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Application of Fuzzy Membership Matrix in Yoga on Insomnia

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Abstract: Fuzzy Set Theory and fuzzy logic are highly suitable and applicable for developing knowledge based system in medicine for the tasks of medical findings. There are variety of models involving fuzzy matrices to deal with different complicated aspects of medical diagnosis. In this paper, procedures are presented for medical diagnosis and for fuzzy decision models.

Keywords: Triangular fuzzy number, Triangular fuzzy number matrix, addition and subtraction, operation on triangular fuzzy number matrix, New membership function, Max-Min composition on fuzzy membership Value matrices, insomnia.

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1. Introduction

Uncertainty is a challenging part in Human's everyday life. The main cause of uncertainty is the information deficiency. Information may be incomplete, not fully reliable, vague, contradictory or deficient in some other way. These various information deficiencies may result in fuzziness or vagueness. Fuzzy set theory introduction by zadeh [8] in 1965 acts as a qualitative computational approach which describes uncertainty. Sanchez [4] formulated the diagnostic models involving fuzzy matrices representing the medical knowledge between the symptoms and diseases. Meenakshi and Kaliraja [3] have extended Sanchez's approach for medical diagnosis using the representation of interval valued fuzzy matrix. Insomnia can be characterized by having a tired feeling after walking up from sleep, frequently waking up during the night and at the same time having trouble when going back to sleep as well as waking up very early during the morning hours. Insomnia can be characterized into two types, primary and secondary. Primary insomnia occurs if an individual has problems sleeping that may not have a direct relationship with some health condition or problem. The secondary insomnia occurs when an individual has sleeping problems because of other reasons such as cancer, heartburn, pain, depression, the use of medication or even the use of some substances such as alcohol.

2. Preliminaries

Definition 2.1. *Triangular fuzzy number is denoted as* $X = (x_1, x_2, x_3)$, $x_1, x_2, x_3 \in R$, $x_1 < x_2 < x_3$.

Definition 2.2. *Triangular fuzzy number matrix of order m x n is defined as* $X = (x_{ij})_{m\times n}$, *where* $x_{ij} = (x_{ijL}, x_{ijM}, x_{ijU})$ *is the* ij^{th} *element of X.* x_{ijL}, x_{ijU} *are the left and right spreads of* x_{ij} *respectively and* x_{ijM} *is the mean value.*

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Definition 2.3. *Let $X = (x_{ij})_{n\times n}$ and $Y = (y_{ij})_{n\times n}$ be two triangular fuzzy number matrices of same order. Then*

(1). Addition Operation

$$X(+)Y = (x_{ij} + y_{ij})_{n\times n}$$

where $x_{ij} + y_{ij} = (x_{ijL} + y_{ijL}, x_{ijM} + y_{ijM}, x_{ijU} + y_{ijU})$ is the ij^{th} element of $X(+)Y$.

(2). Subtraction Operation

$$X(-)Y = (x_{ij} - y_{ij})_{n\times n}$$

where $x_{ij} - y_{ij} = (x_{ijL} - y_{ijL}, x_{ijM} - y_{ijM}, x_{ijU} - y_{ijU})$ is the ij^{th} element of $X(-)Y$.

The same condition holds for triangular fuzzy membership number.

Definition 2.4. *Let $X = (x_{ij})_{m\times p}$ and $Y = (y_{ij})_{p\times n}$ be two triangular fuzzy number matrices, then the multiplication Operation*

$$X(.)B = (Z_{ij})_{m\times n}$$

where, $(Z_{ij}) = \sum_{1,-1}^{P} x_{ik}y_{kj}$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Definition 2.5. *Let F_{mn} denote the set of all $m \times n$ matrices Over F. Elements of F_{mn} are called as fuzzy membership value matrices. For $X = (x_{ij}) \in F_{mp}$ and $Y = (y_{ij}) \in F_{pn}$ the max-min product.*

$$X(.)Y = (Sup_k[\{\inf\{x_{ik}, y_{kj}\}\}]) \in F_{mn}$$

Definition 2.6. *Let $X = (x_{ij})_{n\times n}$ where $X_{ij} = (x_{ijL}, x_{ijM}, x_{ijU})$ and $Y = (y_{ij})_{n\times n}$ where $y_{ij} = (y_{ijL}, y_{ijM}, y_{ijU})$ be two triangular fuzzy number matrices of same order. Then the maximum operation on it is given by $L_{max} = Max(X, Y) = Sup\{x_{ij}, y_{ij}\}$, where $Sup\{x_{ij}, y_{ij}\} = (Sup(x_{ijL}, y_{ijL}), Sup(x_{ijM}, y_{ijM}), Sup(x_{ijU}, y_{ijU}))$ is the ij^{th} element of $max(A, B)$.*

Definition 2.7. *Let $X = (x_1, x_2, x_3)$ be a triangular fuzzy number then $AM(X) = \frac{x_1+x_2+x_3}{3}$. The some condition holds for triangular fuzzy membership number.*

Definition 2.8. *Insomnia, also known as sleepness, is a sleep disorder where people have trouble sleeping. They may have difficulty falling asleep, or staying asleep as long as desired. Insomnia is typically followed by daytime sleepness, low energy, irritabillity and a depressed mood.*

3. Algorithm

Step (1): Construct a triangular fuzzy number Matrix (F, D) over S, where F is a mapping given by $F : D \to \tilde{F}(s)$. $\tilde{F}(s)$ is a set of all triangular fuzzy sets of S. This matrix is denoted by R_0 which is fuzzy occurance matrix of Symptoms-disease triangular fuzzy number matrix.

Step (2): Construct another triangular fuzzy number matrix (F, S) over P, where F_1 is a mapping given by $F_1 \to F(P)_{\frac{n!}{r!(n-r)!}}$. This matrix is denoted by R_s which is the Patient – Symptom triangular fuzzy number matrix.

Step (3): Convert the element of triangular fuzzy number matrix into its membership function as follows : Membership function of $(a_{ij}) = (a_{ijL}, a_{ijM}, a_{ijU})$ is defined as

$$\mu_{aij} = \left(\frac{a_{ijL}}{10} \leq \frac{a_{ijM}}{10} \leq \frac{a_{ijU}}{10}\right), \quad if \;\; 0 \leq a_{ijL} \leq a_{ijM} \leq a_{ijU} \in 10,$$

where $0 \leq \frac{a_{ijL}}{10} \leq \frac{a_{ijM}}{10} \leq \frac{a_{ijU}}{10} \leq 1$. The matrix R_0 and R_s are converted into triangular fuzzy membership matrices namely $(R_0)_{mem}$ and $(R_s)_{mem}$.

Step (4): Compute the following relations,

(1). $R_1 = (R_s)_{mem}(.)(R_0)_{mem}$

(2). $R_2 = (R_s)_{mem}(.)(J(-)(R_0)_{mem})$, where J is the triangular fuzzy membership matrix in which all entries as (1,1,1). $(J(-1)(R_0)_{mem})$ is the complement of $(R_0)_{mem}$ and it is called as non symptom disease triangular fuzzy membership matrix.

(3). $R_3 = (J(-)(R_s)_{mem})(.)(R_0)_{mem}$, where $J - (R_s)_{mem}$ is the complement of R_s and it is called us non patient-symptom triangular fuzzy membership matrix.

(4). R_2 and R_3 calculated using subtraction operation and Definition 2.3.

(5). $R_4 = Max(R_2, R_3)$ it is calculate using Definition 2.6. The element of R_1, R_2, R_3, R_4 is of the form $Y_{ij} = (Y_{ijL}, Y_{ijM}, Y_{ijU})$, where $0 \leq y_{ijL} \leq y_{ijM} \leq y_{ijU} \leq 1$.

(6). $R_5 = R_1(-1)R_4$. By using subtraction operation.

Step (5): Calculate $R_6 = AM(R_{ij})$ and Row = Maximum of i^{th} row which helps the decision maker to strongly confirm the disease for the patient.

4. Case Study

Insomnia sleep is a natural part of life but many people know very little about how important it is and some even try to get by with very little of it. Poor sleep quality can be a cause of insomnia primary and secondary types. A national survery found that over 55% of people who did yoga found that it helped them get better sleep. Yoga will benefit your sleep in many ways. Make sure that you take some kind of physical activity during a day, so the body is naturally ready we consider the set $S = \{S_1, S_2, S_3, S_4\}$ as universal set where S_1, S_2, S_3 and S_4 represent the symptoms respectively and the set $D = \{d_1, d_2\}$ where d_1 and d_2 represent the primary and secondary type of insomnia respectively.

$$F(d_1) = [\langle e_1, (3, 4, 6.00)\rangle; \langle e_2, (4, 6, 6.30)\rangle; \langle e_3, (5.30, 5.45, 6.30)\rangle; \langle e_4, (4, 5.45, 6.00)\rangle; \langle e_5, (5, 5.00, 6.32)\rangle]$$
$$F(d_2) = [\langle e_1, (2, 3.30, 4.20)\rangle; \langle e_2, (4.30, 5.30, 6.00)\rangle; \langle e_3, (2, 3, 5.15)\rangle; \langle e_4, (2.20, 4.25, 5.30)\rangle; \langle e_5, (4.15, 4.45, 5.20)\rangle]$$

The triangular fuzzy number matrix (F, D) is a Parameterized family $(F(d_1))$, $(F(d_2))$ of all triangular fuzzy number matrix over the set S and are determined from expert medical documentation. Thus the triangular fuzzy number matrix (F, D) represents a relation matrix R_0 and it gives an approximate description of the triangular fuzzy number matrix medical knowledge of the two diseases and their symptoms given by,

$$R_0 = \begin{pmatrix} (3, 4, 6.00) & (2, 3.30, 4.20) \\ (4, 6, 6.30) & (4.30, 5.30, 6.00) \\ (5.30, 5.45, 6.30) & (2, 3, 5.15) \\ (4, 5.45, 6.15) & (2.50, 4.25, 5.30) \\ (5, 5.30, 6.35) & (4.15, 4.45, 5.20) \end{pmatrix}$$

Step (2): Again we take $P = \{P_1, P_2, P_3, P_4, P_5\}$ as the universal set where P_1, P_2, P_3, P_4, P_5 represent patients respectively and $S = \{S_1, S_2, S_3, S_4, S_5\}$ as the set of Parameters. Suppose that,

$$F(S_1) = [\langle P_1, (4, 5.15, 6.20)\rangle; \langle P_2, (3, 3.20, 4.15)\rangle; \langle P_3(4.10, 5.40, 7.15)\rangle; \langle P_4(5.14, 6.13, 6.45)\rangle; \langle P_5, (3.40, 5.10, 6.50)\rangle;]$$

$$F(S_2) = [\langle P_1, (2.15, 3.45, 6.15)\rangle; \langle P_2, (4.10, 5.15, 6.14)\rangle; \langle P_3(6.10, 6.45, 7.0)\rangle; \langle P_4(6.12, 6.45, 7.12)\rangle; \langle P_5, (4.20, 5.55, 6.15)\rangle;]$$

$$F(S_3) = [\langle P_1, (4.10, 5.20, 6.15)\rangle; \langle P_2, (4.15, 5.12, 7.45)\rangle; \langle P_3(6, 6.45, 7.45)\rangle; \langle P_4(6.10, 7, 7.45)\rangle; \langle P_5, (5.30, 6.20, 7.0)\rangle;]$$

$$F(S_4) = [\langle P_1, (5.10, 6.20, 6.40)\rangle; \langle P_2, (5.10, 6.20, 6.15)\rangle; \langle P_3(5.25, 6.20, 7.00)\rangle; \langle P_4(4.20, 4.50, 6.20)\rangle; \langle P_5, (5.0, 5.45, 6.30)\rangle;]$$

$$F(S_5) = [\langle P_1, (5.45, 6.12, 6.45)\rangle; \langle P_2, (1.45, 4.35, 5.20)\rangle; \langle P_3(6.30, 7.15, 7.45)\rangle; \langle P_4(6.30, 7.10, 8.0)\rangle; \langle P_5, (6.20, 7.15, 8.00)\rangle;]$$

The triangular fuzzy matrix (F_1, S_1) is another parameterized family of triangular fuzzy number matrix and gives a collection of approximate description of the patient symptoms in the hospital. Thus the triangular fuzzy number matrix (F_1, S) represents a relation matrix R_s called Patient-Symptom matrix given by,

$$\begin{array}{c|ccccc} & S_1 & S_2 & S_3 & S_4 & S_5 \\ \hline P_1 & (4, 5.15, 6.20) & (2.15, 3.45, 6.15) & (4.10, 5.20, 6.15) & (5.10, 6.20, 6.40) & (3.45, 6.12, 6.45) \\ P_2 & (3, 3.20, 4.15) & (4.10, 5.15, 6.14) & (4.15, 5.12, 7.45) & (5.10, 6.20, 6.0) & (1.45, 4.35, 5.20) \\ P_3 & (4.10, 5.40, 7.15) & (6.10, 6.45, 7.0) & (6, 6.45, 7.45) & (5.25, 6.20, 7.00) & (6.3, 7.15, 7.45) \\ P_4 & (5.14, 6.13, 6.45) & (6.12, 6.45, 7.12) & (6.10, 7, 7.45) & (4.20, 4.50, 6.20) & (6.30, 7.10, 8.0) \\ P_5 & (3.40, 5.10, 6.50) & (4.20, 5.55, 6.15) & (5.30, 6.20, 7.0) & (5.0, 5.45, 6.30) & (6.20, 7.15, 8.00) \end{array}$$

Step (3):

$$(R_0)_{mem} = \begin{array}{c|cc} & d_1 & d_2 \\ \hline S_1 & (0.3, 0.4, 0.6) & (0.2, 0.33, 0.42) \\ S_2 & (0.4, 0.6, 0.63) & (0.43, 0.53, 0.60) \\ S_3 & (0.53, 0.54, 0.63) & (0.2, 0.3, 0.51) \\ S_4 & (0.4, 0.54, 0.61) & (0.25, 0.42, 0.53) \\ S_5 & (0.5, 0.53, 0.63) & (0.41, 0.44, 0.52) \end{array}$$

$$(R_s)_{mem} = \begin{array}{c|ccccc} & S_1 & S_2 & S_3 & S_4 & S_5 \\ \hline P_1 & (0.40, 0.51, 0.62) & (0.21, 0.34, 0.61) & (0.41, 0.52, 0.61) & (0.51, 0.62, 0.64) & (0.54, 0.61, 0.64) \\ P_2 & (0.3, 0.32, 0.41) & (0.41, 0.51, 0.61) & (0.41, 0.51, 0.74) & (0.51, 0.62, 0.61) & (0.14, 0.43, 0.52) \\ P_3 & (0.41, 0.54, 0.7) & (0.61, 0.64, 0.70) & (0.60, 0.64, 0.74) & (0.52, 0.62, 0.70) & (0.63, 0.71, 0.74) \\ P_4 & (0.51, 0.61, 0.64) & (0.61, 0.64, 0.71) & (0.61, 0.7, 0.74) & (0.42, 0.45, 0.62) & (0.63, 0.71, 0.80) \\ P_5 & (0.34, 0.51, 0.65) & (0.42, 0.55, 0.6) & (0.53, 0.62, 0.7) & (0.50, 0.54, 0.63) & (0.62, 0.71, 0.80) \end{array}$$

Step (4): Computing the following relation matrices.

$$R_1 = (R_s)_{mem}(.)(R_0)_{mem} = \begin{array}{c|cc} & d_1 & d_2 \\ \hline P_1 & (0.51, 0.54, 0.63) & (0.41, 0.44, 0.60) \\ P_2 & (0.41, 0.54, 0.63) & (0.41, 0.51, 0.61) \\ P_3 & (0.53, 0.6, 0.63) & (0.43, 0.53, 0.60) \\ P_4 & (0.53, 0.54, 0.63) & (0.43, 0.53, 0.60) \\ P_5 & (0.53, 0.55, 0.63) & (0.42, 0.43, 0.60) \end{array}$$

$$R_2 = (R_s)_{mem}(.)(J-(R_0)_{mem}) = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} (0.4, 0.51, 0.62) & (0.48, 0.58, 0.64) \\ (0.39, 0.46, 0.6) & (0.47, 0.58, 0.74) \\ (0.39, 0.54, 0.7) & (0.51, 0.7, 0.74) \\ (0.4, 0.6, 0.64) & (0.51, 0.7, 0.74) \\ (0.39, 0.51, 0.65) & (0.49, 0.54, 0.7) \end{pmatrix} \end{array}$$

$$R_3 = (J-(R_s)_{mem})(.)(R_0)_{mem} = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} (0.39, 0.6, 0.63) & (0.39, 0.53, 0.60) \\ (0.48, 0.60, 0.63) & (0.41, 0.44, 0.59) \\ (0.26, 0.46, 0.6) & (0.26, 0.46, 0.48) \\ (0.3, 0.42, 0.51) & (0.29, 0.42, 0.53) \\ (0.39, 0.54, 0.6) & (0.39, 0.45, 0.58) \end{pmatrix} \end{array}$$

$$R_4 = Max(R_2, R_3) = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} (0.4, 0.6, 0.63) & (0.48, 0.58, 0.64) \\ (0.39, 0.60, 0.63) & (0.47, 0.58, 0.74) \\ (0.39, 0.54, 0.7) & (0.51, 0.7, 0.74) \\ (0.4, 0.6, 0.64) & (0.51, 0.7, 0.74) \\ (0.39, 0.54, 0.65) & (0.49, 0.54, 0.7) \end{pmatrix} \end{array}$$

$$R_5 = (R_1 - R_4) = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} (-0.12, -0.06, 0.23) & (-0.23, -0.14, 0.12) \\ (-0.22, -0.06, 0.22) & (-0.33, -0.07, 0.11) \\ (-0.17, 0.06, 0.24) & (-0.31, -0.17, 0.09) \\ (-0.14, -0.06, 0.23) & (-0.08, -0.17, 0.09) \\ (-0.12, -0.01, 0.12) & (-0.11, 0.11, 0.28) \end{pmatrix} \end{array}$$

Step (5): Row = Maximum of i^{th} row

$$R_5 = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} 0.016 & -0.25 \\ -0.06 & -0.28 \\ 0.13 & -0.39 \\ 0.03 & -0.08 \\ -0.01 & -0.28 \end{pmatrix} \end{array} \begin{array}{c} \\ 0.016 \\ -0.06 \\ 0.13 \\ 0.03 \\ 0.28 \end{array}$$

This can be represented in the form of a graph namely network as follows.

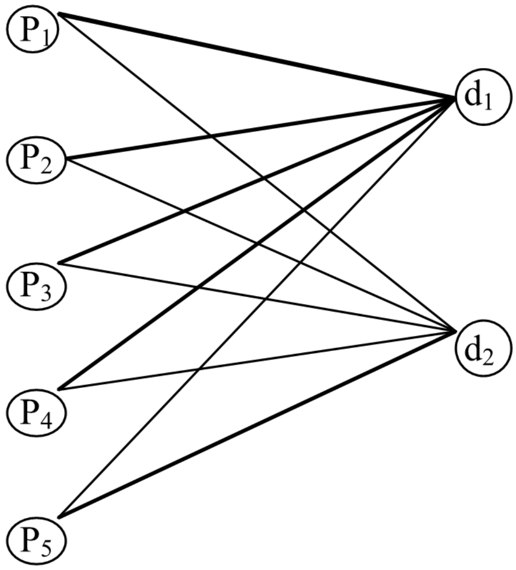

Figure 1. **Fuzzy medical Diagnosis Network**

In the above network, nodes or Vertices denote the patients and diseases, lengths or edges denote the assumption of diseases to the patients. The darken edges denotes the strong confirmation of diseases to the patient.

5. Conclusion

Fuzzy Set framework has been utilized in several different approaches to model the medical diagnostic process and decision making process. From the above analysis it is obvious that, the patient P_1, P_2, P_3 and P_4 suffer from primary insomnia, patient P_5 suffer from secondary insomnia. A good sleep is within reach, and the oftern – recommended treatment is to establish a relaxing bedtime routine. The following yoga stretches can help you relax and enjoy a peaceful and great sleep.

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