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L-Fuzzy Bi-ideal of a Ring

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Abstract: In this paper the concept of L-fuzzy bi-ideal is introduced and some of its properties are discussed and studied.

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1. Introduction

Fuzzy set is a generalization of a Classical set and the membership function is a generalization of the Characteristic function. The notion of fuzzy set was defined by L.A. Zodeh in [1]. In Classical set theory a subset A of a set X can be defined by its Characteristic function $\chi_A : X \to \{0, 1\}$ is defined by $\chi_A(x) = 0$, if $x \notin A$ and $\chi_A(x) = 1$ if $x \in A$ Garrett Birchof [2] introduced the concept of lattice theory. Further J.A Goguen [3] replaced the valuations set $[0, 1]$ by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. Chelvam and Ganesan [4] has introduced the bi-ideal in near ring, later Kuroki [8] introduced the notion of fuzzy bi-ideals in semi groups and Liu[9] studied them in rings. A detail work about bi-ideal and fuzzy bi-ideals in a ring can be found by S.K. Datta [5]. In this paper, we introduce the some theorems in L-fuzzy bi-ideal of a ring.

1.1. Preliminaries

In this section we recall some of the fundamental definitions, which are necessary for this paper.

Definition 1.1. *Let X be a non-empty set. A mapping $\mu : X \to [0, 1]$ is called a fuzzy subset of X.*

Definition 1.2. *Let R be a ring and μ be a fuzzy subset of R. μ is called a fuzzy ideal of R if*

(1). $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$.

(2). $\mu(xy) \geq \max\{\mu(x), \mu(y)\}$.

Definition 1.3. *Let R be a ring. A fuzzy set μ of R is said to be fuzzy subring of R if,*

(1). $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$.

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(2). $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$.

Definition 1.4. *A non empty fuzzy subset μ of a ring R is called an fuzzy bi-ideal of R if*

(1). $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$.

(2). $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$.

(3). $\mu(xyz) \geq \min\{\mu(x), \mu(z)\} \;\; \forall \;\; x, y, z \in R$.

Definition 1.5. *Let R be a ring. A fuzzy set of R is said to be L-fuzzy subring of R if for R the following conditions are satisfied:*

(1). $\mu(x - y) \geq \mu(x) \wedge \mu(y)$.

(2). $\mu(xy) \geq \mu(x) \wedge \mu(y)$.

2. L-Fuzzy Bi-ideal

Definition 2.1. *Let R be a ring and L be a lattice . A fuzzy set of R is said to be L-fuzzy bi-ideal of R if*

(1). $\mu(x - y) \geq \mu(x) \wedge \mu(y)$.

(2). $\mu(xy) \geq \mu(x) \wedge \mu(y)$.

(3). $\mu(xyz) \geq \mu(x) \wedge \mu(z) \;\; \forall \;\; x, y, z \in R$.

Example 2.2. *Consider the fuzzy set μ of R by* $\mu(x) = \begin{cases} 0.7 & \text{if } x \text{ is rational} \\ 0.3 & \text{if } x \text{ is rrational} \end{cases}$. *Then μ is a L-fuzzy bi-ideal of R.*

Definition 2.3. *A fuzzy set μ of a ring R said to be*

(a). L-fuzzy left bi-ideal of R if

- *(1).* $\mu(x - y) \geq \mu(x) \wedge \mu(y)$.
- *(2).* $\mu(xy) \geq \mu(y)$.
- *(3).* $\mu(xry) \geq \mu((xr)y) \geq \mu(y)$.

(b). L-fuzzy right bi-ideal of R if

- *(1).* $\mu(x - y) \geq \mu(x) \wedge \mu(y)$.
- *(2).* $\mu(xy) \geq \mu(x)$.
- *(3).* $\mu(xry) \geq \mu(x(ry)) \geq \mu(x)$.

Theorem 2.4. *If A be L-fuzzy subring of ring R, then,*

(1). $\mu_A(0) \geq \mu_A(x)$.

(2). $\mu_A(-x) = \mu_A(x) \;\; \forall \;\; x, y \in R$.

(3). If R is ring with unity then $\mu_A(x) \leq \mu_A(1)$.

Proof. For x in R and 0 is the identity element of R. Now, (1). $\mu_A(0) = \mu_A(x - x) \geq \mu_A(x) \wedge \mu_A(x)$

$= \mu_A(x)$

$\mu_A(0) \geq \mu_A(x)$

(2). $\mu_A(-x) = \mu_A(0 - x) \geq \mu_A(0) \wedge \mu_A(x)$

$\geq \mu_A(x)$

$\mu_A(-x) = \mu_A(x)$

For x≠0 in R and 1 is the identity element of R.

(3). $\mu_A(1) = \mu_A(xx^{-1}) \geq \mu_A(x) \wedge \mu_A(x^{-1})$

$= \mu_A(x)$

$\mu_A(1) \geq \mu_A(x)$

□

Theorem 2.5. *If A and B be Two L-fuzzy bi-ideal's of a ring R then $A \cap B$ is L-fuzzy bi-ideal of ring R.*

Proof. Let A and B be two L-fuzzy bi-ideal of a ring R. Let $x, y \in A \cap B$ be any element. Then

(1). $\mu_{A\cap B}(x - y) \geq \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$.

(2). $\mu_{A\cap B}(xy) \geq \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$.

(3). $\mu_{A\cap B}(xyz) \geq \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(z)$ for $z \in R$.

Let $x, y \in A \cap B$ be any element Then,

(1). $\mu_{A\cap B(x-y)} \geq \mu_A(x - y) \wedge \mu_B(x - y)$

$\geq \{\mu_A(x) \wedge \mu_A(y)\} \wedge \{\mu_B(x) \wedge \mu_B(y)\}$

$= \{\mu_A(x) \wedge \mu_B(x))\} \wedge \{\mu_A(y) \wedge \mu_B(y)\}$

$= \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$

Thus, $\mu_{A\cap B}(x - y) \geq \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$.

(2). $\mu_{A\cap B}(xy) \geq \mu_A(xy) \wedge \mu_B(xy)$

$\geq \{\mu_A(x) \wedge \mu_A(y)\} \wedge \{\mu_B(x) \wedge \mu_B(y)\}$

$= \{\mu_A(x) \wedge \mu_B(x)\} \wedge \{\mu_A(y) \wedge \mu_B(y)\}$

$= \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$

Thus, $\mu_{A\cap B}\ (xy) \geq \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$.

(3). $\mu_{A\cap B}\ (xyz) \geq \mu_A(xyz) \wedge \mu_B(xyz)$

$\geq \{\mu_A(x) \wedge \mu_A(z)\} \wedge \{\mu_B(x) \wedge \mu_B(z)\}$

$= \{\mu_A(x) \wedge \mu_B(x)\} \wedge \{\mu_A(z) \wedge \mu_B(z)\}$

$= \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(z)$

Thus, $\mu_{A\cap B}\ (xyz) \geq \mu_{A\cap B}\ (x) \wedge \mu_{A\cap B}\ (z)$.

Hence $A \cap B$ is a L-fuzzy bi-ideal of a ring R. □

Definition 2.6. *Let A be a fuzzy set of a ring R. Then α-cut of A is a crisp subset $C_\alpha(A) = \{x \in R : \mu_A(x) \geq \alpha\}$, where $\alpha \in [0, 1]$.*

Theorem 2.7. *Let A be a fuzzy set of a ring R. If A is a L-fuzzy subring of R then $C_\alpha(A)$ is a L-fuzzy subring of R for all $\alpha \in [0, 1]$.*

Proof. Let A be a fuzzy set of a ring R. Suppose A is a L-fuzzy subring of R.

To Prove: $C_\alpha(A)$ is a L-fuzzy subring of R. Let $x, y \in C_\alpha(A)$. By definition $\mu_A(x) \geq \alpha$ and $\mu_A(y) \geq \alpha$. Now,

$$\begin{aligned}
\mu_A(x-y) &\geq \mu_A(x) \wedge \mu_A(y) \\
&\geq \alpha \wedge \alpha \\
&\geq \alpha \\
\mu_A(xy) &\geq \mu_A(x) \wedge \mu_A(y) \\
&\geq \alpha \wedge \alpha \\
&\geq \alpha
\end{aligned}$$

Thus $x - y, xy \in C_\alpha(A)$. $C_\alpha(A)$ is a L-fuzzy subring of R. □

Theorem 2.8. *If A is a L-fuzzy subring of R, then $\mu_A(x-y) = \mu_A(0)$ gives $\mu_A(x) = \mu_A(y)$ for x,y in R the identity 0 in R.*

Proof. Let x and y in R the identity 0 in R. Now,

$$\begin{aligned}
\mu_A(x) &= \mu(x - y + y) \\
&\geq \mu_A[x - y - (-y)] \\
&\geq \mu_A(x-y) \wedge \mu_A(-y) \\
&= \mu_A(x-y) \wedge \mu_A(y) \\
&= \mu_A(0) \wedge \mu_A(y) \\
&= \mu_A(y) \\
\mu_A(y) &= \mu_A[x - (x - y)] \\
&\geq \mu_A(x) \wedge \mu_A(x-y) \\
&= \mu_A(x) \wedge \mu_A(0) \\
&= \mu_A(x)
\end{aligned}$$

□

Theorem 2.9. *If A is an L-fuzzy subring of a ring R. then $H = \{x/x \in R : \mu_A(x) = 1\}$ is either empty or is a subring of R.*

Proof. If no element satisfies this condition then H is empty. If $x, y \in H$ then,

$$\begin{aligned}
\mu_A(x-y) &\geq \mu_A(x) \wedge \mu_A(y) \\
&= 1 \wedge 1 \\
&= 1
\end{aligned}$$

$$\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$$
$$= 1 \wedge 1$$
$$= 1$$

Thus $x - y, xy \in H$. H is a subring of R. Hence H is either empty or a subring of R. □

Theorem 2.10. *Let A be a fuzzy set of a ring R. If A is a L-fuzzy bi-ideal of R then $C_\alpha(A)$ is a L-fuzzy bi-ideal of R for all* $[0, 1]$.

Proof. Let A be a fuzzy set of a ring R. Suppose A is a L-fuzzy bi-ideal of R. Let $x, y, z \in C_\alpha(A)$. By definition $\mu_A(x) \geq \alpha$, $\mu_A(y) \geq \alpha$, $\mu_A(z) \geq \alpha$. Now,

$$\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y)$$
$$\geq \alpha \wedge \alpha$$
$$\geq \alpha$$
$$\mu_A(x - y) \geq \alpha$$
$$x - y \in C_\alpha(A)$$
$$\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$$
$$\geq \alpha \wedge \alpha$$
$$\mu_A(xy) \geq \alpha$$

Therefore, $xy \in C_\alpha(A)$.

$$\mu_A(xyz) \geq \mu_A(x) \wedge \mu_A(z)$$
$$\geq \alpha \wedge \alpha$$
$$\geq \alpha$$
$$\mu_A(xyz) \geq \alpha$$
$$xyz \in C_\alpha(A)$$

$C_\alpha(A)$ is a L-fuzzy bi-ideal of R. □

Theorem 2.11. *Let A be L-fuzzy left bi-ideal and B be L-fuzzy right bi-ideal of a ring R, then $A \cap B$ is L-fuzzy bi-ideal of ring R.*

Proof. Let $x, y \in A \cap B$ be any element. Then

$$\mu_{A\cap B}(x - y) \geq \mu_A(x - y) \wedge \mu_B(x - y)$$
$$\geq \{\mu_A(x) \wedge \mu_A(y)\} \wedge \{\mu_B(x) \wedge \mu_B(y)\}$$
$$= \{\mu_A(x) \wedge \mu_B(x)\} \wedge \{\mu_A(y) \wedge \mu_B(y)\}$$
$$= \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)\mu_{A\cap B}(x - y) \qquad \geq \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$$

Also,

$$\mu_{A\cap B}(xy) \geq \mu_A(xy) \wedge \mu_B(xy) \tag{1}$$

Since A is L-fuzzy left bi-ideal and B is L-fuzzy right bi-ideal of the ring R. Therefore, we have $\mu_A(xy) \geq \mu_A(y)$ and $\mu_B(xy) \geq \mu_B(x)$.

$$\mu_A(xy) \wedge \mu_B(xy) \geq \mu_A(y) \wedge \mu_B(x) \tag{2}$$

As $A \cap B \subseteq A$ and $A \cap B \subseteq B$. So $\mu_{A\cap B}(y) \leq \mu_A(y)$ and $\mu_{A\cap B}(x) \leq \mu_B(x)$.

$$\mu_A(y) \wedge \mu_B(x) \geq \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y) \tag{3}$$

From (1), (2) and (3) we have

$$\begin{aligned}\mu_{A\cap B}(xy) &\geq \mu_A(xy) \wedge \mu_B(xy)\\ &\geq \mu_A(y) \wedge \mu_B(x)\\ &\geq \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)\end{aligned}$$

Further let $x, y \in A \cap B$ and $r \in R$ then

$$\mu_{A\cap B}(xry) \geq \mu_A(xry) \wedge \mu_B(xry) \tag{4}$$

But

$$\mu_A(xry) \geq \mu_A((xr)y) \geq \mu_A(y)$$
$$\mu_B(xry) \geq \mu_B(x(ry)) \geq \mu_B(x)$$
$$\mu_A(xry) \wedge \mu_B(xry) \geq \mu_A(y) \wedge \mu_B(x) \tag{5}$$

As $A \cap B \subseteq A$ and $A \cap B \subseteq B$. So $\mu_{A\cap B}(y) \leq \mu_A(y)$ and $\mu_{A\cap B}(x) \leq \mu_B(x)$.

$$\mu_A(y) \wedge \mu_B(x) \geq \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y) \tag{6}$$

From (4), (5) and (6) we have

$$\mu_{A\cap B}(xry) \geq \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y)$$

Hence $A \cap B$ is L-fuzzy bi-ideal of a ring R. □

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