{1}{2}\sum_{v_i, v_j \epsilon V} d(v_i, v_j) + \frac{1}{2}\sum_{v_i \epsilon V', v_j \epsilon V} d(v_i, v_j) + \frac{1}{2}\sum_{v_i \epsilon V, v'_j \epsilon V'} d(v_i, v_j) + \frac{1}{2}\sum_{v'_i, v'_j \epsilon V'} d(v'_i, v'_j) \\
&= \frac{1}{2}\left\{\begin{array}{l}
d(v_1, v_1) + d(v_1, v_2) + ... + d(v_1, v_n) \\
+d(v_2, v_1) + d(v_2, v_2) + ... + d(v_2, v_n) \\
+... + ... + ... + ...+ \\
+d(v_n, v_1) + d(v_n, v_2) + ... + d(v_n, v_n)
\end{array}\right\}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}\left\{\begin{array}{l} d(v_1',v_1)+d(v_1',v_2)+...+d(v_1',v_n)\\ +d(v_2',v_1)+d(v_2',v_2)+...+d(v_2',v_n)\\ +...+...+.........+..................+\\ +d(v_n',v_1)+d(v_n',v_2)+...+d(v_n',v_n)\end{array}\right\}\\
&+\frac{1}{2}\left\{\begin{array}{l} d(v_1,v_1')+d(v_1,v_2')+...+d(v_1,v_n')\\ +d(v_2,v_1')+d(v_2,v_2')+...+d(v_2,v_n')\\ +...+...+..........+.............+\\ +d(v_n,v_1')+d(v_n,v_2')+...+d(v_n,v_n')\end{array}\right\}\\
&+\frac{1}{2}\left\{\begin{array}{l} d(v_1',v_1')+d(v_1',v_2')+...+d(v_1',v_n')\\ +d(v_2',v_1')+d(v_2',v_2')+...+d(v_2',v_n')\\ +...+...+.........+..................+\\ +d(v_n',v_1')+d(v_n',v_2')+...+d(v_n',v_n')\end{array}\right\}\\
=\;& W(C_n)+\frac{1}{2}[2W(C_n)+n.diam(C_n)]+\frac{1}{2}[2W(C_n)+n.diam(C_n)]+\frac{1}{2}\left[2W(C_n)+\frac{n(n-1)}{2}\right]\\
=\;& 4W(C_n)+\frac{n(n-1)}{2}+n(diam(T_2(C_n))).
\end{aligned}
$$

Case (1): For even cycle, $diam(T_2(C_n))=\frac{n}{2}$. Therefore,

$$
\begin{aligned}
W(T_2(C_n)) &= 4W(C_n)+\frac{n(n-1)}{2}+n(diam(T_2(C_n)))\\
&= 4(\frac{n^3}{8})+\frac{n(n-1)}{2}+n(\frac{n}{2})=\frac{n^3+2n^2-n}{2}.
\end{aligned}
$$

Case (2): For odd cycle, $diam(T_2(C_n))=\frac{n+1}{2}$. Therefore,

$$
\begin{aligned}
W(T_2(C_n)) &= 4W(C_n)+\frac{n(n-1)}{2}+n(diam(T_2(C_n)))\\
&= 4(\frac{n^3-n}{8})+\frac{n(n-1)}{2}+n\frac{n+1}{2}=\frac{n^3+2n^2-n}{2}.
\end{aligned}
$$

From the above two cases $W(T_2(C_n))=\frac{n^3+2n^2-n}{2}$. □

Theorem 2.11. *If G is a connected graph of order $n\geq 2$, then $W(G)<W(T_2(G))$.*

Proof. Let G be a connected graph with n vertices and m edges. Then the semitotal point graph $T_2(G)$ has $n_1=n+m$ vertices and $m_1=3m$ edges. Since, the Wiener index of a graph increases when new vertices are added to it and G is induced subgraph of $T_2(G)$. Hence, Wiener index of semitotal point graph $T_2(G)$ is greater than Wiener index of a graph G. Therefore $W(G)<W(T_2(G))$. □

Lemma 2.12. *For any connected graph G of order n,*

$$(4n^2-9n+5)\leq W(T_2(G))\leq\frac{1}{8}(3n^4-2n^3-3n^2+2n).$$

Upper bound attains if G is a complete graph and lower bound attains if G is a star.

Proof. Let G be a graph of order n and size m. Then $T_2(G)$ has $n+m$ vertices and $3m$ edges. Any graph G of order n has maximum number of edges if and only if $G\cong K_n$ and $T_2(G)$ has maximum number of vertices if and only if $G\cong K_n$. We know that Wiener index of a graph G increases when new vertices are added to the graph and $T_2(K_n)$ has maximum number of vertices compared with any other $T_2(G)$, where G is a graph of order n. Therefore, $W(T_2(G)) \leq W(T_2(K_n))$. From Corollary 2.5, $W(T_2(K_n)) = \frac{1}{8}(3n^4 - 2n^3 - 3n^2 + 2n)$. Therefore,

$$W(T_2(G)) \leq \frac{1}{8}(3n^4 - 2n^3 - 3n^2 + 2n). \tag{3}$$

with equality in (3) if and only if $G \cong K_n$. Any graph G of order n has minimum number of edges if and only if $G \cong T$ and $T_2(G)$ has minimum number of vertices if and only if $G \cong T$. Therefore, $W(T_2(T)) \leq W(T_2(G))$. From Corollary 2.9, $W(T_2(S_n)) = (4n^2 - 9n + 5) \leq W(T_2(T))$. Therefore,

$$4n^2 - 9n + 5 \leq W(T_2(G)) \tag{4}$$

with equality in (4) if and only if $G \cong S_n$. From (3) and (4), we have

$$(4n^2 - 9n + 5) \leq W(T_2(G)) \leq \frac{1}{8}(3n^4 - 2n^3 - 3n^2 + 2n).$$

Upper bound attains if G is a complete graph and lower bound attains if G is a star. □

3. Hosoya Polynomial of $\overline{T_2(G)}$

In this section, we obtain the Hosoya polynomial and Wiener index of the complement of the semitotal point graph $\overline{T_2(G)}$ of a graph G.

Theorem 3.1. *For any graph G, the complement of the semitotal point graph of G is connected if and only if G is not a star.*

Proof. Suppose $G = K_{1,n}$, $n \geq 1$ and u is the central vertex of the star. Then u' is an isolated vertex of $\overline{T_2(G)}$, and is not connected. On the other hand, if G is not connected, then $\overline{G}$ ($\subseteq \overline{T_2(G)}$) is connected. Since every edge $e \in E(G)$ is not incident with each vertex $v \in V(G)$ not in the same component. By definition, v' and e' are adjacent in $\overline{T_2(G)}$. Now suppose G is connected. Since G is not a star, each vertex $v \in V(G)$ is not incident with at least one edge $e \in E(G)$. By definition, v' and e' are adjacent in $\overline{T_2(G)}$. Thus $\overline{T_2(G)}$ is connected. □

Theorem 3.2. *If G is not a star, then $diam(\overline{T_2(G)}) \leq 3$, and the equality holds if and only if $G \cong P_4, K_3$.*

Proof. For $e_1, e_2 \in E(G)$, since all the line-vertices of $\overline{T_2(G)}$ form a complete subgraph, e_1' and e_2' are adjacent in $\overline{T_2(G)}$. For $u \in V(G)$ and $e \in E(G)$, if they are not incident in G, then u' and e' are adjacent in $\overline{T_2(G)}$. Otherwise, there is an edge e_i not incident with u, then u', e_i', e' is a path in $\overline{T_2(G)}$. For $u, v \in V(G)$, if they are not adjacent in G, then u' and v' are adjacent in $\overline{T_2(G)}$. Suppose u and v are adjacent in G. If u and v are not adjacent to a common vertex w in G, then u', w', v' is a path in $\overline{T_2(G)}$. If u and v are not incident with a common edge e_i in G, then u', e_i', v' is a path in $\overline{T_2(G)}$. Otherwise, u and v are not incident with e_1 and e_2 respectively in G, then u', e_1', e_2', v' is a path in $\overline{T_2(G)}$.
It is easy to see that if $G \cong P_4, K_3$, then $diam(\overline{T_2(G)}) = 3$. On the other hand, $diam(\overline{T_2(G)}) = 3$. Let $r', s' \in V(\overline{T_2(G)})$ such that the distance of r', s' in $\overline{T_2(G)}$ is 3. By the above argument, we see that $r, s \in V(G)$, r and s are adjacent and neither not adjacent to a common vertex nor not incident with a common edge. Hence the only possibility is $G \cong P_4$ or K_3. □

Theorem 3.3. *Let $G \ncong S_n, P_4, K_3$ be a graph of order n and size m. Then*

$$W(\overline{T_2(G)}; q) = \left(\binom{n+m}{2} - \frac{1}{2}(n(n-1) + m(m-7)) - mn\right) q^2 + \left(\frac{1}{2}(n(n-1) + m(m-7)) + mn\right) q$$

and

$$W(\overline{T_2(G)}) = (n+m)(n+m-1) - \frac{1}{2}(n(n-1) + m(m-7)) - mn.$$

Proof. The complement of the semitotal point graph $\overline{T_2(G)}$ has the diameter ≤ 2, for any graph G other than S_n, P_4, K_3. Therefore, from Theorem 1.2 we have the Hosoya polynomial and the Wiener index as

$$W(\overline{T_2(G)}; q) = \left(\binom{n+m}{2} - \frac{1}{2}(n(n-1) + m(m-7)) - mn\right) q^2 + \left(\frac{1}{2}(n(n-1) + m(m-7)) + mn\right) q$$

and

$$W(\overline{T_2(G)} = (n+m)(n+m-1) - \frac{1}{2}(n(n-1) + m(m-7)) - mn. \tag{5}$$

□

The Hosoya polynomial and Wiener index of complement of semitotal point graph of some standard classes of graphs are obtained analogously.

Corollary 3.4. *For a nontrivial tree $T \ncong S_n, P_4$ of order n,*

$$W(\overline{T_2(T)}; q) = 3(n-1)q^2 + 2(n-1)(n-2)q \text{ and } W(\overline{T_2(T)}) = 2(n^2-1).$$

Corollary 3.5. *For a cycle C_n of order $n > 3$,*

$$W(\overline{T_2(C_n)}; q) = 3nq^2 + 2n(n-2)q \text{ and } W(\overline{T_2(C_n)}) = n(2n-5).$$

Corollary 3.6. *For a complete graph K_n of order n,*

$$W(\overline{T_2(K_n)}; q) = \frac{3}{2}n(n-1)q^2 + \frac{1}{8}(n^4 + 2n^3 - 13n^2 + 10n)q$$

and

$$W(\overline{T_2(K_n)}) = \frac{1}{8}(n^4 + 2n^3 + 11n^2 - 14n).$$

Corollary 3.7. *For a complete bipartite graph $K_{r,s} \ncong S_n$,*

$$W(\overline{T_2(K_{r,s})}; q) = 3rsq^2 + \frac{1}{2}(rs(2r + 2s + rs - 3) + (r-s)^2 - r - s)q$$

and

$$W(\overline{T_2(K_{r,s})}) = \frac{1}{2}(rs(2r + 2s + rs + 9) + (r-s)^2 - r - s).$$

Corollary 3.8. *For a wheel graph W_n,*

$$W(\overline{T_2(W_n)}; q) = 6(n-1)q^2 + \frac{3}{2}(3n^2 - 9n + 6)q$$

and

$$W(\overline{T_2(W_n)}) = \frac{3}{2}(3n^2 - n - 2).$$

Lemma 3.9. *For any connected graph $G \ncong P_4, K_3$ of order n,*

$$2(n^2 - 1) \leq W(\overline{T_2(G)}) \leq \frac{1}{8}(n^4 + 2n^3 + 11n^2 - 14n).$$

Upper bound attains if G is a complete graph and lower bound attains if G is a path.

Proof. Let G be a graph of order n and size m. Then $\overline{T_2(G)}$ has $n+m$ vertices and $3m$ edges. Any graph G of order n has maximum number of edges if and only if $G \cong K_n$ and $\overline{T_2(G)}$ has maximum number of vertices if and only if $G \cong K_n$. We know that Wiener index of a graph G increases when new vertices are added to the graph and $\overline{T_2(K_n)}$ has maximum number of vertices compared with any other $\overline{T_2(G)}$, where G is a graph of order n. Therefore, $W(\overline{T_2(G)}) \leq W(\overline{T_2(K_n)})$. From Corollary 3.6, $W(\overline{T_2(K_n)}) = \frac{1}{8}(n^4 + 2n^3 + 11n^2 - 14n)$. Therefore,

$$W(\overline{T_2(G)}) \leq \frac{1}{8}(n^4 + 2n^3 + 11n^2 - 14n). \tag{6}$$

with equality in (6) if and only if $G \cong K_n$. Any graph G of order n has minimum number of edges if and only if $G \cong T$ and $\overline{T_2(G)}$ has minimum number of vertices if and only if $G \cong T$ other than P_4. Therefore, $W(\overline{T_2(T)}) \leq W(\overline{T_2(G)})$. From Corollary 3.4, $W(\overline{T_2(P_n)}) = 2(n^2 - 1) \leq W(\overline{T_2(T)})$. Therefore,

$$2(n^2 - 1) \leq W(\overline{T_2(G)}) \tag{7}$$

with equality in (7) if and only if $G \cong S_n$. From (6) and (7), we have

$$2(n^2 - 1) \leq W(\overline{T_2(G)}) \leq \frac{1}{8}(n^4 + 2n^3 + 11n^2 - 14n).$$

Upper bound attains if G is a complete graph and lower bound attains if $G \ncong P_4$ is a path. □

The following theorem gives the Nordhaus-Gaddum type inequality for Wiener index of semitotal point graph.

Theorem 3.10. *For a connected graph G of order $n \geq 4$,*

$$3(2n^2 - 3n + 1) \leq W(T_2(G)) + W(\overline{T_2(G)}) \leq \frac{n}{2}(n^3 + 2n - 3).$$

Proof. This bound is immediate from Lemmas 2.12 and 3.9. □

References

[1] F. Harary, *Graph Theory*, Addison-Wesley, Reading, Mass (1969).

[2] H. Hosoya, *On some counting polynomials in chemistry*, Discrete Appl. Math., 19(1988), 239-257.

[3] M. Randić, *In search for graph invariants of chemical interest*, J. Mol. Struct., 300(1993), 551-571.

[4] B. E. Sagan, Y. N. Yeh and P. Zang, *The Wiener polynomial of a graph*, Int. J. Quant. Chem., 60(5)(2009), 959-969.

[5] E. Sampathkumar and S. B. Chikkodimath, *Semitotal graphs of a graph-I*, J. Karnatak Univ. Sci., 18(1973), 274-280.

[6] H. Wiener, *Strucural determination of paraffin boiling points*, J. Amer. Chem. Soc., 69(1947), 17-20.

[7] L. Zhang and B. Wu, *The Nordhaus-Gaddum-type inequalities for some chemical indices*, MATCH Commun. Math. Comput. Chem., 54(2005), 185-194.