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A Study on Magneto-Thermodiffusive Wave Propagation in Dual-Phase-Lag Thermoelasticity with Temperature-Dependent Properties

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Abstract: The present paper is concerned with the analysis of magneto-thermodiffusive wave propagation in a homogeneous, isotropic, thermally and perfectly conducting elastic medium with temperature dependent mechanical properties. The formulation is established under the purview of dual-phase-lag thermoelasticity theory with diffusion. The modulus of elasticity is taken as a linear function of reference temperature. The resulting non-dimensional coupled equations are applied to a specific problem of a half-space whose surface is subjected to a time dependent mechanical shock. The analytical expressions for the displacement components, stresses, concentration and temperature field in the physical domain are obtained by employing normal mode analysis. Finally, numerical solution is carried out for copper material and corresponding graphs are plotted to illustrate and compare theoretical results. Discussions have been made to highlight the effects of phase lag parameters, temperature dependent modulus of elasticity, frequency and time on the physical fields. Some particular cases of interest have been deduced from the present investigation. The phenomenon of a finite speed of propagation is observed graphically for each field.

MSC: 74A15, 80A20.

Keywords: Dual-phase-lag; Diffusion; Magneto-thermoelasticity; Temperature-dependent elastic modulus; Normal mode analysis.

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Nomenclature

σ_{ij}	: Components of stress tensor
λ, μ	: Lame's constants
β_1	: $(3\lambda + 2\mu)\,\alpha_t$
β_2	: $(3\lambda + 2\mu)\,\alpha_c$
α_t	: Coefficient of linear thermal expansion
α_c	: Coefficient of linear diffusion expansion
u_i	: Components of the displacement vector
ρ	: Density of the medium
e_{ij}	: Components of the strain tensor
e	: Cubical dilatation

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K	: Thermal conductivity
c_E	: Specific heat at constant strain
a	: Measure of thermodiffusion effect
b	: Measure of diffusive effect
D	: Thermodiffusion constant
τ_q, τ_T	: Thermal relaxation times of dual-phase-lag theory
$\hat{\tau}_q, \hat{\tau}_T$	: Diffusion relaxation times of dual-phase-lag theory
θ	: $T - T_0$
T	: Absolute temperature
T_0	: Temperature of the medium in its natural state assumed to be $\left\lvert \frac{\theta}{T_0} \right\rvert \ll 1$
δ_{ij}	: Kronecker delta
p	: $P - P_0$
P	: Chemical potential per unit mass at non equilibrium conditions
P_0	: Chemical potential per unit mass of natural state
c	: $C - C_0$
C	: Non-equilibrium concentration
C_0	: Mass concentration at natural state
μ_e	: Magnetic permeability
ε_0	: Electric permittivity
$\vec{H}$	: Applied magnetic field
$\vec{E}$	: Induced electric field
$\vec{h}$	: Induced magnetic field
$\vec{J}$	: Current density vector.

1. Introduction

Two well-known generalized thermoelastic models that drew attention of researchers are [11] model and [6] model. In Lord-Shulman (L-S) model, a thermal relaxation time parameter is introduced in the Fouriers law of heat conduction and this model is also known as single-phase-lag model, whereas in the model of Green-Lindsay (G-L), two different relaxation times are introduced in the constitutive relations and it is known as temperature rate dependent thermoelasticity. In the next generalization to thermoelasticity, [7–9, 9] provided sufficient basic modifications in the constitutive equations that permit treatment of a much wider class of heat flow problems. These models are labeled as GN-I, GN-II and GN-III.

Another important generalization is known as dual-phase-lag thermoelasticity, which is developed by [26] and [2]. Tzou [26] introduced two different phase lags, one for the heat flux vector and the other for the temperature gradient. According to this model, the classical Fourier's law $\vec{q} = -K\vec{\nabla}T$ has been replaced by $\vec{q}(P, t + \tau_q) = -K\vec{\nabla}T(P, t + \tau_T)$, where the temperature gradient $\vec{\nabla}T$ at a point P of the material at time $t + \tau_T$ corresponds to the heat flux vector $\vec{q}$ at the same point at time $t + \tau_q$. The delay time τ_T is supposed to be caused by the microstructural interactions and it is also called the phase lag of the temperature gradient. The other delay time τ_q is interpreted as the relaxation time due to the fast transient effects of the thermal inertia and is called the phase lag of heat flux. Quintanilla [21] discussed the stability of dual-phase-lag heat conduction. Kalkal [10] investigated three-dimensional thermoelastic problem with temperature-dependent modulus of elasticity in the dual-phase-lag model by employing normal mode analysis. Othman [18] established a three dimensional model of generalized thermoelasticity in a homogeneous, isotropic elastic half-space under the effect of gravity field in the context of dual-phase-lag model. (Deswal et al., 2018) analyzed the thermodynamical interactions in a two-temperature micropolar thermoelasticity with gravity using normal mode analysis. The formulation is applied to the dual-phase-lag thermoelasticity theory.

The theory of magneto-thermoelasticity is concerned with the effect of magnetic field on the elastic and thermoelastic deformations of solid body and has received the attention of researchers. Paria [20] discussed the theoretical outline of the development of magneto-thermoelasticity and also studied the propagation of plane magneto-thermoelastic waves in an isotropic unbounded medium under the influence of a magnetic field acting transversely to the direction of propagation. Sherief [23] examined a two-dimensional half-space problem subjected to a non-uniform thermal shock in the context of electromagneto-thermoelasticity theory. Sinha [25] investigated the effects of rotation and relaxation time on wave propagation using the methodology of Laplace transformation and eigenvalue approach. Mukhopadhyay [12] applied the technique of Laplace transform to analyze the thermoelastic interactions in a homogeneous and isotropic medium with a cylindrical cavity in the purview of different theories of thermoelasticity. Effects of rotation and magnetic field on two-dimensional thermoelastic interactions in the purview of three theories, namely, L-S, GN-II and coupled theory, has been studied by Othman [16]. Abbas, [1] solved a three-dimensional problem of generalized thermoelasticity using normal mode analysis and eigenvalue approach in the context of Green-Naghdi model-II with temperature dependent material properties. Said, [22] applied normal mode technique to analyze the influences of rotation and magnetic field on a thermoelastic medium in the context of three-phase-lag model. Othman [19] discussed the effect of rotation and gravitational field on a micropolar magneto-thermoelastic solid in the context of dual-phase-lag model by using normal mode analysis.

Thermodiffusion in an elastic solid is due to the coupling of the fields of temperature, mass diffusion and that of strain. Nowacki, [13–15, 15] developed the theory of thermoelastic diffusion by using a coupled thermoelastic model. Sherief [24] extended the theory of thermoelastic diffusion and derived the governing equations for the generalized thermoelastic diffusion problem in an elastic solid which allows the finite speeds of propagation for thermoelastic and diffusive waves. Under different kind of loads, Deswal [3] examined various problems employing the theory of generalized thermoelastic diffusion. Othman [17] investigated the disturbances in a homogeneous, isotropic reference temperature-dependent elastic medium with fractional order generalized thermoelastic diffusion by employing normal mode analysis. Deswal [4] discussed an axi-symmetric generalized thermoelastic diffusion problem with two-temperature and initial stress under fractional order heat conduction using Laplace and Hankel transforms.

The current manuscript is concerned with the investigation of disturbances in a homogeneous, isotropic, temperature-dependent elastic medium with dual-phase-lag theory under the effect of diffusion. Normal mode analysis is used to obtain the exact solutions for displacement components, stresses, temperature field and concentration. These expressions are calculated numerically for a copper-like material and depicted graphically to observe the effects of phase lags, temperature dependent parameter, frequency and time. The present study is motivated by the importance of thermoelastic diffusion process in the field of oil extraction. This model is not only of theoretical interest, but may have practical applications in various fields such as geomechanics, earthquake engineering, seismology, soil dynamics and other related topics.

2. Field Equations and Constitutive Relations

Consider the field equations and constitutive relations in the context of homogeneous, isotropic, thermally and perfectly conducting elastic medium with dual-phase-lags and diffusion as:

(i). the equation of motion

$$\rho \ddot{u}_i = \sigma_{ji,j} + F_i. \tag{1}$$

(ii). heat conduction equation with dual-phase-lags

$$K\left(1+\tau_T\frac{\partial}{\partial t}\right)\nabla^2\theta = \left(1+\tau_q\frac{\partial}{\partial t}+\frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2}\right)(\rho c_E\dot{\theta}+\beta_1 T_0\dot{e}+aT_0\dot{c}). \tag{2}$$

(iii). the constitutive equations

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}(\lambda e - \beta_1\theta - \beta_2 c), \tag{3}$$

$$p = -\beta_2 e + bc - a\theta, \tag{4}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{5}$$

(iv). equation of mass diffusion

$$D\beta_2\left(1+\hat{\tau}_T\frac{\partial}{\partial t}\right)e_{,ii} + Da\left(1+\hat{\tau}_T\frac{\partial}{\partial t}\right)\theta_{,ii} + \frac{\partial}{\partial t}\left(1+\hat{\tau}_q\frac{\partial}{\partial t}+\frac{1}{2}\hat{\tau}_q^2\frac{\partial^2}{\partial t^2}\right)c = Db\left(1+\hat{\tau}_T\frac{\partial}{\partial t}\right)c_{,ii}\,. \tag{6}$$

In the above equations, a comma denotes material derivative and the summation convention is used. Linearized Maxwell equations governing the electromagnetic field for a perfectly conducting medium are taken as

$$\text{curl}\,\vec{h} = \vec{J} + \epsilon_0\frac{\partial \vec{E}}{\partial t}, \tag{7}$$

$$\text{curl}\,\vec{E} = -\mu_e\frac{\partial \vec{h}}{\partial t}, \tag{8}$$

$$\vec{E} = -\mu_e\left(\frac{\partial \vec{u}}{\partial t}\times\vec{H}\right), \tag{9}$$

$$\text{div}\,\vec{h} = 0. \tag{10}$$

3. Problem Formulation

Consider a homogeneous, isotropic, thermally and perfectly conducting elastic medium with diffusion and temperature-dependent modulus of elasticity. We shall use the rectangular cartesian co-ordinate system (x, y, z), having the surface of the half-space as the plane $z = 0$, with z-axis pointing vertically inwards as shown in Fig. A. The orientation of the primary magnetic field $\vec{H} = (0, H_0, 0)$ is taken towards the positive direction of y-axis. Due to the application of this magnetic field, there arises in the medium an induced magnetic field $\vec{h}$ and an induced electric field $\vec{E}$. Further $\vec{h}$ and $\vec{E}$ are small in magnitude in accordance with the assumptions of the linear theory of thermoelasticity. We restrict our analysis to x-z plane. Thus all the quantities in the medium are independent of the variable y. So the displacement vector $\vec{u}$ will have the components

$$u = u_x = u(x,z,t), \quad v = v_y = 0, \quad w = w_z = w(x,z,t). \tag{11}$$

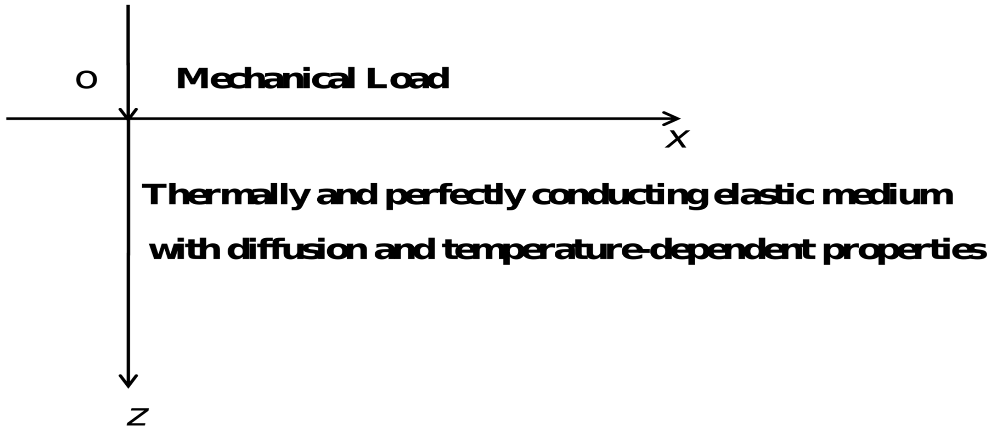

Fig. A. Geometry of the problem.

Also, the medium is supposed to be initially at rest and the undisturbed state is maintained at uniform temperature. The components of the initial magnetic field vector $\vec{H}$ are

$$H_x = 0, \qquad H_y = H_0, \qquad H_z = 0. \tag{12}$$

The electric intensity vector is normal to both the magnetic intensity and the displacement vector. Also, the electric intensity vector $\vec{E}$ is parallel to the current density vector $\vec{J}$, thus

$$E_x = E_1, \quad E_y = 0, \quad E_z = E_3, \quad J_x = J_1, \quad J_y = 0, \quad J_z = J_3. \tag{13}$$

From (7)-(10), one can obtain

$$E_1 = \mu_e H_0 \frac{\partial w}{\partial t}, \qquad E_2 = 0, \qquad E_3 = -\mu_e H_0 \frac{\partial u}{\partial t}, \tag{14}$$

$$h_1 = 0, \qquad h_2 = -H_0 e, \quad h_3 = 0, \tag{15}$$

$$J_1 = H_0 \frac{\partial e}{\partial z} - \epsilon_0 \mu_e H_0 \frac{\partial^2 w}{\partial t^2}, \quad J_2 = 0, \qquad J_3 = -H_0 \frac{\partial e}{\partial x} + \epsilon_0 \mu_e H_0 \frac{\partial^2 u}{\partial t^2}. \tag{16}$$

Lorentz's force $\vec{F}$ is given by the relation

$$\vec{F} = \mu_e(\vec{J} \times \vec{H}). \tag{17}$$

Inserting (12) and (16) in (17), we can obtain the components of the Lorentz's force $\vec{F}$ as

$$F_x = \mu_e H_0^2 \left(\frac{\partial e}{\partial x} - \epsilon_0 \mu_e \frac{\partial^2 u}{\partial t^2} \right), \qquad F_y = 0, \qquad F_z = \mu_e H_0^2 \left(\frac{\partial e}{\partial z} - \epsilon_0 \mu_e \frac{\partial^2 w}{\partial t^2} \right). \tag{18}$$

Our goal is to investigate the effect of the temperature-dependence of the modulus of elasticity keeping the other elastic and thermal parameters as constant. Therefore, we assume

$$E = E_0 f(\theta), \quad \lambda = E_0 \lambda_0 f(\theta), \quad \mu = E_0 \mu_0 f(\theta), \quad \beta_1 = E_0 \beta_{10} f(\theta) \quad \beta_2 = E_0 \beta_{20} f(\theta), \tag{19}$$

where $f(\theta) = 1 - \alpha^* T_0$ is a given non-dimensional function of temperature, $\lambda_0 = \frac{v}{(1+v)(1-2v)}$, $\mu_0 = \frac{1}{2(1+v)}$, $\beta_{10} = \frac{\alpha_t}{(1-2v)}$, $\beta_{20} = \frac{\alpha_c}{(1-2v)}$ and v is the Poisson's ratio. In case of a temperature-independent modulus of elasticity, $f(\theta) = 1$ and $E = E_0$. Taking into consideration (19), stress components are reduced to the forms

$$\sigma_{zz} = E_0 f(\theta)[(\lambda_0 + 2\mu_0)\frac{\partial w}{\partial z} + \lambda_0 \frac{\partial u}{\partial x} - \beta_{10}\theta - \beta_{20}c], \tag{20}$$

$$\sigma_{xx} = E_0 f(\theta)[(\lambda_0 + 2\mu_0)\frac{\partial u}{\partial x} + \lambda_0 \frac{\partial w}{\partial z} - \beta_{10}\theta - \beta_{20}c], \tag{21}$$

$$\sigma_{zx} = \mu_0 E_0 f(\theta)\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right). \tag{22}$$

Plugging the components of Lorentz force and stresses into the equation of motion along with the consideration of two-dimensional problem, the field equation (1) converts to

$$\rho \ddot{u} = E_0 f(\theta)\left[(\lambda_0 + \mu_0)\frac{\partial e}{\partial x} + \mu_0 \nabla^2 u - \beta_{10}\frac{\partial \theta}{\partial x} - \beta_{20}\frac{\partial c}{\partial x}\right] + \mu_e H_0^2 \left[\frac{\partial e}{\partial x} - \epsilon_0 \mu_e \frac{\partial^2 u}{\partial t^2}\right], \tag{23}$$

$$\rho \ddot{w} = E_0 f(\theta)\left[(\lambda_0 + \mu_0)\frac{\partial e}{\partial z} + \mu_0 \nabla^2 w - \beta_{10}\frac{\partial \theta}{\partial z} - \beta_{20}\frac{\partial c}{\partial z}\right] + \mu_e H_0^2 \left[\frac{\partial e}{\partial z} - \epsilon_0 \mu_e \frac{\partial^2 w}{\partial t^2}\right]. \tag{24}$$

Heat conduction equation (2) and diffusion equation (6), in view of expressions (19), take the form

$$K\left(1 + \tau_T \frac{\partial}{\partial t}\right)\nabla^2\theta = \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2 \frac{\partial^2}{\partial t^2}\right)(\rho c_E \dot{\theta} + E_0 \beta_{10} T_0 f(\theta)\dot{e} + a T_0 \dot{c}), \tag{25}$$

$$Db\left(1 + \hat{\tau}_T \frac{\partial}{\partial t}\right)\nabla^2 c = D\beta_{20} E_0 f(\theta)\left(1 + \hat{\tau}_T \frac{\partial}{\partial t}\right)\nabla^2 e + Da\left(1 + \hat{\tau}_T \frac{\partial}{\partial t}\right)\nabla^2\theta + \left(1 + \hat{\tau}_q \frac{\partial}{\partial t} + \frac{1}{2}\hat{\tau}_q^2 \frac{\partial^2}{\partial t^2}\right)\frac{\partial c}{\partial t}, \tag{26}$$

where $e = \operatorname{div} \vec{u} = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}$ and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator. Now, we will use the following non-dimensional variables to transform the above equations into non-dimensional forms:

$$(x', z', u', w') = c_0\eta_0(x, z, u, w), \quad (t', \tau_T', \tau_q', \hat{\tau}_T', \hat{\tau}_q') = c_0^2\eta_0(t, \tau_T, \tau_q, \hat{\tau}_T, \hat{\tau}_q), \quad \sigma_{ij}' = \frac{\sigma_{ij}}{\rho c_0^2}, \quad \theta' = \frac{\beta_{10}E_0}{\rho c_0^2}\theta, \quad c' = \frac{\beta_{20}E_0}{\rho c_0^2}c. \tag{27}$$

where $\eta_0 = \frac{\rho c_E}{K}$, $c_0^2 = \frac{(\lambda_0 + 2\mu_0)E_0}{\rho}$. Using Helmholtz decomposition, the displacement components can be written as

$$u = \frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial z}, \qquad w = \frac{\partial q}{\partial z} - \frac{\partial \psi}{\partial x} \quad \text{and} \quad \psi = (-\vec{U})_y \tag{28}$$

where $q(x, z, t)$ and $\psi(x, z, t)$ are scalar potential functions and $\vec{U}(x, z, t)$ is the vector potential function.

Introducing the above dimensionless parameters and potentials functions, equations (23)-(26) recast into the following forms (dropping the primes)

$$\left(\gamma_3 \nabla^2 - \delta_0 \frac{\partial^2}{\partial t^2}\right)\psi = 0, \tag{29}$$

$$\left(\gamma_1 \nabla^2 - \gamma_2 \frac{\partial^2}{\partial t^2}\right)q - \theta - c = 0, \tag{30}$$

$$\left[\left(1 + \tau_T \frac{\partial}{\partial t}\right)\nabla^2 - \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2 \frac{\partial^2}{\partial t^2}\right)\frac{\partial}{\partial t}\right]\theta - \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2 \frac{\partial^2}{\partial t^2}\right)\frac{\partial}{\partial t}(\gamma_4 \nabla^2 q - \gamma_5 c) = 0, \tag{31}$$

$$\xi_1\left(1 + \hat{\tau}_T \frac{\partial}{\partial t}\right)\nabla^4 q + \xi_2\left(1 + \hat{\tau}_T \frac{\partial}{\partial t}\right)\nabla^2\theta + \left[\xi_3\left(1 + \hat{\tau}_q \frac{\partial}{\partial t} + \frac{1}{2}\hat{\tau}_q^2 \frac{\partial^2}{\partial t^2}\right)\frac{\partial}{\partial t} - \left(1 + \hat{\tau}_T \frac{\partial}{\partial t}\right)\nabla^2\right]c = 0, \tag{32}$$

where

$$\gamma_1 = 1 + \frac{\alpha_0 \mu_e H_0^2}{\rho c_0^2}, \quad \gamma_2 = \alpha_0 \delta_0, \quad \gamma_3 = \frac{\mu_0 E_0}{\rho c_0^2 \alpha_0}, \quad \gamma_4 = \frac{E_0^2 \beta_{10}^2 T_0}{K \rho \eta_0 \alpha_0 c_0^2},$$

$$\gamma_5 = \frac{a T_0 \beta_{10}}{K \eta_0 \beta_{20}}, \quad \xi_1 = \frac{\beta_{20}^2 E_0^2}{\rho b c_0^2 \alpha_0}, \quad \xi_2 = \frac{a \beta_{20}}{b \beta_{10}}, \quad \xi_3 = \frac{1}{\eta_0 D b} \quad \text{and} \quad \delta_0 = 1 + \frac{\epsilon_0 \mu_e^2 H_0^2}{\rho}.$$

4. Normal Mode Analysis

The solutions of the physical variables can be decomposed in terms of normal modes in the following form

$$[u, w, q, \psi, \theta, \sigma_{ij}, c, p_1](x, z, t) = [u^*, w^*, q^*, \psi^*, \theta^*, \sigma^*_{ij}, c^*, p^*_1](z) \exp(\omega t + \iota m x), \tag{33}$$

where ω is the complex time constant (frequency), ι is the imaginary unit, m is the wave number in x-direction and $u^*, w^*, q^*, \psi^*, \theta^*, \sigma^*_{ij}, c^*$ and p^*_1 are the amplitudes of the functions. By virtue of (33), equations (29)-(32) transform to the forms

$$(D^2 - \epsilon_1)\psi^*(z) = 0, \tag{34}$$

$$(\gamma_1 D^2 - \epsilon_2)q^*(z) - \theta^*(z) - c^*(z) = 0, \tag{35}$$

$$(\epsilon_3 D^2 - \epsilon_5)\theta^*(z) - \omega\epsilon_4\gamma_4(D^2 - m^2)q^*(z) + \gamma_5\omega\epsilon_4 c^*(z) = 0, \tag{36}$$

$$\epsilon_6(D^4 - 2m^2 D^2 + m^4)q^*(z) + \epsilon_9\xi_2(D^2 - m^2)\theta^*(z)(\epsilon_9 D^2 - \epsilon_8)c^*(z) = 0, \tag{37}$$

where $D = \dfrac{d}{dz}$, $\epsilon_1 = m^2 + \dfrac{\delta_0\omega^2}{\gamma_3}$, $\epsilon_2 = \gamma_1 m^2 + \gamma_2\omega^2$, $\epsilon_3 = 1 + \tau_T\omega$,
$\epsilon_4 = 1 + \tau_q\omega + \dfrac{1}{2}\tau_q^2\omega^2$, $\epsilon_5 = \epsilon_3 m^2 + \epsilon_4\omega$, $\epsilon_6 = \xi_1(1 + \hat{\tau}_T\omega)$,
$\epsilon_7 = \xi_3\omega(1 + \hat{\tau}_q\omega + \dfrac{1}{2}\hat{\tau}_q^2\omega^2)$, $\epsilon_8 = m^2\epsilon_9 + \epsilon_7$, $\epsilon_9 = 1 + \hat{\tau}_T\omega$.
Eliminating $q^*(z), c^*(z)$ and $\theta^*(z)$ from equations (35)-(37), we get the following six order differential equation

$$[D^6 + PD^4 + QD^2 + R][q^*(z), \theta^*(z), c^*(z)] = 0, \tag{38}$$

where

$$P = \frac{A_1B_4 - B_2\epsilon_3 - A_3B_1}{B_1\epsilon_3},$$
$$Q = \frac{B_3\epsilon_3 + A_3B_2 - A_1B_5 - A_2B_4}{B_1\epsilon_3},$$
$$R = \frac{A_2B_5 - A_3B_3}{B_1\epsilon_3},$$
$$A_1 = \epsilon_4\omega(\gamma_1\gamma_5 - \gamma_4),\quad A_2 = \epsilon_4\omega(\epsilon_2\gamma_5 - m^2\gamma_4),\quad A_3 = \epsilon_5 + \omega\epsilon_4\gamma_5,$$
$$B_1 = \epsilon_9\gamma_1 - \epsilon_6,\quad B_2 = \epsilon_9\epsilon_2 + \epsilon_8\gamma_1 - 2m^2\epsilon_6 \quad B_3 = \epsilon_2\epsilon_8 - \epsilon_6 m^4,$$
$$B_4 = \epsilon_9(1 + \xi_2),\quad B_5 = \epsilon_8 + m^2\epsilon_9\xi_2.$$

Equation (38) can be factorized as

$$[(D^2 - \lambda_1^2)(D^2 - \lambda_2^2)(D^2 - \lambda_3^2)][q^*(z), \theta^*(z), c^*(z)] = 0, \tag{39}$$

where $\lambda_n^2, (n = 1, 2, 3)$ are the roots of the characteristic equation

$$\lambda^6 + P\lambda^4 + Q\lambda^2 + R = 0, \tag{40}$$

and are given by

$$\lambda_1 = \sqrt{\frac{1}{3}[2p\sin(q) - P]},$$
$$\lambda_2 = \sqrt{\frac{1}{3}[-P - p(\sqrt{3}\cos(q) + \sin(q)]},$$

$$\lambda_3 = \sqrt{\frac{1}{3}[-P + p(\sqrt{3}\cos(q) - \sin(q)]},$$
$$p = \sqrt{P^2 - 3Q}, \qquad q = \frac{\sin^{-1}(r)}{3}, \qquad r = \frac{-2P^3 + 9PQ - 27R}{2p^3}.$$

The solution of equation (38), which is bounded as $z \to \infty$, is given by

$$q^*(z) = \sum_{n=1}^{3} M_n(m,\omega)e^{-\lambda_n z}, \tag{41}$$
$$\theta^*(z) = \sum_{n=1}^{3} M'_n(m,\omega)e^{-\lambda_n z}, \tag{42}$$
$$c^*(z) = \sum_{n=1}^{3} M''_n(m,\omega)e^{-\lambda_n z}, \tag{43}$$

where M_n, M'_n and M''_n are some parameters depending upon m and ω. Using solutions (41)-(43) in equations (35)-(37), we get the following relations

$$\theta^*(z) = \sum_{n=1}^{3} H_{1n} M_n(m,\omega)e^{-\lambda_n z}, \tag{44}$$
$$c^*(z) = \sum_{n=1}^{3} H_{2n} M_n(m,\omega)e^{-\lambda_n z}, \tag{45}$$

where

$$H_{1n} = \frac{\omega\epsilon_4[\gamma_4(\lambda_n^2 - m^2) - \gamma_5(\gamma_1\lambda_n^2 - \epsilon_2)]}{\epsilon_3\lambda_n^2 - \epsilon_5 - \omega\epsilon_4\gamma_5}, \qquad H_{2n} = \gamma_1\lambda_n^2 - \epsilon_2 - H_{1n}.$$

The solution of equation (34) can be written as

$$\psi^*(z) = M_4(m,\omega)e^{-\lambda_4 z}, \tag{46}$$

where

$$\lambda_4 = \sqrt{m^2 + \frac{\delta_0\omega^2}{\gamma_3}}.$$

5. Application

We consider a homogeneous, isotropic, thermally and perfectly conducting elastic medium with diffusion and temperature-dependent modulus of elasticity occupying the half space $z \geq 0$. The constants M_n's will be determined by imposing the proper boundary conditions.

Mechanical load on the surface of the half-space

The surface $z = 0$ is taken to be isothermal. Hence, the boundary conditions in this case are

$$\sigma_{zz}(x,0,t) + \bar{\sigma}_{zz}(x,0,t) = -P_1(x,t), \tag{47}$$
$$\sigma_{zx}(x,0,t) + \bar{\sigma}_{zx}(x,0,t) = 0, \tag{48}$$
$$\theta(x,0,t) = 0, \tag{49}$$
$$c(x,0,t) = 0, \tag{50}$$

where $P_1(x,t)$ is a given function of x and t and $\bar{\sigma}_{zj}(j = x,y,z)$ is the Maxwell stress given in the form

$$\bar{\sigma}_{zj} = \mu_e(H_z h_j + H_j h_z - H_k h_k \delta_{zj}). \tag{51}$$

Application of non-dimensionalization and normal mode analysis techniques defined in (27) and (33) respectively along with $P_1' = \frac{P_1}{\rho C_0^2}$ transform the above boundary conditions to the forms

$$\sigma_{zz}^*(z) + \bar{\sigma}_{zz}^*(z) = -P_1^*, \tag{52}$$

$$\sigma_{zx}^*(z) + \bar{\sigma}_{zx}^*(z) = 0, \tag{53}$$

$$\theta^*(z) = 0, \tag{54}$$

$$c^*(z) = 0, \quad \text{at} \ \ z = 0, \tag{55}$$

where

$$\bar{\sigma}_{zz}^*(z) = \sum_{n=1}^{3} H_{3n}' M_n e^{-\lambda_n z}, \quad \bar{\sigma}_{zx}^*(z) = 0 \ \ and \ \ H_{3n}' = \mu_e H_0^2(\lambda_n^2 - m^2).$$

With the aid of non-dimensional quantities defined in (27), the expressions of stresses (20), (22) and displacement components (28), in combination with relations (41), (44)-(46), recast into the following forms:

$$u^*(z) = \sum_{n=1}^{3} (\iota m M_n e^{-\lambda_n z}) - \lambda_4 M_4 e^{-\lambda_4 z}, \tag{56}$$

$$w^*(z) = \sum_{n=1}^{3} (-\lambda_n M_n e^{-\lambda_n z}) - \iota m M_4 e^{-\lambda_4 z}, \tag{57}$$

$$\sigma_{zz}^*(z) = \sum_{n=1}^{3} (H_{3n} M_n e^{-\lambda_n z}) - H_{34} M_4 e^{-\lambda_4 z}, \tag{58}$$

$$\sigma_{zx}^*(z) = \sum_{n=1}^{3} (-H_{4n} M_n e^{-\lambda_n z}) + H_{44} M_4 e^{-\lambda_4 z}, \tag{59}$$

where

$$H_{3n} = \frac{\lambda_n^2 - \epsilon_{10} m^2 - H_{1n} - H_{2n}}{\alpha_0}, \qquad H_{34} = \frac{\iota m \lambda_4(\epsilon_{10} - 1)}{\alpha_0}, \qquad H_{4n} = 2m\iota\epsilon_{11}\lambda_n,$$
$$H_{44} = \epsilon_{11}(\lambda_4^2 + m^2), \qquad \epsilon_{10} = \frac{\lambda_0 E_0}{\rho c_0^2}, \qquad \epsilon_{11} = \frac{\mu_0 E_0}{\alpha_0 \rho c_0^2}, \qquad (n = 1, 2, 3).$$

The boundary conditions (52)-(55), with the help of expressions (44), (45), (58) and (59), yield a non-homogeneous system of four equations, which can be written in matrix form as

$$\begin{bmatrix} H_{31} + H_{31}' & H_{32} + H_{32}' & H_{33} + H_{33}' & -H_{34} \\ H_{41} & H_{42} & H_{43} & -H_{44} \\ H_{11} & H_{12} & H_{13} & 0 \\ H_{21} & H_{22} & H_{23} & 0 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{bmatrix} = \begin{bmatrix} -P_1^* \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{60}$$

The expressions of M_n, $(n = 1, 2, 3)$ and M_4 obtained by solving the system (60), when substituted in (44), (45) and (56)-(59), provide us the following expressions of field variables

$$u^*(z) = \frac{1}{\Delta}[\iota m \Delta_1 e^{-\lambda_1 z} + \iota m \Delta_2 e^{-\lambda_2 z} + \iota m \Delta_3 e^{-\lambda_3 z} - \lambda_4 \Delta_4 e^{-\lambda_4 z}], \tag{61}$$

$$w^*(z) = -\frac{1}{\Delta}[\lambda_1 \Delta_1 e^{-\lambda_1 z} + \lambda_2 \Delta_2 e^{-\lambda_2 z} + \lambda_3 \Delta_3 e^{-\lambda_3 z} + \iota m \Delta_4 e^{-\lambda_4 z}], \tag{62}$$

$$\theta^*(z) = \frac{1}{\Delta}[H_{11} \Delta_1 e^{-\lambda_1 z} + H_{12} \Delta_2 e^{-\lambda_2 z} + H_{13} \Delta_3 e^{-\lambda_3 z}], \tag{63}$$

$$\sigma_{zz}^*(z) = \frac{1}{\Delta}[H_{31} \Delta_1 e^{-\lambda_1 z} + H_{32} \Delta_2 e^{-\lambda_2 z} + H_{33} \Delta_3 e^{-\lambda_3 z} - H_{34} \Delta_4 e^{-\lambda_4 z}], \tag{64}$$

$$\sigma_{zx}^*(z) = \frac{1}{\Delta}[-H_{41} \Delta_1 e^{-\lambda_1 z} - H_{42} \Delta_2 e^{-\lambda_2 z} - H_{43} \Delta_3 e^{-\lambda_3 z} + H_{44} \Delta_4 e^{-\lambda_4 z}], \tag{65}$$

$$c^*(z) = \frac{1}{\Delta}[H_{21} \Delta_1 e^{-\lambda_1 z} + H_{22} \Delta_2 e^{-\lambda_2 z} + H_{23} \Delta_3 e^{-\lambda_3 z}], \tag{66}$$

where

$$\Delta = (H_{31} + H'_{31})d_1 - (H_{32} + H'_{32})d_2 + (H_{33} + H'_{33})d_3 + H_{34}d_4,$$
$$\Delta_1 = -P_1^* d_1, \quad \Delta_2 = P_1^* d_2, \quad \Delta_3 = -P_1^* d_3, \quad \Delta_4 = P_1^* d_4,$$
$$d_1 = -H_{44}(H_{12}H_{23} - H_{13}H_{22}),$$
$$d_2 = -H_{44}(H_{11}H_{23} - H_{13}H_{21}),$$
$$d_3 = -H_{44}(H_{11}H_{22} - H_{21}H_{12}),$$
$$d_4 = H_{41}(H_{12}H_{23} - H_{13}H_{22}) - H_{42}(H_{11}H_{23} - H_{21}H_{13}) + H_{43}(H_{11}H_{22} - H_{12}H_{21}).$$

6. Particular Cases

6.1. Case 1: without dual-phase-lags

To discuss the problem of wave propagation in a thermally and perfectly conducting elastic medium with diffusion in the context of L-S theory, it is sufficient to set the values of τ_q, $\hat{\tau}_q$, τ_T, $\hat{\tau}_T$, ϵ_4 and ϵ_7 as $\tau_q = \tau_0$, $\hat{\tau}_q = \tau^0$, $\tau_T = 0$, $\hat{\tau}_T = 0$, $\epsilon_4 = 1 + \tau_0\omega$ and $\epsilon_7 = \xi_3\omega(1 + \tau^0\omega)$. The corresponding expressions for displacements, stresses, temperature field and concentration can be obtained from expressions (61)-(66).

6.2. Case 2: without temperature dependent parameters

If we remove the temperature dependence effect, then we shall be dealing a half-space problem in electromagneto-thermoelastic medium with diffusion under dual-phase-lag theory. So, by putting $\alpha^* = 0$ in (19), which implies $f(\theta) = 1$, the corresponding expressions for displacements, stresses, temperature field and concentration can be obtained from expressions (61)-(66).

7. Numerical results and discussion

With an aim to illustrate the theoretical results obtained in the preceding sections, we now present some numerical results. The numerical work has been carried out with the help of computer programming using the software MATLAB. Material chosen for this purpose is copper, the physical data for which is given as Othman *et al.* (2013):

$\rho = 8954\ kg\ m^{-3}$, $\epsilon_0 = \frac{10^{-9}}{36\pi}\ Fm^{-1}$, $H_0 = \frac{10^7}{4\pi}\ Am^{-1}$ $\mu_e = 4\pi(10)^{-7}\ Hm^{-1}$

$\alpha_t = 1.78 \times 10^{-5} K^{-1}$, $\alpha_c = 1.98 \times 10^{-4} m^3\ kg^{-1}$, $K = 386\ W\ m^{-1} K^{-1}$,

$c_E = 383.1\ J\ kg^{-1} K^{-1}$, $\tau_q = 0.02\ s$, $\tau_T = 0.015\ s$, $\hat{\tau_T} = 0.15\ s$, $\hat{\tau_q} = 0.2\ s$, $a = 1.2 \times 10^4\ m^2 s^{-2} K^{-1}$, $b = 0.9 \times 10^6\ m^5 kg^{-1} s^{-2}$, $T_0 = 293\ K$,

$D = 0.85 \times 10^{-8}\ kg\ s\ m^{-3}$, $E_0 = 10.4 \times 10^{10}\ kg\ m^{-1}\ s^{-2}$, $v = 0.33$.

Utilizing the above values of parameters, values of the non-dimensional field variables have been evaluated and the results are displayed in the form of graphs at different positions of z at $x = 1.0$. From application point of view, we have divided the graphs into two categories. In first category (Figures 1-4), all the field quantities have been examined for four different cases: (i) Dual-phase-lag magneto-thermoelastic diffusion theory with temperature dependence at $\omega = 1.0$ [DPL ($\omega = 1.0$), solid line], (ii) Dual-phase-lag magneto-thermoelastic diffusion theory with temperature dependence at $\omega = 1.3$ [DPL ($\omega = 1.3$), long-dashed line], (iii) Lord-Shulman magneto-thermoelastic diffusion theory with temperature dependence at $\omega = 1.0$ [L-S ($\omega = 1.0$), small-dashed line] and (iv) Lord-Shulman magneto-thermoelastic diffusion theory with temperature dependence at $\omega = 1.3$ [L-S ($\omega = 1.3$), dotted line]. All the figures in first category are displayed at $t = 0.01$ and $\alpha^* = 0.001$. In second category (Figures 5-8), the comparisons of the dimensionless physical quantities are made for the four different cases: (i) with temperature dependent elastic moduli at $t = 0.01$ ($\alpha^* = 0.001, t = 0.01$, solid line), (ii) with temperature dependent elastic moduli at $t = 0.05$ ($\alpha^* = 0.001, t = 0.05$, long-dashed line), (iii) without temperature dependent elastic moduli at $t = 0.01$ ($\alpha^* = 0.0, t = 0.01$, small-dashed line) and (iv) without temperature dependent elastic moduli at $t = 0.05$ ($\alpha^* = 0.0, t = 0.05$, dotted line). All the figures in second category describing the effects of temperature dependence and time are displayed at $\omega = 1.0$.

7.1. Category I

Figure 1 depicts the distribution of displacement component w with distance z in the context of the two theories DPL and L-S at $\omega = 1.0$ and 1.3. It is observed that the distribution of displacement w begins with positive values, thereafter decreases smoothly and ultimately tends to zero for $z \geq 3.6$. The behaviour of displacement field based on the DPL and LS theories for the two different value of ω is similar and it can also be noticed from the plot that the displacement distribution for frequency ($\omega = 1.0$) has large values in comparison to the values at frequency ($\omega = 1.3$). Hence frequency has a decreasing effect on the profile of displacement distribution, whereas phase lags are having a very small effect on this distribution. Variation in normal stress σ_{zz} with spatial coordinate z has been displayed in Figure 2. We observe that the behaviour of normal stress field is similar in nature for both theories at $\omega = 1.0$ and 1.3 i.e it begins with negative values on the boundary of half space, then decreases to minimum and thereafter tends to zero. We can see from the figure that the value of normal stress for L-S theory is less (in magnitude) as compared to DPL and the stress decreases as we increase the value of frequency ω.

In Figure 3, we have plotted the variation of temperature θ with distance z for the theories DPL and L-S at different values of frequency ω. It starts with a zero value which is completely in agreement with the boundary conditions. It can be seen that the maximum impact zone of frequency ω is around $0.3 \leq z \leq 1.0$. The value of temperature θ is zero initially, gradually increases in the range $0.0 \leq z \leq 0.6$, attaining maximum value at $z = 0.6$ and then ultimately approaches to zero for both the theories. Figure 4 shows the distribution of mass concentration c with distance z for both the theories (DPL and L-S) at different values of frequency ($\omega = 1.0, 1.3$). It starts with a zero value which is completely in agreement with the boundary conditions. In the beginning, the vibration amplitude quickly rises to its maximum value in the range $0.0 \leq z \leq 0.7$. It can also be noticed from the plot that the mass concentration for DPL theory has large values in comparison to L-S theory, which illuminates that the phase lags are having a significant increasing effect on the profile of concentration field. Similar effects of frequency can also be noticed from the figure.

7.2. Category II

Figure 5 depicts the effects of temperature dependence and time on the variations of displacement component w with distance z. Displacement field starts with positive values having magnitudes $4.196191, 4.367441, 3.453728$ and 3.594678 for four different cases respectively, which confirms the significant impact of time as well as temperature dependent parameter on the profile. We see that the increment in the time increases the magnitude of displacement component w. Hence it has an increasing effect on the profile of normal displacement. For temperature dependent and temperature independent cases at $t = 0.01$ and 0.05, the effect is more pronounced in the range $0.0 \leq z \leq 0.7$. Figure 6 exhibits that the distribution of normal stress σ_{zz} versus distance begins from negative values in the absence and presence of temperature dependence at different times (i.e $t = 0.01, 0.05$). We see that the increment in time as well as the absence of temperature dependence increase the magnitude of stress component σ_{zz}. It is also seen that all the curves show similar trends and the temperature dependent properties have more significant effects on variations of normal stress as compared to time.

Figure 7 shows the variation of temperature field θ with distance $0.0 \leq z \leq 5.6$. As expected, temperature field is having a coincident starting point of zero magnitude for all the four cases, which is in quite good agreement with the boundary conditions. It can be noticed from the plot that the temperature distribution for $t = 0.05$ has large values in comparison to the values at time $t = 0.01$ for both temperature dependent and temperature independent properties. Temperature field has a qualitative similar behaviour for all the four cases. As can be seen from the figure, the maximum impact zone of time is around the range $0.5 \leq z \leq 0.9$. We further observe that the temperature field exhibits significant sensitivity towards the presence and absence of temperature dependence. Presence of temperature dependent elastic moduli has lowered down the values of temperature distribution. Figure 8 has been plotted to illustrate the influences of temperature dependence and time on the profile of mass concentration distribution. It is evident that (i) the values of mass concentration field recorded in temperature independent case are less than those values recorded in temperature dependent case, (ii) with the increase in time t, values of mass concentration increases. Also, the effect of the temperature dependence is significant for $0.3 \leq z \leq 3.2$ and the influence of time t is prominent in the range $0.5 \leq z \leq 1.6$.

8. Conclusions

By using normal mode analysis, behaviour of normal displacement, normal stress, temperature and mass concentration in a homogeneous, isotropic, thermally and perfectly conducting elastic medium with diffusion and temperature-dependent modulus of elasticity has been examined within the framework of dual-phase-lag theory. The theoretical and numerical results reveal that all the considered parameters have significant effects on the field variables. According to the above analysis, we can conclude the following points:

- All the considered fields are found to be sensitive towards the dual-phase-lag parameters except the displacement field. In the presence of these parameters, the magnitudes of stress and mass concentration field increase but the magnitude of temperature field decreases.
- In all the figures, it is clear that all the fields are restricted in a limited region which is in accordance with the notion of generalized thermoelasticity theory and supports the physical facts.
- An increment in the value of frequency causes a decrement in the values of displacement and stress fields. On the other hand an increment in the value of frequency causes an increment in the values of temperature and mass concentration fields.
- We observe that the temperature dependent properties have a significant effect on all the physical fields. When we remove these properties, stress and temperature fields increase in magnitude, mass concentration field decreases, where as displacement field has both increasing and decreasing effects.

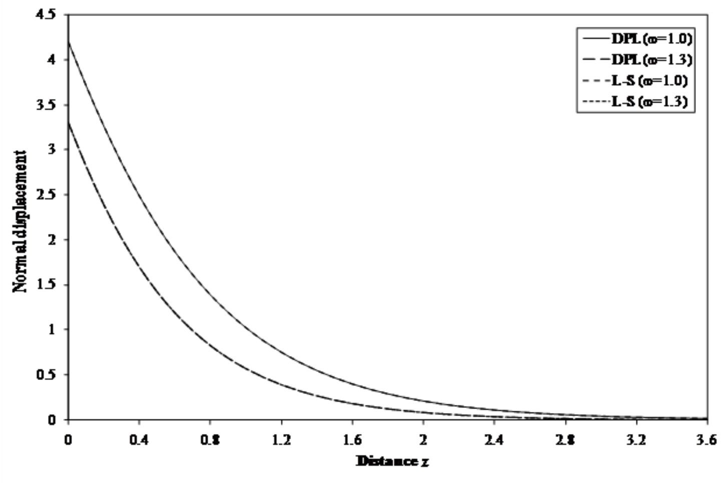

Figure 1. Normal displacement vs. distance

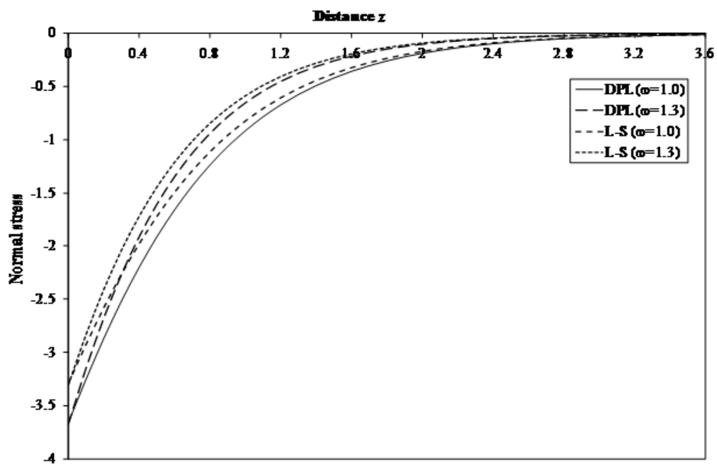

Figure 2. Normal stress vs. distance

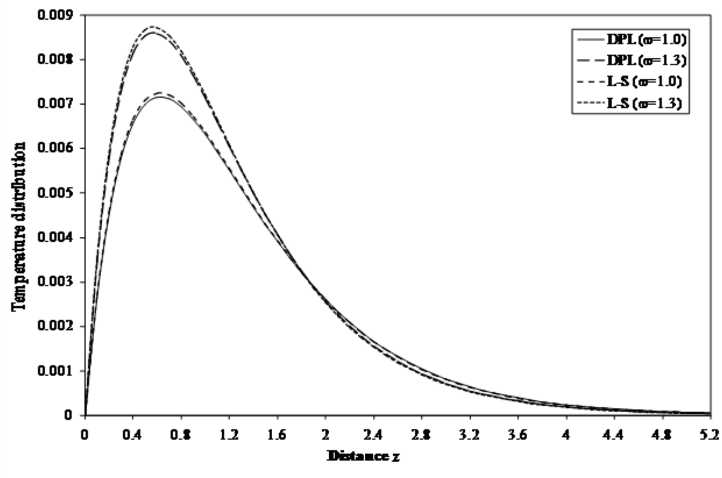

Figure 3. Temperature vs. distance

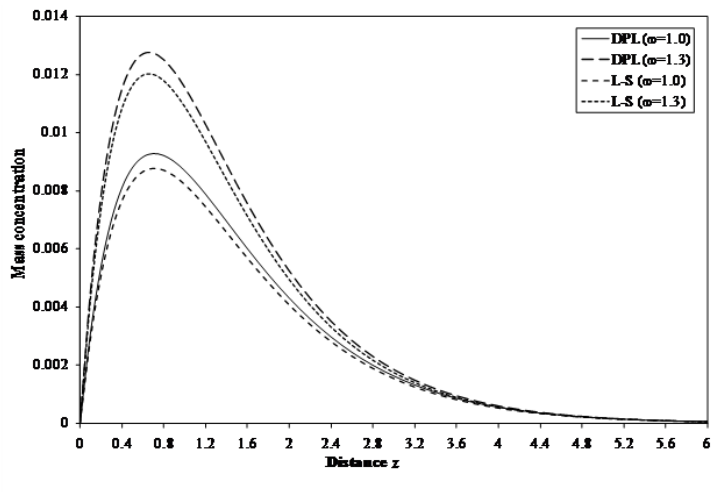

Figure 4. **Mass concentration vs. distance**

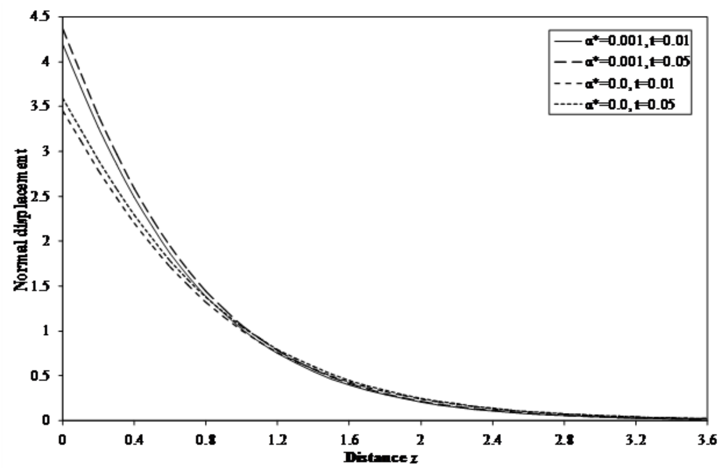

Figure 5. **Effect of temperature dependence and time on normal displacement**

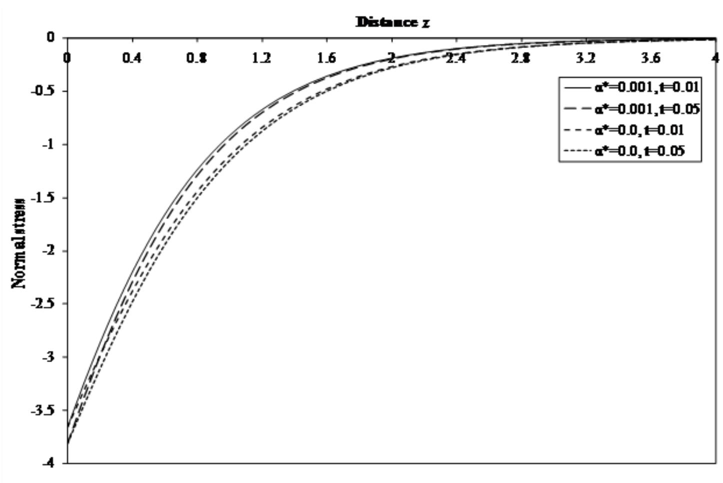

Figure 6. **Effect of temperature dependence and time on normal stress**

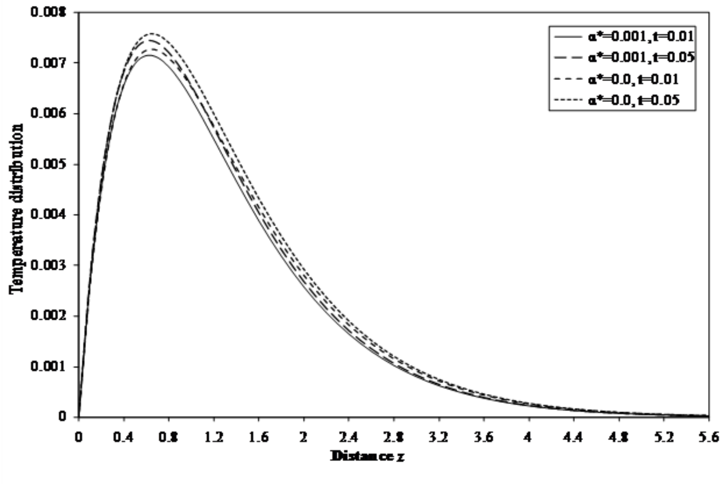

Figure 7. Effect of temperature dependence and time on temperature

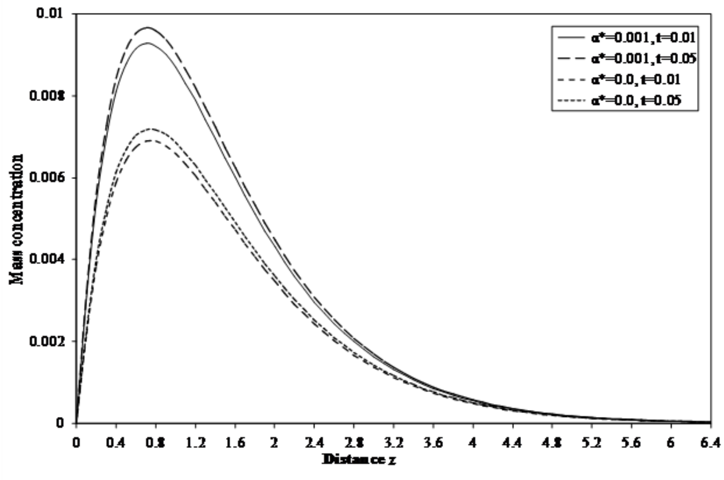

Figure 8. Effect of temperature dependence and time on mass concentration

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