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Interval Valued Fuzzy Ideals of Γ-Semirings

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Abstract: In this paper, we introduce the concept of interval valued fuzzy ideals of Γ-Semirings. We also characterize some of its properties and illustrate with examples of interval valued fuzzy ideals of Γ-Semirings.

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1. Introduction

The notion of fuzzy sets was introduced by Zadeh [8] in 1965 and he [9] also generalized it to interval valued fuzzy subsets (shortly i-v fuzzy subsets) whose of membership values are closed sub intervals of [0,1]. H.S.Vandiver [7] introduced Semirings. In 1995, M.K.Rao [5] introduced the notion of Γ-Semiring. V.Chinnadurai and K. Arulmozhi worked on interval valued fuzzy ideals of Γ-near rings [2]. In this direction, we define a notion of interval valued fuzzy ideals of Γ-Semirings. We also characterize some of its properties and illustrate with examples.

2. Preliminaries

In this section, we list some basic concepts and well known results of interval valued fuzzy set theory.

Definition 2.1 ([3]). *Let S and Γ be two additive commutative semigroups. Then S is called a Γ-semiring if there exists a mapping $(a, \alpha, b) \to a\alpha b$, where $a, b \in S$ and $\alpha \in \Gamma$ satisfying the following conditions:*

(1). $(a + b)\alpha c = a\alpha c + b\alpha c$,

(2). $a\alpha(b + c) = a\alpha b + a\alpha c$,

(3). $a\,(\alpha + \beta)\,b = a\alpha b + a\beta b$,

(4). $a\alpha(b\beta c) = (a\alpha b)\beta c$

for all $a, b, c \in S$ and for all $\alpha, \beta \in \Gamma$.

Definition 2.2 ([3]). *A non empty subset T of a Γ-semiring S is called a left (respectively right) ideal of S if T is a sub semigroup of $(S, +)$ and $x\alpha a \in T$ (respectively $a\alpha x \in T$) for all $a \in T$, $x \in S$ and $\alpha \in \Gamma$.*

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Definition 2.3 ([8]). *Let S be a non-empty set. A mapping $\mu : S \to [0,1]$ is called a fuzzy subset of S.*

Definition 2.4 ([4]). *Let λ be a non-empty fuzzy subset of a Γ-semiring S (i.e., $\lambda(x) \neq 0$ for some $x \in S$). Then λ is called a fuzzy left ideal (respectively fuzzy right ideal) of S if*

(1). $\lambda(x+y) \geq \min\{\lambda(x), \lambda(y)\}$,

(2). $\lambda(x\alpha y) \geq \lambda(y)$ *(respectively* $\lambda(x\alpha y) \geq \lambda(x)$*) for all* $x, y \in S$ *and* $\alpha \in \Gamma$.

Definition 2.5 ([6]). *An interval valued number $\overline{a}$ is a closed sub interval of [0,1] i.e) $\overline{a} = [a^-, a^+]$ such that $0 \leq a^- \leq a^+ \leq 1$ where a^- and a^+ are the lower and upper limits of $\overline{a}$ respectively. In this notation, $\overline{0} = [0,0]$ and $\overline{1} = [1,1]$. For any two interval numbers $\overline{a} = [a^-, a^+]$ and $\overline{b} = [b^-, b^+]$, we define*

(1). $\overline{a} \leq \overline{b} \Leftrightarrow a^- \leq b^-$ *and* $a^+ \leq b^+$,

(2). $\overline{a} = \overline{b} \Leftrightarrow a^- = b^-$ *and* $a^+ = b^+$,

(3). $\overline{a} < \overline{b} \Leftrightarrow \overline{a} \leq \overline{b}$ *and* $\overline{a} \neq \overline{b}$ *and*

(4). $k\overline{a} = [ka^-, ka^+]$ *whenever* $0 < k < 1$.

Definition 2.6 ([1]). *Let X be a non-empty set. A mapping $\overline{\mu} : X \to D[0,1]$ is called an interval valued fuzzy subset (briefly, i-v fuzzy subset) of X, where $D[0,1]$ denotes the family of all closed sub-intervals of [0,1] and $[\mu(x), \mu^+(x)] = \left[\mu^-(x), \mu^+(x)\right]$, where μ^- and μ^+ are fuzzy subsets of X such that $\mu^-(x) \leq \mu^+(x)$ for all $x \in X$. Thus $\overline{\mu}(x)$ is an interval (a closed subset of [0,1]).*

Definition 2.7 ([1]). *The minimum of any two interval valued numbers $\overline{a}$ and $\overline{b}$ in $D[0,1]$ is defined as* $\min^i\{\overline{a}, \overline{b}\} = [\min\left\{a^-, b^-\right\}, \min\left\{a^+, b^+\right\}]$.

Definition 2.8 ([1]). *The maximum of any two interval valued numbers $\overline{a}$ and $\overline{b}$ in $D[0,1]$ is defined as* $\max^i\{\overline{a}, \overline{b}\} = [\max\left\{a^-, b^-\right\}, \max\left\{a^+, b^+\right\}]$.
Also, for any interval numbers $\overline{a}_j = \left[a_j^-, a_j^+\right]$, $\overline{b}_j = \left[b_j^-, b_j^+\right] \in D[0,1]$, $j \in \Omega$ (where Ω is an index set), we define

(1). $\inf^i \overline{a_j} = [\inf_{j\in\Omega} a_j^-, \inf_{j\in\Omega} a_j^+]$.

(2). $\sup^i \overline{a_j} = [\sup_{j\in\Omega} a_j^-, \sup_{j\in\Omega} a_j^+]$.

For any interval numbers $\overline{a}$ and $\overline{b}$ in $D[0,1]$, the following are true.

(1). $\min^i\{\overline{a}, \overline{a}\} = \overline{a}$ *and* $\max^i\{\overline{a}, \overline{a}\} = \overline{a}$.

(2). $\min^i\left\{\overline{a}, \overline{b}\right\} = \min^i\left\{\overline{b}, \overline{a}\right\}$ *and* $\max^i\left\{\overline{a}, \overline{b}\right\} = \max^i\left\{\overline{b}, \overline{a}\right\}$.

(3). If $\overline{a} \geq \overline{b}$, *then* $\min^i\{\overline{a}, \overline{c}\} \geq \min^i\left\{\overline{b}, \overline{c}\right\}$ *and* $\max^i\{\overline{a}, \overline{c}\} \geq \max^i\left\{\overline{b}, \overline{c}\right\}$ *for all* $\overline{c} \in D[0,1]$.

Definition 2.9 ([1]). *Let $\overline{\mu}$ be an i-v fuzzy subset of a set X and $[t_1, t_2] \in D[0,1]$. Then the set $\overline{U}(\overline{\mu} : [t_1, t_2]) = \{x \in X : \overline{\mu}(x) \geq [t_1, t_2]\}$ is called the upper level set of $\overline{\mu}$.*
Note that

$$\begin{aligned}
\overline{U}(\overline{\mu} : [t_1, t_2]) &= \{x \in X : \overline{\mu}(x) \geq [t_1, t_2]\} \\
&= \{x \in X : [\mu^-(x), \mu^+(x)] \geq [t_1, t_2]\} \\
&= \{x \in X : \mu^-(x) \geq t_1\} \cap \{x \in X : \mu^+(x) \geq t_2\} \\
&= (U(\mu^- : t_1)) \cap (U(\mu^+ : t_2)).
\end{aligned}$$

Definition 2.10 ([1]). *Let $\overline{\mu}, \overline{\nu}, \overline{\mu}_i (i \in \Omega)$ be i-v fuzzy subsets of X. Then the following are defined by*

(1). $\overline{\mu} \leq \overline{\nu} \Leftrightarrow \overline{\mu}(x) \leq \overline{\nu}(x)$ for all $x \in X$,

(2). $\overline{\mu} = \overline{\nu} \Leftrightarrow \overline{\mu}(x) = \overline{\nu}(x)$ for all $x \in X$,

(3). $(\overline{\mu} \cup \overline{\nu})(x) = \max^i\{\overline{\mu}(x), \overline{\nu}(x)\}$ for all $x \in X$,

(4). $(\overline{\mu} \cap \overline{\nu})(x) = \min^i\{\overline{\mu}(x), \overline{\nu}(x)\}$ for all $x \in X$,

(5). $\cup_{i\in\Omega}\overline{\mu}_i(x) = \sup^i\{\overline{\mu}_i(x) : i \in \Omega\}$ for all $x \in X$ and

(6). $\cap_{i\in\Omega}\overline{\mu}_i(x) = \inf^i\{\overline{\mu}_i(x) : i \in \Omega\}$ for all $x \in X$.

Here $\sup^i\{\overline{\mu}_i(x) : i \in \Omega\} = [\sup_{i\in\Omega}\{\mu_i^-(x)\}, \sup_{i\in\Omega}\{\mu_i^+(x)\}]$ is the i-v supremum norm and $\inf^i\{\overline{\mu}_i(x) : i \in \Omega\} = [\inf_{i\in\Omega}\{\mu_i^-(x)\}, \inf_{i\in\Omega}\{\mu_i^+(x)\}]$ is the i-v infimum norm.

3. Interval Valued Fuzzy Ideals of Γ-Semirings

In this section we introduce the notion of interval valued fuzzy ideal of Γ-Semiring S and obtain some characterizations of interval valued fuzzy ideal of S.

Definition 3.1. *An i-v fuzzy subset $\overline{\lambda}$ of a Γ-Semiring S is called an i-v fuzzy ideal of S if*

(1). $\overline{\lambda}(x+y) \geq \min^i\{\overline{\lambda}(x), \overline{\lambda}(y)\}$,

(2). $\overline{\lambda}(xay) \geq \overline{\lambda}(y)$,

(3). $\overline{\lambda}(x\alpha y) \geq \overline{\lambda}(x)$ for all $x, y \in S$ and $\alpha \in \Gamma$.

Note that $\overline{\lambda}$ is an i-v fuzzy left ideal of S if it satisfies (1) and (2) and $\overline{\lambda}$ is an i-v fuzzy right ideal of S if it satisfies (1) and (3).

Example 3.2. *Let N_0 be a set of non-negative integers. Let $\Gamma = N_0$. Then N_0, Γ are additive commutative semigroups. Define the mapping $N_0 \times \Gamma \times N_0 \longrightarrow N_0$ by $a\alpha b$ as usual product of a, α, b for all $a, b \in N_0$ and $\alpha \in \Gamma$. Now define $\overline{\lambda} : N_0 \longrightarrow D[0,1]$ by*

$$\overline{\lambda}(x) = \begin{cases} \overline{1}, & \text{if } x = 0 \\ [0.5, 0.6], & \text{if } x \text{ is even} \\ [0.3, 0.4], & \text{if } x \text{ is odd} \end{cases}$$

for all $x \in N_0$. Then $\overline{\lambda}$ is an i-v fuzzy ideal of N_0.

Example 3.3. *Let S be a set of positive integers. Let Γ be the set of positive even integers. Then S and Γ are additive commutative semigroups. Define the mapping $S \times \Gamma \times S \longrightarrow S$ by $a\alpha b$ as usual product of a, α, b for all $a, b \in S$ and $\alpha \in \Gamma$. Now define $\overline{\lambda} : S \longrightarrow D[0,1]$ by*

$$\overline{\lambda}(x) = \begin{cases} [0.4, 0.5], & \text{if } x \text{ is even} \\ [0.2, 0.3], & \text{if } x \text{ is odd} \end{cases}$$

for all $x \in S$. Then $\overline{\lambda}$ is an i-v fuzzy ideal of S.

Example 3.4. *Consider the sets* $Z_4 = \{0,1,2,3\}$ *and* $\Gamma = \{0,2\}$*. Then* Z_4 *and* Γ *are additive modulo 4 commutative semigroups from the following table (a). Define the mapping* $Z_4 \times \Gamma \times Z_4 \longrightarrow Z_4$ *by* $a\alpha b$ *as multiplication modulo 4 for all* $a, b \in Z_4$ *and* $\alpha \in \Gamma$*. The map is well defined by the tables (b) and (c).*

$\oplus$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

(a)

$\alpha = 0$	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0

(b)

$\alpha = 2$	0	1	2	3
0	0	0	0	0
1	0	2	0	2
2	0	0	0	0
3	0	2	0	2

(c)

Now, Define $\overline{\lambda} : Z_4 \longrightarrow D[0,1]$ *by*

$$\overline{\lambda}(x) = \begin{cases} [0.8, 0.9], & \text{if } x = 0 \\ [0.6, 0.7], & \text{if } x = 1 \\ [0.1, 0.2], & \text{otherwise} \end{cases}$$

for all $x \in Z_4$*. When* $x = 1, y = 1$ *we get* $\overline{\lambda}(x+y) = [0.1, 0.2]$ *and* $\min^i \{\overline{\lambda}(x), \overline{\lambda}(y)\} = [0.6, 0.7]$*. This shows that* $\overline{\lambda}(x+y) < \min^i\{\overline{\lambda}(x), \overline{\lambda}(y)\}$*. Therefore Then* $\overline{\lambda}$ *is not an i-v fuzzy ideal of* Z_4*.*

Theorem 3.5. *Let* S *be a* Γ*- Semiring and* $\overline{\lambda}$ *be an i-v fuzzy subset of* S*. Then* $\overline{\lambda} = [\lambda^-, \lambda^+]$ *is an i-v fuzzy ideal of* S *if and only if* λ^- *and* λ^+ *are fuzzy ideals of* S*.*

Proof. Assume that $\overline{\lambda}$ is an i-v fuzzy ideal of S. For any $x, y \in S$ and $\alpha \in \Gamma$ we have

$$\begin{aligned} \left[\lambda^-(x+y), \lambda^+(x+y)\right] &= \overline{\lambda}(x+y) \\ &\geq \min^i\{\overline{\lambda}(x), \overline{\lambda}(y)\} \\ &= \min^i\{\left[\lambda^-(x), \lambda^+(x)\right], \left[\lambda^-(y), \lambda^+(y)\right]\} \\ &= \left[\min\left\{\lambda^-(x), \lambda^-(y)\right\}, \min\left\{\lambda^+(x), \lambda^+(y)\right\}\right] \end{aligned}$$

It follows that $\lambda^-(x+y) \geq \min\left\{\lambda^-(x), \lambda^-(y)\right\}$ and $\lambda^+(x+y) \geq \min\left\{\lambda^+(x), \lambda^+(y)\right\}$. Also,

$$\begin{aligned} \left[\lambda^-(xay), \lambda^+(x\alpha y)\right] &= \overline{\alpha}(x\alpha y) \\ &\geq \overline{\lambda}(y) \\ &= \left[\lambda^-(y), \lambda^+(y)\right] \end{aligned}$$

It follows that $\lambda^-(xay) \geq \lambda^-(y)$ and $\lambda^+(xay) \geq \lambda^+(y)$. Also

$$\begin{aligned} \left[\lambda^-(xay), \lambda^+(xay)\right] &= \overline{\lambda}(x\alpha y) \\ &\geq \overline{\lambda}(x) \\ &= \left[\lambda^-(x), \lambda^+(x)\right] \end{aligned}$$

It follows that $\lambda^-(x\alpha y) \geq \lambda^-(x)$ and $\lambda^+(x\alpha y) \geq \lambda^+(x)$. Thus λ^- and λ^+ are fuzzy ideals of S. Conversely, assume that λ^- and λ^+ are fuzzy ideals of S. Let $x, y \in S$ and $\alpha \in \Gamma$. Then

$$\begin{aligned} \overline{\lambda}(x+y) &= \left[\lambda^-(x+y), \lambda^+(x+y)\right] \\ &\geq \left[\min\left\{\lambda^-(x), \lambda^-(y)\right\}, \min\left\{\lambda^+(x), \lambda^+(y)\right\}\right] \end{aligned}$$

$$= \min^i\{[\lambda^-(x), \lambda^+(x)], [\lambda^-(y), \lambda^+(y)]\}$$
$$= \min^i\{\overline{\lambda}(x), \overline{\lambda}(y)\}$$

This implies that $\overline{\lambda}(x+y) \geq \min^i\{\overline{\lambda}(x), \overline{\lambda}(y)\}$. Since $\lambda^-(x\alpha y) \geq \lambda^-(y)$ and $\lambda^+(x\alpha y) \geq \lambda^+(y)$, we have $[\lambda^-(x\alpha y), \lambda^+(x\alpha y)] \geq [\lambda^-(y), \lambda^+(y)]$. This implies that $\overline{\lambda}(x\alpha y) \geq \overline{\lambda}(y)$. Since $\lambda^-(x\alpha y) \geq \lambda^-(x)$ and $\lambda^+(x\alpha y) \geq \lambda^+(x)$, we have $[\lambda^-(x\alpha y), \lambda^+(x\alpha y)] \geq [\lambda^-(x), \lambda^+(x)]$. This implies that $\overline{\lambda}(x\alpha y) \geq \overline{\lambda}(x)$. Thus $\overline{\lambda}$ is an i-v fuzzy ideal of S. □

Remark 3.6. *In particular ,we have $\overline{\lambda}$ is an i-v fuzzy left (right) ideal of S if and only if λ^- and λ^+ are fuzzy left(right) ideals of S.*

Theorem 3.7. *Let S be a Γ-Semiring and $\overline{\lambda}$ be an i-v fuzzy subset of S. Then $\overline{\lambda}$ is an i-v fuzzy ideal of S if and only if $\overline{U}(\overline{\lambda} : [t_1, t_2])$ is an ideal of S for all $[t_1, t_2] \in D[0, 1]$.*

Proof. Assume that $\overline{\lambda}$ is an i-v fuzzy ideal of S. Let $[t_1, t_2] \in D[0, 1]$ such that $x, y \in \overline{U}(\overline{\lambda} : [t_1, t_2])$. Then $\overline{\lambda}(x+y) \geq \min^i\{\overline{\lambda}(x), \overline{\lambda}(y)\} \geq \min^i\{[t_1, t_2], [t_1, t_2]\} = [t_1, t_2]$. Thus $x+y \in \overline{U}(\overline{\lambda} : [t_1, t_2])$. Therefore $\overline{U}(\overline{\lambda} : [t_1, t_2])$ is a sub semigroup of $(S, +)$. Let $x \in S$ and $y \in \overline{U}(\overline{\lambda} : [t_1, t_2])$. Then $\overline{\lambda}(x\alpha y) \geq \overline{\lambda}(y) \geq [t_1, t_2]$. Therefore $x\alpha y \in \overline{U}(\overline{\lambda} : [t_1, t_2])$. Also $\overline{\lambda}(y\alpha x) \geq \overline{\lambda}(y) \geq [t_1, t_2]$. Therefore $y\alpha x \in \overline{U}(\overline{\lambda} : [t_1, t_2])$. Thus $\overline{U}(\overline{\lambda} : [t_1, t_2])$ is an ideal of S for all $[t_1, t_2] \in D[0, 1]$. Conversely, assume that $\overline{U}(\overline{\lambda} : [t_1, t_2])$ is an ideal of S for all $[t_1, t_2] \in D[0, 1]$. Let $x, y \in S$. If $\overline{\lambda}(x+y) < \min^i\{\overline{\lambda}(x), \overline{\lambda}(y)\}$, choose an interval $\overline{a} = [a_1, a_2] \in D[0, 1]$ such that $\overline{\lambda}(x+y) < [a_1, a_2] < \min^i\{\overline{\lambda}(x), \overline{\lambda}(y)\}$. This implies that $\overline{\lambda}(x) > [a_1, a_2]$ and $\overline{\lambda}(y) > [a_1, a_2]$. Then we have $x, y \in \overline{U}(\overline{\lambda} : [a_1, a_2])$. Since $\overline{U}(\overline{\lambda} : [a_1, a_2])$ is a sub semigroup of $(S, +)$, we have $x+y \in \overline{U}(\overline{\lambda} : [a_1, a_2])$. This implies that $\overline{\lambda}(x+y) \geq [a_1, a_2]$. This contradiction shows that $\overline{\lambda}(x+y) \geq \min^i\{\overline{\lambda}(x), \overline{\lambda}(y)\}$. Again if $\overline{\lambda}(x\alpha y) < \overline{\lambda}(y)$, choose an interval $\overline{a} = [a_1, a_2] \in D[0, 1]$ such that $\overline{\lambda}(x\alpha y) < [a_1, a_2] < \overline{\lambda}(y)$. Then we have $y \in \overline{U}(\overline{\lambda} : [a_1, a_2])$. Since $\overline{U}(\overline{\lambda} : [a_1, a_2])$ is a left ideal of S we have $x\alpha y \in \overline{U}(\overline{\lambda} : [a_1, a_2])$. This implies that $\overline{\lambda}(x\alpha y) \geq [a_1, a_2]$ which is a contradiction. Thus $\overline{\lambda}(x\alpha y) \geq \overline{\lambda}(y)$. In the same way, we can show that $\overline{\lambda}(x\alpha y) \geq \overline{\lambda}(x)$. Thus $\overline{\lambda}$ is an i-v fuzzy ideal of S. □

Theorem 3.8. *Let S be a Γ-Semiring and I be an ideal of S. Then for any $\overline{a} \in D[0, 1]$, there exists an i-v fuzzy ideal $\overline{\lambda}$ of S such that $\overline{U}(\overline{\lambda} : \overline{a}) = I$.*

Proof. Let I be an ideal of S. Define an i-v fuzzy set $\overline{\lambda}$ in S by

$$\overline{\lambda}(x) = \begin{cases} \overline{a} & \text{if } x \in I \\ \overline{0} & \text{if } x \notin I \end{cases}$$

for all $x \in S$. Clearly, $\overline{U}(\overline{\lambda} : \overline{a}) = I$. Let $x, y \in S$ and $\alpha \in \Gamma$. If $x, y \in I$ then $x+y \in I$ and so $\overline{\lambda}(x+y) = \overline{a} = \min^i\{\overline{a}, \overline{a}\} = \min^i\{\overline{\lambda}(x), \overline{\lambda}(y)\}$. If $x \notin I$, $y \notin I$, then $\overline{\lambda}(x) = \overline{0} = \overline{\lambda}(y)$ and thus $\overline{\lambda}(x+y) \geq \overline{0} = \min^i\{\overline{0}, \overline{0}\} = \min^i\{\overline{\lambda}(x), \overline{\lambda}(y)\}$. Suppose that only one of x, y belongs to I, say x. Then $\overline{\lambda}(x+y) \geq \overline{0} = \min^i\{\overline{a}, \overline{0}\} = \min^i\{\overline{\lambda}(x), \overline{\lambda}(y)\}$. Thus in all the cases, we have $\overline{\lambda}(x+y) \geq \min^i\{\overline{\lambda}(x), \overline{\lambda}(y)\}$. Suppose $\overline{\lambda}(x\alpha y) < \overline{\lambda}(y)$ for some $x, y \in S$. Since $\overline{\lambda}$ is two valued, we have $\overline{\lambda}(x\alpha y) = \overline{0}$ and $\overline{\lambda}(y) = \overline{a}$. This implies that $x\alpha y \notin I$ and $y \in I$. This is a contradiction since I is a left ideal of S. Therefore $\overline{\lambda}(x\alpha y) \geq \overline{\lambda}(y)$. Suppose $\overline{\lambda}(x\alpha y) < \overline{\lambda}(x)$ for some $x, y \in S$. Since $\overline{\lambda}$ is two valued, we have $\overline{\lambda}(x\alpha y) = \overline{0}$ and $\overline{\lambda}(x) = \overline{a}$. This implies that $x\alpha y \notin I$ and $x \in I$. This is a contradiction since I is a right ideal of S. Therefore $\overline{\lambda}(x\alpha y) \geq \overline{\lambda}(x)$. Thus $\overline{\lambda}$ is an i-v fuzzy ideal of S. □

Theorem 3.9. *If $\{\overline{\lambda}_j : j \in \Omega\}$ is a family of i-v fuzzy ideals of Γ-Semiring S then $\cap_{j\in\Omega}\overline{\lambda}_j$ is also an i-v fuzzy ideal of S where Ω is an index set.*

Proof. Let $x, y, z \in S$. Then for any $j \in \Omega$,

$$\begin{aligned} \cap\overline{\lambda}_j\left(x+y\right) &= \inf{}^i\{\overline{\lambda}_j\left(x+y\right)\} \\ &\geq \overset{i}{\inf}\{\overset{i}{\min}\{\overline{\lambda}_j\left(x\right), \overline{\lambda}_j(y)\}\} \\ &= \min{}^i\{\inf{}^i\left\{\overline{\lambda}_j\left(x\right)\right\}, \inf{}^i\left\{\overline{\lambda}_j\left(y\right)\right\}\} \\ &= \min{}^i\{\cap\overline{\lambda}_i\left(x\right), \cap\overline{\lambda}_j\left(y\right)\} \end{aligned}$$

Also

$$\begin{aligned} \cap_{j\in\Omega}\overline{\lambda}_j\left(x\alpha y\right) &= \inf{}^i\{\overline{\lambda}_j\left(x\alpha y\right) : j \in \Omega\} \\ &\geq \overset{i}{\inf}\{\overline{\lambda}_j\left(y\right) : j \in \Omega\} \\ &= \cap_{j\in\Omega}\overline{\lambda}_j\left(y\right) \end{aligned}$$

Also,

$$\begin{aligned} \cap_{j\in\Omega}\overline{\lambda}_j\left(x\alpha y\right) &= \inf{}^i\{\overline{\lambda}_j\left(x\alpha y\right) : j \in \Omega\} \\ &\geq \overset{i}{\inf}\{\overline{\lambda}_j\left(x\right) : j \in \Omega\} \\ &= \cap_{j\in\Omega}\overline{\lambda}_j\left(x\right) \end{aligned}$$

Thus $\cap_{j\in\Omega}\overline{\lambda}_j$ is an i-v fuzzy ideal of S. ☐

Theorem 3.10. *Let I be a subset of a Γ-Semiring S. Then the characteristic function $\overline{f}_I : S \longrightarrow D[0,1]$ is an i-v fuzzy ideal of S if and only if I is an ideal of S.*

Proof. Assume that $\overline{f}_I$ is an i-v fuzzy ideal of S where $\overline{f}_I : S \longrightarrow D[0,1]$ defined by

$$\overline{f}_I\left(x\right) = \begin{cases} \overline{1}, & \text{if } x \in I \\ \overline{0}, & \text{if } x \notin I \end{cases}$$

for all $x \in S$. Let $x, y \in I$ and $\alpha \in \Gamma$. Now $\overline{f}_I\left(x+y\right) \geq \min^i\left\{\overline{f}_I\left(x\right), \overline{f}_I\left(y\right)\right\} = \min^i\left\{\overline{1}, \overline{1}\right\} = \overline{1}$ and so $\overline{f}_I\left(x+y\right) = \overline{1}$. This implies that $x + y \in I$. Therefore I is a sub semigroup of $(S, +)$. Let $x \in S$ and $y \in I$. Also $\overline{f}_I\left(x\alpha y\right) \geq \overline{f}_I\left(y\right) = \overline{1}$ and so $\overline{f}_I\left(x\alpha y\right) = \overline{1}$. This implies that $x\alpha y \in I$. Since $\overline{f}_I\left(y\alpha x\right) \geq \overline{f}_I\left(y\right)$ and $\overline{f}_I\left(y\right) = \overline{1}$, we have $\overline{f}_I\left(y\alpha x\right) = \overline{1}$ and hence $y\alpha x \in I$. Thus I is an ideal of S.

Conversely, assume that I is an ideal of S. Let $x, y \in S$ and $\alpha \in \Gamma$. If $x, y \in I$ then $x + y \in I$ and so $\overline{f}_I\left(x+y\right) = \overline{1} = \min^i\{\overline{1}, \overline{1}\} = \min^i\left\{\overline{f}_I\left(x\right), \overline{f}_I\left(y\right)\right\}$ If $x \notin I$, $y \notin I$, then $\overline{f}_I\left(x\right) = \overline{0} = \overline{f}_I\left(y\right)$ and thus $\overline{f}_I\left(x+y\right) \geq \overline{0} = \min^i\left\{\overline{0}, \overline{0}\right\} = \min^i\left\{\overline{f}_I\left(x\right), \overline{f}_I\left(y\right)\right\}$.

Suppose that only one of x, y belongs to I, say x. Then $\overline{f}_I\left(x+y\right) \geq \overline{0} = \min^i\{\overline{1}, \overline{0}\} = \min^i\left\{\overline{f}_I\left(x\right), \overline{f}_I\left(y\right)\right\}$. Thus in all these cases, we have $\overline{f}_I\left(x+y\right) \geq \min^i\left\{\overline{f}_I\left(x\right), \overline{f}_I\left(y\right)\right\}$. Suppose $\overline{f}_I\left(x\alpha y\right) < \overline{f}_I\left(y\right)$ for some $x, y \in S$. Since $\overline{f}_I$ is two valued, we have $\overline{f}_I\left(x\alpha y\right) = \overline{0}$ and $\overline{f}_I\left(y\right) = \overline{1}$. This implies that $x\alpha y \notin I$ and $y \in I$. This is a contradiction since I is a left ideal of S. Therefore $\overline{f}_I\left(x\alpha y\right) \geq \overline{f}_I\left(y\right)$. Suppose $\overline{f}_I\left(x\alpha y\right) < \overline{f}_I\left(x\right)$ for some $x, y \in S$. Since $\overline{f}_I$ is two valued, we have $\overline{f}_I\left(x\alpha y\right) = \overline{0}$ and $\overline{f}_I\left(x\right) = \overline{1}$. This implies that $x\alpha y \notin I$ and $x \in I$. This is a contradiction since $x \in I$ implies that $x\alpha y \in I$ for all $y \in S$. Therefore $\overline{f}_I\left(x\alpha y\right) \geq \overline{f}_I(x)$. Thus $\overline{f}_I$ is an i-v fuzzy ideal of S. ☐

Theorem 3.11. *If $\overline{\lambda}$ is an i-v fuzzy ideal of a Γ-Semiring S then the set $S_{\overline{\lambda}} = \{x \in S : \overline{\lambda}(x) \geq \overline{\lambda}(0)\}$ is an ideal of S.*

Proof. Let $\overline{\lambda}$ be an i-v fuzzy ideal of S. We claim that $S_{\overline{\lambda}}$ is an ideal of S. Let $x, y \in S_{\overline{\lambda}}$. Then $\overline{\lambda}(x) \geq \overline{\lambda}(0)$, $\overline{\lambda}(y) \geq \overline{\lambda}(0)$. Now $\overline{\lambda}(x+y) \geq \min^i \left\{\overline{\lambda}(x), \overline{\lambda}(y)\right\} \geq \min^i \left\{\overline{\lambda}(0), \overline{\lambda}(0)\right\} = \overline{\lambda}(0)$. This implies that $x + y \in S_{\overline{\lambda}}$. Thus $S_{\overline{\lambda}}$ is a sub semigroup of S. Let $x \in S$, $y \in S_{\overline{\lambda}}$ and $\alpha \in \Gamma$. Then $\overline{\lambda}(y) \geq \overline{\lambda}(0)$. Now, $\overline{\lambda}(xay) \geq \overline{\lambda}(y) \geq \overline{\lambda}(0)$. This implies $x\alpha y \in S_{\overline{\lambda}}$. Also, $\overline{\lambda}(y\alpha x) \geq \overline{\lambda}(y) \geq \overline{\lambda}(0)$. This implies $y\alpha x \in S_{\overline{\lambda}}$. Thus $S_{\overline{\lambda}}$ is an ideal of S. □

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