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Interval Valued Fuzzy Ideals and Anti Fuzzy Ideals of Γ-Near-ring

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Abstract: The aim of this paper is to study the notion of an interval valued fuzzy ideal of a near ring and interval valued anti fuzzy ideal of a near-ring and to discuss some of their properties.

Keywords: Near-ring, Near-subring, Fuzzy Ideals of near-ring, Fuzzy subring, Interval Valued Fuzzy ideals of near-ring, Interval Valued Anti fuzzy ideals of near-ring.

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1. Introduction

A near-ring satisfying all axioms of an associative ring excepts commutativity of addition of one of the two distributive laws. After the introduction of fuzzy sets by Zadeh [18], there have been a number of generalizations of this fundamental concept. Salah Abou-Zaid [1] introduced the theory of a fuzzy subnear-ring and fuzzy ideals of a near-ring. Fuzzy ideals of a ring and a characterization of a regular ring studied by Lui. The notion of fuzzy ideals of near rings with interval valued membership functions introduced by B. Davvaz [4] in 2001. In [3], R. Biswas defined interval-valued fuzzy subgroups of the same nature of Rosenfelds's fuzzy subgroups. A comprehensive review of theory of fuzzy ideals of near-rings and anti fuzzy ideal of near-rings can be found in [5, 6, 8]. K. Murugalingam and K. Arjunan [9] introduced interval valued fuzzy subsemirings of semi-rings. Y. B. Jun and K. H. Kim [7] discussed interval-valued R-subgroups in terms of near-rings. N. Thillaigovindan [15, 16] have studied interval valued fuzzy ideals and anti fuzzy ideals of near-rings. Abou-Zaid [1] proposed the concept of fuzzy sub near-rings and ideals. T. Srinivas [13, 14] studied the notion of anti fuzzy ideals of Γ-near-rings and anti fuzzy near-algebra over anti fuzzy fields. T. Nagaiah [12] proposed interval-valued fuzzy ideals of Γ-near-rings. The aim of this paper is to study the notion of a interval valued fuzzy ideals of a near ring and interval valued anti fuzzy ideals of near-ring and to discuss some of their properties.

2. Preliminaries

For the sake of continuity we recall some basic definition.

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Definition 2.1. *A set N together with two binary operations $+$ (called addition) and $\cdot$ (called multiplication) is called a (right) near-ring if:*

A1: N is a group (not necessarily abelian) under addition;

A2: multiplication is associative (so N is a semigroup under multiplication); and

A3: multiplication distributes over addition on the right: for any x, y, z in N, it holds that $(x+y)\cdot z = (x\cdot z)+(y\cdot z)$.

This near-ring will be termed as right near-ring. If $x.(y+z) = x.y + x.z$ instead of condition A3 the set N satisfies, then we call N a left near-ring. Near-rings are generalised rings: addition needs not be commutative and (more important) only one distributive law is postulated.

Definition 2.2. *A Γ-near-ring is a triple $(M, +, \Gamma)$ where*

(1). $(M, +)$ is a group.

(2). Γ is a nonempty set of binary operations on M such that for each $\alpha \in \Gamma$, $(M, +, \alpha)$ is a near-ring.

(3). $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Definition 2.3. *A subset A of a Γ-near-ring M is called a left (respectively right) ideal of M if*

(1). $(A, +)$ is a normal divisor of $(M, +)$.

(2). $u\alpha(x+v) - u\alpha v \in A$ (respectively $x\alpha u \in A$) for all $x \in A$, $\alpha \in \Gamma$ and $u, v \in M$.

A fuzzy set in a set M is a function $\mu : M \to [0,1]$. We shall use the notation μ_t, called a level subset of μ, for $\{x \in M | \mu(x) \geq t\}$, where $t \in [0,1]$.

Definition 2.4. *Let R be a near-ring and μ be a fuzzy subset of R. We say a fuzzy subnear-ring of R if*

(1). $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$,

(2). $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in R$.

Definition 2.5. *Let R be a near-ring and μ be a fuzzy subset of R. μ is called a fuzzy left ideal of R if μ is a fuzzy subnear-ring of R and satisfies: for all $x, y \in R$.*

(1). $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$,

(2). $\mu(y+x-y) \geq \mu(x)$,

(3). $\mu(xy) \geq \mu(y)$ or $\mu(xy) \geq \mu(x)$

Definition 2.6. *Let R be a near-ring and μ be a fuzzy subset of R. μ is called a fuzzy right ideal of R if μ is a fuzzy subnear-ring of R and satisfies: for all $x, y \in R$.*

(1). $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$,

(2). $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$,

(3). $\mu(y+x-y) \geq \mu(x)$,

(4). $\mu((x+i)y - xy) \geq \mu(i)$.

Definition 2.7. *A fuzzy set μ in a Γ-near-ring M is called a fuzzy left (respectively right) ideal of M if*

(1). μ is a fuzzy normal divisor with respect to the addition.

(2). $\mu(ua(x+v)-uav) \geq \mu(x)$ (respectively $\mu(x\alpha u) \geq \mu(x)$) for all $x, u, v \in M$ and $\alpha \in \Gamma$.

The condition (1) of Definition 2.7 means that μ satisfies:

(1). $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$,

(2). $\mu(y+x-y) \geq \mu(x)$.

Theorem 2.8. *Let M be a Γ-near-ring and μ be a fuzzy left (respectively right) ideal of M. Then the set $M_\mu := \{x \in M | \mu(x) = \mu(0)\}$ is a left (respectively right) ideal of M.*

Definition 2.9. *Let I be an ideal of a Γ-near-ring N. For each $x+I, y+I$ in the factor group $\frac{N}{I}$ and $\alpha \in \Gamma$, we define $(x+I)+(y+I) = (x+y)+I$ and $(x+I)\alpha(y+I) = (x\alpha y)+I$. Then $\frac{N}{I}$ is a Γ-near-ring which we call the residue class Γ-near-ring of N with respect to I.*

Definition 2.10. *An interval-valued number $\tilde{a}$ on [0,1] is a closed subinterval of [0,1], that is $\tilde{a} = [a^-, a^+]$ such that $0 \leq a^- \leq a^+ \leq 1$, where a^- and a^+ are lower and upper limits of $\tilde{a}$ respectively. The set of all closed sub intervals of [0,1] is denoted by $D[0,1]$. In this notation $\tilde{0} = [0^-, 0^+]$ and $\tilde{1} = [1^-, 1^+]$. We also identify the interval $[a, a]$ by the number $a \in [0,1]$. For any two interval numbers $\tilde{a} = [a^-, a^+]$ and $\tilde{b} = [b^-, b^+]$ on [0,1], we define*

(1). $\tilde{a} \leq \tilde{b} \Leftrightarrow a^- \leq b^-$ and $a^+ \leq b^+$.

(2). $\tilde{a} = \tilde{b} \Leftrightarrow a^- = b^-$ and $a^+ = b^+$.

(3). $\tilde{a} < \tilde{b} \Leftrightarrow \tilde{a} \leq \tilde{b}$ and $\tilde{a} \neq \tilde{b}$.

(4). $k\tilde{a} = [ka^-, ka^+]$, for $0 \leq k \leq 1$.

Definition 2.11.

(1). Min-Norm: A mapping $\min^i : D[0,1] \times D[0,1] \to D[0,1]$ defined by $\min^i(\tilde{a}, \tilde{b}) = [\min(a^-, b^-), \min(a^+, b^+)]$ for all $\tilde{a}, \tilde{b} \in D[0,1]$ is called an interval min-norm.

(2). Max-Norm: A mapping $\max^i : D[0,1] \times D[0,1] \to D[0,1]$ defined by $\max^i(\tilde{a}, \tilde{b}) = [\max(a^-, b^-), \max(a^+, b^+)]$ for all $\tilde{a}, \tilde{b} \in D[0,1]$ is called an interval max-norm.

Let $\min^i$ and $\max^i$ be the interval-valued min-norm and interval-valued max-norm on $D[0,1]$ respectively. Then the following are true:

(1). $\min^i(\tilde{a}, \tilde{a}) = \tilde{a}$ and $\max^i(\tilde{a}, \tilde{a}) = \tilde{a}$ $\forall$ $\tilde{a} \in D[0,1]$.

(2). $\min^i(\tilde{a}, \tilde{b}) = \min^i(\tilde{b}, \tilde{a})$ and $\max^i(\tilde{a}, \tilde{b}) = \max^i(\tilde{b}, \tilde{a})$ $\forall$ $\tilde{a}, \tilde{b} \in D[0,1]$.

(3). If $\forall$ $\tilde{a}, \tilde{b}, \tilde{c} \in D[0,1], \tilde{a} \geq \tilde{b}$, then $\min^i(\tilde{a}, \tilde{c}) \geq \min^i(\tilde{b}, \tilde{c})$ and $\max^i(\tilde{a}, \tilde{c}) \leq \max^i(\tilde{b}, \tilde{c})$.

Definition 2.12. *Let $\tilde{\mu}$ be an interval valued fuzzy subset of a set X and $[t_1, t_1] \in D[0,1]$. Then the set $\tilde{U}(\tilde{\mu} : [t_1, t_1]) = \{x \in X | \tilde{\mu}(x) \geq [t_1, t_1]\}$, is called the upper level set of $\tilde{\mu}$. Note that*

$$\begin{aligned}\tilde{U}(\tilde{\mu} : [t_1, t_1]) &= \{x \in X | [\mu^-(x), \mu^+(x)] \geq [t_1, t_1]\} \\ &= \{x \in X | \mu^-(x) \geq t_1\} \cap \{x \in X | \mu^+(x) \geq t_2\} \\ &= (U(\mu^- : t_1)) \cap (U(\mu^+ : t_2))\end{aligned}$$

3. Interval Valued Fuzzy Ideals of Γ-Near-Ring

Definition 3.1. *An interval valued fuzzy subset $\tilde{\mu}$ of a Γ-near-ring N is called interval valued fuzzy sub Γ-near-ring of N if*

(1). $\tilde{\mu}(x-y) \geq \min^{i}\{\tilde{\mu}(x), \tilde{\mu}(y)\}$

(2). $\tilde{\mu}(xay) \geq \min^{i}\{\tilde{\mu}(x), \tilde{\mu}(y)\} \ \ \forall \ \ x, y \in N.$

Definition 3.2. *An interval valued fuzzy subset $\tilde{\mu}$ of a Γ-near-ring N is called interval valued fuzzy ideal Γ-near-ring of N if $\tilde{\mu}$ is an interval valued fuzzy sub Γ-near-ring N and*

(1). $\tilde{\mu}(y+x-y) \geq \tilde{\mu}(x)$

(2). $\tilde{\mu}(xay) \geq \tilde{\mu}(y)$

(3). $\tilde{\mu}((x+z)\alpha y - x\alpha y) \geq \tilde{\mu}(z) \ \ \forall \ \ x, y, z \in N, \alpha \in \Gamma.$

Theorem 3.3. *Let N be a Γ-near-ring and $\{\tilde{\mu}_i : i \in I\}$ a non-empty family of subsets of N. If $\{\tilde{\mu}_i : i \in I\}$ is an interval valued fuzzy ideal of N then $\bigcap_{i\in I} \tilde{\mu}_i$ is an interval valued fuzzy ideal of N.*

Proof. Let $\{\tilde{\mu}_i \ : i \in I\}$ be an interval valued fuzzy ideal of N. Let $x, y, z \in N$ and $\alpha \in \Gamma$. Then we have $\forall \ \ i \in I$

$$\begin{aligned}
\left(\bigcap_{i\in I} \tilde{\mu}_i\right)(x-y) &= \overset{i}{\inf}\{\tilde{\mu}_i(x-y)\}_{i\in I} \\
&\geq \overset{i}{\inf}\{\overset{i}{\min}\{\tilde{\mu}_i(x), \tilde{\mu}_i(y)\}\}_{i\in I} \\
&= \overset{i}{\min}\{\overset{i}{\inf}(\tilde{\mu}_i(x)), \overset{i}{\inf}(\tilde{\mu}_i(y))\}_{i\in I} \\
&= \overset{i}{\min}\left\{\left(\bigcap_{i\in I} \tilde{\mu}_i\right)(x), \left(\bigcap_{i\in I} \tilde{\mu}_i\right)(y)\right\} \\
\left(\bigcap_{i\in I} \tilde{\mu}_i\right)(xay) &= \overset{i}{\inf}\{\tilde{\mu}_i(x\alpha y)\}_{i\in I} \\
&\geq \overset{i}{\inf}\{\overset{i}{\min}\{\tilde{\mu}_i(x), \tilde{\mu}_i(y)\}\}_{i\in I} \\
&= \overset{i}{\min}\{\overset{i}{\inf}(\tilde{\mu}_i(x)), \overset{i}{\inf}(\tilde{\mu}_i(y))\}_{i\in I} \\
&= \overset{i}{\min}\left\{\left(\bigcap_{i\in I} \tilde{\mu}_i\right)(x), \left(\bigcap_{i\in I} \tilde{\mu}_i\right)(y)\right\} \\
\left(\bigcap_{i\in I} \tilde{\mu}_i\right)(y+x-y) &= \overset{i}{\inf}\{\tilde{\mu}_i(y+x-y)\}_{i\in I} \\
&\geq \overset{i}{\inf}\{\tilde{\mu}_i(x)\}_{i\in I} \\
&= \left(\bigcap_{i\in I} \tilde{\mu}_i\right)(y) \\
\left(\bigcap_{i\in I} \tilde{\mu}_i\right)(x\alpha y) &= \overset{i}{\inf}\{\tilde{\mu}_i(xay)\}_{i\in I} \\
&\geq \overset{i}{\inf}\{\tilde{\mu}_i(y)\}_{i\in I} \\
&= \left(\bigcap_{i\in I} \tilde{\mu}_i\right)(y) \\
\left(\bigcap_{i\in I} \tilde{\mu}_i\right)((x+z)\alpha y - x\alpha y) &= \overset{i}{\inf}\{\tilde{\mu}_i((x+z)\alpha y - x\alpha y)\}_{i\in I} \\
&\geq \overset{i}{\inf}\{\tilde{\mu}_i(z)\}_{i\in I}
\end{aligned}$$

$$= \left(\bigcap_{i \in I} \tilde{\mu}_i \right)(z)$$

Therefore $\left(\bigcap_{i\in I} \tilde{\mu}_i\right)$ is an interval valued anti fuzzy ideal of a Γ-near-ring N □

Theorem 3.4. *Let $\tilde{\mu}$ be an interval valued fuzzy subset of Γ-near-ring N. $\tilde{\mu} = [\mu^-, \mu^+]$ is an interval valued fuzzy left (right) ideal of Γ-near-ring N if and only if μ^+, μ^- are fuzzy left (right) ideals of Γ-near-ring N.*

Proof. (A) Assume that $\tilde{\mu}$ is an interval valued fuzzy left (right) ideal of Γ-near-ring N.. For any $x, y, z \in N$ and $\alpha \in \Gamma$, then we have

(1). $$\left[\mu^-(x-y), \mu^+(x-y)\right] = \tilde{\mu}(x-y) \geq \min{}^i\{\tilde{\mu}(x), \tilde{\mu}(y)\}$$
$$= \min{}^i\{\left[\mu^-(x), \mu^+(x)\right], \left[\mu^-(y), \mu^+(y)\right]\}$$
$$= \left[\min\left\{\mu^-(x), \mu^-(y)\right\}, \min\left\{\mu^+(x), \mu^+(y)\right\}\right].$$

It follows that $\mu^-(x-y) \geq \min\{\mu^-(x), \mu^-(y)\}$ *and*

$$\mu^+(x-y) \geq \min\{\mu^+(x), \mu^+(y)\}$$

(2). $$\left[\mu^-(xay), \mu^+(xay)\right] = \tilde{\mu}(x\alpha y) \geq \min{}^i\{\tilde{\mu}(x), \tilde{\mu}(y)\}$$
$$= \min{}^i\{\left[\mu^-(x), \mu^+(x)\right], \left[\mu^-(y), \mu^+(y)\right]\}$$
$$= \left[\min\left\{\mu^-(x), \mu^-(y)\right\}, \min\left\{\mu^+(x), \mu^+(y)\right\}\right].$$

It follows that $\mu^-(xay) \geq \min\{\mu^-(x), \mu^-(y)\}$ *and*

$$\mu^+(xay) \geq \min\{\mu^+(x), \mu^+(y)\}$$

(3). $$\left[\mu^-(y+x-y), \mu^+(y+x-y)\right] = \tilde{\mu}(y+x-y)$$
$$\geq \tilde{\mu}(x)$$
$$= \left[\mu^-(x), \mu^+(x)\right]$$

It follows that $\mu^-(y+x-y) \geq \mu^-(x)$ *and*

$$\mu^+(y+x-y) \geq \mu^+(x)$$

(4). $$\left[\mu^-(x\alpha y), \mu^+(x\alpha y)\right] = \tilde{\mu}(x\alpha y)$$
$$\geq \tilde{\mu}(y)$$
$$= \left[\mu^-(x), \mu^+(x)\right]$$

It follows that $\mu^-(x\alpha y) \geq \mu^-(y)$ *and*

$$\mu^+(x\alpha y) \geq \mu^+(y)$$

(5). $$\left[\mu^-((x+z)\alpha y - x\alpha y), \mu^+((x+z)\alpha y - x\alpha y)\right] = \tilde{\mu}((x+z)\alpha y - x\alpha y)$$
$$\geq \tilde{\mu}(z)$$
$$= \left[\mu^-(z), \mu^+(z)\right]$$

It follows that $\mu^-((x+z)\alpha y - x\alpha y) \geq \mu^-(z)$ *and*

$$\mu^+((x+z)\alpha y - x\alpha y) \geq \mu^+(z)$$

Hence μ^+, μ^- are fuzzy left (right) ideals of Γ-near-ring N.

(B). Conversely, assume that μ^-, μ^+ are fuzzy left (right) ideals of Γ-near-ring N. Let $x, y, z \in R$. Then

(1). $\tilde{\mu}(x-y) = [\mu^{-}(x-y), \mu^{+}(x-y)]$

$\geq [\min\{\mu^{-}(x), \mu^{-}(y)\}, \min\{\mu^{+}(x), \mu^{+}(y)] .$

$= \min^{i}\{[\mu^{-}(x), \mu^{+}(x)], [\mu^{-}(y), \mu^{+}(y)]\}$

$= \min^{i}\{\tilde{\mu}(x), \tilde{\mu}(y)\}$

(2). $\tilde{\mu}(x\alpha y) = [\mu^{-}(x\alpha y), \mu^{+}(x\alpha y)]$

$\geq [\min\{\mu^{-}(x), \mu^{-}(y)\}, \min\{\mu^{+}(x), \mu^{+}(y)] .$

$= \min^{i}\{[\mu^{-}(x), \mu^{+}(x)], [\mu^{-}(y), \mu^{+}(y)]\}$

$= \min^{i}\{\tilde{\mu}(x), \tilde{\mu}(y)\}$

(3). $\tilde{\mu}(y+x-y) = [\mu^{-}(y+x-y), \mu^{+}(y+x-y)]$

$\geq [\mu^{-}(x), \mu^{+}(x)]$

$= \tilde{\mu}(x)$

(4). $\tilde{\mu}(x\alpha y) = [\mu^{-}(x\alpha y), \mu^{+}(x\alpha y)]$

$\geq [\mu^{-}(x), \mu^{+}(x)]$

$= \tilde{\mu}(y)$

(5). $\tilde{\mu}((x+z)\alpha y - x\alpha y) = [\mu^{-}((x+z)\alpha y - x\alpha y), \mu^{+}((x+z)\alpha y - x\alpha y)]$

$\geq [\mu^{-}(z), \mu^{+}(z)]$

$= \tilde{\mu}(z)$

Hence $\tilde{\mu}$ is an interval valued fuzzy left (right) ideal of a Γ-near-ring N. □

Theorem 3.5 ([15]). *Let $\tilde{\mu}$ be an interval valued fuzzy subset of R. $\tilde{\mu}$ is an interval valued fuzzy left (right) ideal of R if and only if $\tilde{U}(\tilde{\mu} : [t_1, t_1])$ is a left (right) ideal of R, for all $[t_1, t_1] \in D[0, 1]$.*

4. Interval Valued Anti Fuzzy Ideals of Γ-Near-Ring

Definition 4.1. *An interval valued fuzzy subset $\tilde{\mu}$ of a Γ-near-ring N is called interval valued anti fuzzy sub Γ-near-ring of N if*

(1). $\tilde{\mu}(x-y) \leq \max_i\{\tilde{\mu}(x), \tilde{\mu}(y)\}$,

(2). $\tilde{\mu}(xay) \leq \max_i\{\tilde{\mu}(x), \tilde{\mu}(y)\} \quad \forall \quad x, y \in N$.

Definition 4.2. *An interval valued fuzzy subset $\tilde{\mu}$ of a Γ-near-ring N is called interval valued anti fuzzy ideal Γ-near-ring of N if $\tilde{\mu}$ is an interval valued anti fuzzy sub Γ-near-ring N and*

(1). $\tilde{\mu}(y+x-y) \leq \tilde{\mu}(x)$,

(2). $\tilde{\mu}(xay) \leq \tilde{\mu}(y)$,

(3). $\tilde{\mu}((x+z)\alpha y - x\alpha y) \leq \tilde{\mu}(z) \quad \forall \quad x, y, z \in N, \ \alpha \in \Gamma$.

Theorem 4.3. *Let N be a Γ-near-ring and $\{\tilde{\mu}_i \ : i \in I\}$ a non-empty family of subsets of N. If $\{\tilde{\mu}_i \ : i \in I\}$ is an interval valued anti fuzzy ideal of N then $\bigcup_{i \in I} \tilde{\mu}_i$ is an interval valued anti fuzzy ideal of N.*

Proof. Let $\{\tilde{\mu}_i : i \in I\}$ be an interval valued anti fuzzy ideal of N. Let $x, y, z \in N$ and $\alpha \in \Gamma$. Then we have $\forall\ i \in I$

$$\begin{aligned}
\left(\bigcup_{i\in I}\tilde{\mu}_i\right)(x-y) &= \sup^i\{\tilde{\mu}_i(x-y)\}_{i\in I} \\
&\leq \sup^i\{\max_i\{\tilde{\mu}_i(x), \tilde{\mu}_i(y)\}\}_{i\in I} \\
&= \max_i\{\sup^i(\tilde{\mu}_i(x)), \sup^i(\tilde{\mu}_i(y))\}_{i\in I} \\
&= \max_i\left\{\left(\bigcup_{i\in I}\tilde{\mu}_i\right)(x), \left(\bigcup_{i\in I}\tilde{\mu}_i\right)(y)\right\} \\
\left(\bigcup_{i\in I}\tilde{\mu}_i\right)(x\alpha y) &= \sup^i\{\tilde{\mu}_i(x\alpha y)\}_{i\in I} \\
&\leq \sup^i\{\max_i\{\tilde{\mu}_i(x), \tilde{\mu}_i(y)\}\}_{i\in I} \\
&= \max_i\{\sup^i(\tilde{\mu}_i(x)), \sup^i(\tilde{\mu}_i(y))\}_{i\in I} \\
&= \max_i\left\{\left(\bigcup_{i\in I}\tilde{\mu}_i\right)(x), \left(\bigcup_{i\in I}\tilde{\mu}_i\right)(y)\right\} \\
\left(\bigcup_{i\in I}\tilde{\mu}_i\right)(y+x-y) &= \sup^i\{\tilde{\mu}_i(y+x-y)\}_{i\in I} \\
&\leq \sup^i\{\tilde{\mu}_i(x)\}_{i\in I} \\
&= \left(\bigcup_{i\in I}\tilde{\mu}_i\right)(y) \\
\left(\bigcup_{i\in I}\tilde{\mu}_i\right)(x\alpha y) &= \sup^i\{\tilde{\mu}_i(x\alpha y)\}_{i\in I} \\
&\leq \sup^i\{\tilde{\mu}_i(y)\}_{i\in I} \\
&= \left(\bigcup_{i\in I}\tilde{\mu}_i\right)(y) \\
\left(\bigcup_{i\in I}\tilde{\mu}_i\right)((x+z)\alpha y - x\alpha y) &= \sup^i\{\tilde{\mu}_i((x+z)ay - xay)\}_{i\in I} \\
&\leq \sup^i\{\tilde{\mu}_i(z)\}_{i\in I} \\
&= \left(\bigcup_{i\in I}\tilde{\mu}_i\right)(z)
\end{aligned}$$

Therefore $\bigcup_{i\in I}\tilde{\mu}_i$ is an interval valued anti fuzzy ideal of a Γ-near-ring N. □

Theorem 4.4. *Let $\tilde{\mu}$ be an interval valued fuzzy subset of Γ-near-ring N. $\tilde{\mu} = [\mu^-, \mu^+]$ is an interval valued anti fuzzy left (right) ideal of Γ-near-ring N if and only if μ^+, μ^- are anti fuzzy left (right) ideals of Γ-near-ring N.*

Proof. (A). Assume that $\tilde{\mu}$ is an interval valued anti fuzzy left (right) ideal of Γ-near-ring N.. For any $x, y, z \in N$ and $\alpha \in \Gamma$, then we have

(1). $$\begin{aligned}
\left[\mu^-(x-y), \mu^+(x-y)\right] &= \tilde{\mu}(x-y) \leq max^i\{\tilde{\mu}(x), \tilde{\mu}(y)\} \\
&= \max^i\{[\mu^-(x), \mu^+(x)], [\mu^-(y), \mu^+(y)]\} \\
&= \left[\max\{\mu^-(x), \mu^-(y)\}, \max\{\mu^+(x), \mu^+(y)\}\right].
\end{aligned}$$

It follows that $\mu^-(x-y) \leq \max\{\mu^-(x), \mu^-(y)\}$ *and*

$\mu^+(x-y) \leq \max\{\mu^+(x), \mu^+(y)\}$

(2). $$\left[\mu^{-}(x\alpha y),\mu^{+}(x\alpha y)\right]=\tilde{\mu}(x\alpha y)$$
$$\leq \max{}^{i}\{\tilde{\mu}(x),\tilde{\mu}(y)\}$$
$$=\max{}^{i}\{\left[\mu^{-}(x),\mu^{+}(x)\right],\left[\mu^{-}(y),\mu^{+}(y)\right]\}$$
$$=\left[\max\left\{\mu^{-}(x),\mu^{-}(y)\right\},\max\left\{\mu^{+}(x),\mu^{+}(y)\right\}\right]$$

It follows that $\mu^{-}(x\alpha y)\leq \max\{\mu^{-}(x),\mu^{-}(y)\}$ *and*

$$\mu^{+}(x\alpha y)\alpha\leq \max\{\mu^{+}(x),\mu^{+}(y)\}$$

(3). $$\left[\mu^{-}(y+x-y),\mu^{+}(y+x-y)\right]\alpha=\tilde{\mu}(y+x-y)$$
$$\leq \tilde{\mu}(x)$$
$$=\left[\mu^{-}(x),\mu^{+}(x)\right]$$

It follows that $\mu^{-}(y+x-y)\leq \mu^{-}(x)$ *and*

$$\mu^{+}(y+x-y)\leq \mu^{+}(x)$$

(4). $$\left[\mu^{-}(x\alpha y),\mu^{+}(x\alpha y)\right]=\tilde{\mu}(x\alpha y)$$
$$\leq \tilde{\mu}(y)$$
$$=\left[\mu^{-}(x),\mu^{+}(x)\right]$$

It follows that $\mu^{-}(x\alpha y)\leq \mu^{-}(y)$ *and*

$$\mu^{+}(x\alpha y)\leq \mu^{+}(y)$$

(5). $$\left[\mu^{-}((x+z)\alpha y-x\alpha y),\mu^{+}((x+z)\alpha y-x\alpha y)\right]=\tilde{\mu}((x+z)\alpha y-x\alpha y)$$
$$\leq \tilde{\mu}(z)$$
$$=\left[\mu^{-}(z),\mu^{+}(z)\right]$$

It follows that $\mu^{-}((x+z)\alpha y-x\alpha y)\leq \mu^{-}(z)$ *and*

$$\mu^{+}((x+z)\alpha y-x\alpha y)\leq \mu^{+}(z)$$

Hence μ^{+}, μ^{-} are fuzzy left (right) ideals of Γ-near-ring N.

(B). Conversely, assume that μ^{-}, μ^{+} are anti fuzzy left (right) ideals of Γ-near-ring N. Let $x,y,z\in R$. Then

(1). $$\tilde{\mu}(x-y)=\left[\mu^{-}(x-y),\mu^{+}(x-y)\right]$$
$$\leq \left[\max\left\{\mu^{-}(x),\mu^{-}(y)\right\},\max\left\{\mu^{+}(x),\mu^{+}(y)\right\}\right]$$
$$=\max{}^{i}\{\left[\mu^{-}(x),\mu^{+}(x)\right],\left[\mu^{-}(y),\mu^{+}(y)\right]\}$$
$$=\max{}^{i}\{\tilde{\mu}(x),\tilde{\mu}(y)\}$$

(2). $$\tilde{\mu}(x\alpha y)=\left[\mu^{-}(x\alpha y),\mu^{+}(x\alpha y)\right]$$
$$\leq \left[\max\left\{\mu^{-}(x),\mu^{-}(y)\right\},\max\left\{\mu^{+}(x),\mu^{+}(y)\right\}\right].$$
$$=\max{}^{i}\{\left[\mu^{-}(x),\mu^{+}(x)\right],\left[\mu^{-}(y),\mu^{+}(y)\right]\}$$
$$=\max{}^{i}\{\tilde{\mu}(x),\tilde{\mu}(y)\}$$

(3). $$\tilde{\mu}(y+x-y)=\left[\mu^{-}(y+x-y),\mu^{+}(y+x-y)\right]$$
$$\leq \left[\mu^{-}(x),\mu^{+}(x)\right]$$
$$=\tilde{\mu}(x)$$

(4). $\tilde{\mu}(x\alpha y) = [\mu^-(x\alpha y), \mu^+(x\alpha y)]$

$$\leq [\mu^-(x), \mu^+(x)]$$

$$= \tilde{\mu}(y)$$

(5). $\tilde{\mu}((x+z)\alpha y - x\alpha y) = [\mu^-((x+z)\alpha y - x\alpha y), \mu^+((x+z)\alpha y - x\alpha y)]$

$$\leq [\mu^-(z), \mu^+(z)]$$

$$= \tilde{\mu}(z)$$

Hence $\tilde{\mu}$ is an interval valued anti fuzzy left (right) ideal of a Γ-near-ring N. □

References

[1] S. Abou-Zaid, *On fuzzy subnear-rings and ideals*, Fuzzy sets and Systems, 44(1991), 139-146.

[2] R. Biswas, *Fuzzy subgroups and anti fuzzy subgroups*, Fuzzy Sets and Systems, 44(1990), 121-124.

[3] R. Biswas, *Rosenfeld's fuzzy subgroups with interval valued membership functions*, Fuzzy sets and systems, 63(1994), 87-90.

[4] B. Davvaz, *Fuzzy ideals of near-rings with interval valued membership functions*, Journal of Sciences, Islamic republic of Iran, 12(2)(2001), 171-175.

[5] T. K. Dutta and B. K. Biswas, *Fuzzy ideals of near-rings*, Bull. Calcutta Math. Soc., 89(1997), 447-456.

[6] S.M. Hong, Y.B. Jun and H.S. Kim, *Fuzzy ideals in near-rings*, Bulletin of Korean Mathematical Society, 35(3)(1998), 455-464.

[7] Y.B. Jun and K.H. Kim, *Interval valued fuzzy R-subgroups of near-rings*, Indian. J. Pure appl. Math., 33(2002), 71-80.

[8] K. H. Kim and Y.B. Jun, *Anti fuzzy ideals in near rings*, Iranian Journal of Fuzzy Systems, 2(2005), 71-80.

[9] K. Murugalingam and K. Arjunan, *A study on interval valued fuzzy subsemirings of semirings*, International Journal of Applied Mathematics and Modeling, 1(2013), 1-6.

[10] T. Nagaiah, K. Vijay kumar, A. Iampan and T. Srinivas, *A study of fuzzy ideals of PO-Gamma semigroups*, Palestina Journal of Mathematics, 5(1)(2016), 1-7.

[11] T.L. Dewangan and M.M. Singh, *Fuzzy ideals and anti fuzzy ideals of near-rings*, International Journal for Research in Applied Science & Engineering Technology.

[12] T. Nagaiah, K. Vijay kumar, P. Narasimha Swamy and T. Srinivas, *Interval valued fuzzy ideals of* Γ*-near-rings*, Bulletin of Culcutta Mathematical Society.

[13] T. Srinivas, T. Nagaiah and P. Narasimha Swamy, *Anti fuzzy ideals of* Γ*-near-rings*, Annals of Fuzzy Mathematics and Informatics, 3(2)(2012), 255-266.

[14] T. Srinivas, P. Narasimha Swamy and T. Nagaiah, *Anti fuzzy near-algebras over anti fields*, Annals of Fuzzy Mathematics and Informatics, 4(2)(2012), 243-252.

[15] N. Thillaigovindan, V. Chinnadurai and K. Kadalarasi, *Interval valued fuzzy ideals of near-rings*, The Journal of fuzzy Mathematics, 23(2015), 471-484.

[16] N. Thillaigovindan, V. Chinnadurai and K. Lenin Muthu Kumaran, *Some remarks on interval valued anti fuzzy ideals of near-rings*, Annals of Fuzzy Mathematics and Informatics, 11(3)(2011), 461-474.

[17] L. A. Zadeh, *The concepts of a lingustic variable and its application to approximate reasoning-I*, Information and Sciences, 8(1975), 199-249.

[18] L.A. Zadeh, Fuzzy sets, Inform and Control, 8(1965), 338-353.