

Some Results on Super Heronian Mean Number of Graphs

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Abstract: We add to some fresh outcomes for Super Heronian Mean Number of graphs. It has been found that the graphs obtained by the collection of Super Heronian Mean Number of Ladder, Triangular snake, Double Triangular snake also admit Super Heronian Mean Number.

Keywords: Super Heronian mean number, $S_{hm}(L_n \odot K_{1,2})$, $S_{hm}(TL_n \odot K_1)$, $S_{hm}(T_n \odot K_1)$, $S_{hm}(D(T_n) \odot K_1)$.

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1. Introduction

The graphs which are used here are finite, undirected graphs. Here $V(G)$ indicates vertices and $E(G)$ indicates edges. For all described view of Graph Labeling we refer to J.A. Gillian [1] and we follow Harary [2] for all other standard terminology and notations in Graph Theory. We will provide short summary and definitions which are useful for the present investigation.

Definition 1.1. Let G be a graph and let $f : V(G) \rightarrow \{1, 2, \dots, n\}$ be a function such that the label of the edge uv is defined by, $f^*(e = uv) = \left\lfloor \frac{f(u) + \sqrt{f(u)f(v) + f(v)}}{3} \right\rfloor$ (or) $\left\lceil \frac{f(u) + \sqrt{f(u)f(v) + f(v)}}{3} \right\rceil$ and $\{f(V(G))\} \cup \{f^*(e) : e \in G\} \subseteq \{1, 2, \dots, n\}$. If n is the smallest positive integer satisfying these conditions that all the vertex and edge labels are distinct, and there is no common vertex and edge labels, then 'n' is called the Super Heronian Mean Number of a graph G and it is denoted by $S_{hm}(G)$.

Theorem 1.2. Path are Super Heronian Mean Number.

Theorem 1.3. Ladders are Super Heronian Mean Number.

2. Main Results

Theorem 2.1. $S_{hm}(L_n \odot K_{1,2}) = 13n - 1$.

Proof. Let L_n be a Ladder and w_i, x_i be the pendant vertices adjacent to u_i and y_i and also x_i be the pendant vertex adjacent to v_i . Define a function $f : V(L_n \odot K_{1,2}) \rightarrow \{1, 2, \dots, n\}$ by

$$f(u_i) = 13i - 1; \quad 1 \leq i \leq n - 1$$

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$$\begin{aligned}
 f(v_i) &= 13i - 6; & 1 \leq i \leq n \\
 f(w_i) &= 13i - 7; & 1 \leq i \leq n \\
 f(x_i) &= 13i - 3; & 1 \leq i \leq n \\
 f(y_i) &= 13i - 12; & 1 \leq i \leq n \\
 f(z_i) &= 13i - 11; & 1 \leq i \leq n \\
 f(u_n) &= 13n - 1; & 1 \leq i \leq n
 \end{aligned}$$

Clearly the vertex and edge labels are distinct. Hence $S_{hm}(L_n \odot K_{1,2}) = 13n - 1$. □

Example 2.2. Super Heronian mean number of $n = 4$, $S_{hm}(L_4 \odot K_{1,2}) = 13n - 1$.

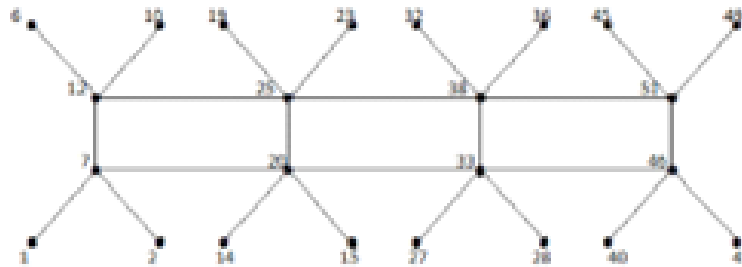


Figure 1.

Theorem 2.3. $S_{hm}(TL_n \odot K_1) = 10n - 2$.

Proof. Let TL_n be a Triangular Ladder. Let $u_1u_2 \dots u_n$ and $v_1v_2 \dots v_n$ be two path of length n in the graph $TL_n \odot K_1$ and w_i, x_i be the pendant vertices. Define a function $f : V(TL_n \odot K_1) \rightarrow \{1, 2, \dots, n\}$ by

$$\begin{aligned}
 f(u_i) &= 10i - 4; & 1 \leq i \leq n \\
 f(v_i) &= 10i - 2; & 1 \leq i \leq n - 1 \\
 f(w_1) &= 1; \\
 f(w_i) &= 10i - 11; & 2 \leq i \leq n \\
 f(x_i) &= 10i - 6; & 1 \leq i \leq n \\
 f(v_n) &= 10n - 2
 \end{aligned}$$

Clearly the vertex and edge labels are distinct. Hence $S_{hm}(TL_n \odot K_1) = 10n - 2$. □

Example 2.4. Super Heronian mean number of $n = 5$, $S_{hm}(TL_5 \odot K_1) = 10n - 2$.

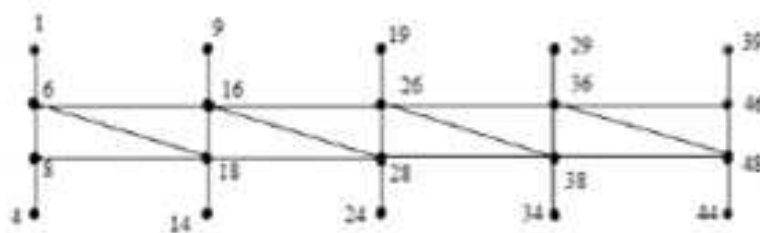


Figure 2.

Theorem 2.5. $S_{hm}(T_n \odot K_1) = 9n - 5$.

Proof. Let T_n be a Triangular snake and u_i, v_i be the vertices of a triangular snake and also w_i, x_i be the pendant vertices. Define a function $f : V(T_n \odot K_1) \rightarrow \{1, 2, \dots, n\}$ by

$$\begin{aligned} f(u_1) &= 1; & f(u_i) &= 9i - 7; & 2 \leq i \leq n \\ f(v_i) &= 9i; & & & 1 \leq i \leq n - 1 \\ f(w_i) &= 9i - 5; & & & 1 \leq i \leq n - 1 \\ f(x_i) &= 9i - 2; & & & 1 \leq i \leq n - 1 \\ f(w_n) &= 9n - 5 \end{aligned}$$

Clearly the vertex and edge labels are distinct. Hence $S_{hm}(T_n \odot K_1) = 9n - 5$. □

Example 2.6. Super Heronian mean number of $S_{hm}(T_4 \odot K_1) = 9n - 5$.

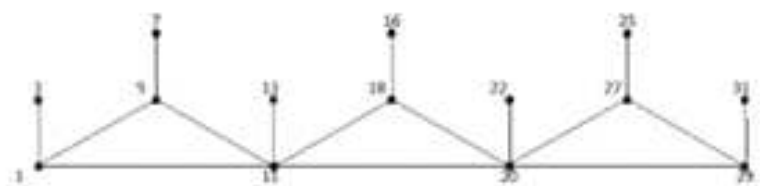


Figure 3.

Theorem 2.7. $S_{hm}(D(T_n) \odot K_1) = 16n - 10$.

Proof. Let $D(T_n)$ be a double Triangular snake. Let u_i, v_i, w_i be the vertices of a double triangular snake and x_i, y_i, s_i, t_i be the pendant vertices. Define a function $f : V(D(T_n) \odot K_1) \rightarrow \{1, 2, \dots, n\}$ by

$$\begin{aligned} f(u_i) &= 16i - 14; & 1 \leq i \leq n \\ f(v_i) &= 16i - 6; & 1 \leq i \leq n - 1 \\ f(w_i) &= 16i; & 1 \leq i \leq n - 1 \\ f(x_i) &= 16i - 11; & 1 \leq i \leq n \end{aligned}$$

$$f(y_i) = 16i - 4; \quad 1 \leq i \leq n - 1$$

$$f(s_i) = 16i - 9; \quad 1 \leq i \leq n - 1$$

$$f(s_n) = 16n - 10$$

$$f(t_i) = 16i - 2; \quad 1 \leq i \leq n - 1$$

Clearly the vertex and edge labels are distinct. Hence $S_{hm}(D(T_n) \odot K_1) = 16n - 10$. □

Example 2.8. Super Heronian mean number of $S_{hm}(D(T_4) \odot K_1)$ is given below.

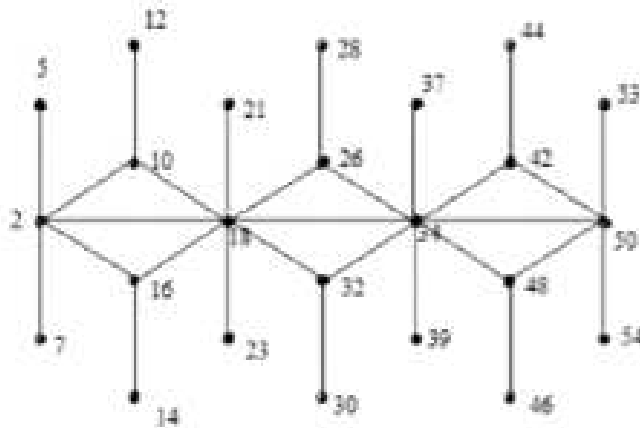


Figure 4.

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