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SFS and SFS-2 Domination

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Abstract: Let $G_{A,V}$ be a fuzzy soft graph and $S \subseteq V$ is a fuzzy soft dominating set in $G_{A,V}$, then S is said to be a secure fuzzy soft dominating set if for each vertex $x_i \in V - S$ is adjacent to a vertex $x_j \in S$ such that $(S - \{x_j\}) \cup \{x_i\}$ is a dominating set for all $e \in A$ and the minimum fuzzy soft cardinality taken over all minimal secure fuzzy soft dominating set is called secure fuzzy soft domination number and is denoted by $\gamma_{sefs}(G_{A,V})$.

Keywords: Secure Fuzzy soft domination, Secure Fuzzy soft domination number, total secure fuzzy soft domination, fuzzy soft 2-dominating set, total fuzzy soft 2-dominating set, secure fuzzy soft 2-dominating set, total secure fuzzy soft 2-dominating set.

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1. Introduction

The concept of domination in fuzzy graphs was first introduced by A Somasundaram and S Somasundaram [8]. The same concept in terms of strong arcs was described by A Nagoorgani [5]. The notion of fuzzy soft graphs was introduced by Sumit Mohinta and Samanta [7] and later on Muhammed Akram and Saira Nawas [6] introduced different types of fuzzy soft graphs and their properties.S. The concept of secure domination in fuzzy graphs was introduced by M G Karunambigai, S Sivasankar and K Palanivel [4]. In 2017 T K Mathew varkey and Rani Rajeevan [9] introduced the concept of domination in fuzzy soft graphs.

2. Preliminaries

Definition 2.1 ([3]). *Assume X be a universal set, S be the set of parameters and $P(X)$ denote the power set of X. If there is a mapping $F : S \to P(X)$, then we call the pair (F, S), a soft set over X.*

Definition 2.2 ([3]). *Let X be a universal set, S be the set of parameters and $E \subset S$. If there is a mapping $F : E \to I^X$, I^X be the set of all fuzzy subsets of S, then we say that (F, E) is a fuzzy soft set over X.*

Definition 2.3 ([7]). *Let $V = \{x_1, x_2, x_3, \cdots x_n\}$ (non empty set) E (parameters set) and $A \subseteq E$. Also let*

(1). $\rho : A \to F(V)$, collection of all fuzzy subsets in V and each element e of A is mapped to $\rho(e) = \rho_e$ (say) and $\rho_e : V \to [0, 1]$, each element x_i is mapped to $\rho_e(x_i)$ and we call (A, ρ), a fuzzy soft vertex.

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(2). $\mu : A \to F(V \times V)$, *collection of all fuzzy subsets in* $V \times V$, *which mapped each element* e *to* $\mu(e) = \mu_e$ *(say) and* $\mu_e : V \times V \to [0,1]$, *which mapped each element* (x_i, x_j) *to* $\mu_e(x_i, x_j)$, *and we call* (A, μ) *as a fuzzy soft edge.*

Then $((A,\rho),(A,\mu))$, *is called a fuzzy soft graph if and only if* $\mu_e(x_i,x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)\ \ \forall\ \ e \in A$ *and* $\forall\ \ i,j = 1,2,3,\ldots n$, *this fuzzy soft graph is denoted by* $G_{A,V}$.

Definition 2.4 ([6]). *A fuzzy soft graph* $H_{A,V}(\tau_e, \sigma_e)$ *is called a fuzzy soft sub graph of* $G_{A,V}(\rho_e, \mu_e)$ *if* $\forall\ \ e \in A \tau_e(x_i) \leq \rho_e(x_i)\ \ \forall\ \ x_i \in V$ *and* $\sigma_e(x_i,x_j) \leq \mu_e(x_i,x_j)\ \ \forall\ \ x_i, x_j \in V$.

Definition 2.5 ([7]). *The underlying crisp graph of a fuzzy soft graph* $G_{A,V} = ((A,\rho),(A,\mu))$ *is denoted by* $G^* = (\rho^*, \mu^*)$, *where* $\rho^* = \{x_i \in V;\ \rho_e(x_i) > 0\ \ \text{for some}\ \ e \in E\}$ *and* $\mu^* = \{(x_i,x_j) \in V \times V;\ \mu_e(x_i,x_j) > 0\ \ \text{for some}\ \ e \in E\}$.

Definition 2.6 ([6]). *A fuzzy soft graph* $G_{A,V}$ *is called a strong fuzzy soft graph if* $\mu_e(x_i,x_j) = \rho_e(x_i) \wedge \rho_e(x_j)\ \ \forall\ \ (x_i,x_j) \in \mu,\ \ \forall\ \ e \in A$ *and is called a complete fuzzy soft graph if* $\mu_e(x_i,x_j) = \rho_e(x_i) \wedge \rho_e(x_j)\ \ \forall\ \ x_i, x_j \in \rho,\ \ \forall\ \ e \in A$.

Definition 2.7 ([6]). *Let* $G_{A,V}$ *be a fuzzy soft graph. Then the order of* $G_{A,V}$ *is defined as* $O(G_{A,V}) = \sum_{e \in A} \left(\sum_{x_i \in V} \rho_e(x_i) \right)$ *and size of* $G_{A,V}$ *is defined as* $S(G_{A,V}) = \sum_{e \in A} \left(\sum_{x_i, x_j \in V} \mu_e(x_i,x_j) \right)$ *and the degree of a vertex* x_i *is defined as* $d_{(G_{A,V})}(x_i) = \sum_{e \in A} \left(\sum_{x_j \in V\ \ x_i \neq x_j} \mu_e(x_i,x_j) \right)$.

Definition 2.8 ([9]). *Degree of a fuzzy soft graph* $G_{A,V}$ *is defined as* $D_{G_{A,V}} = \max\{d_{G_{A,V}}(x_i); x_i \in V\}$.

Definition 2.9 ([9]). *A fuzzy soft graph* $G_{A,V}$ *is said to be regular fuzzy soft graph if the fuzzy graph corresponding to each parameter* $e \in A$ *is a regular fuzzy graph.*

Definition 2.10 ([9]). *A fuzzy soft sub graph* $H_{A,V}(\tau_e, \sigma_e)$ *is said to be a spanning fuzzy soft sub graph if* $\forall\ \ e \in A \tau_e(x_i) = \rho_e(x_i)\ \ \forall\ \ x_i \in V$ *and* $\sigma_e(x_i,x_j) \leq \mu_e(x_i,x_j)\ \ \forall\ \ x_i, x_j \in V$.

Definition 2.11 ([9]). *A fuzzy soft sub graph* $H_{A,V}(\tau_e, \sigma_e)$ *is said to be an induced fuzzy soft graph if* $\forall\ \ e \in A \sigma_e(x_i,x_j) = \tau_e(x_i) \wedge \tau_e(x_j) \wedge \mu_e(x_i,x_j)\ \ \forall\ \ x_i, x_j \in V$ *and is denoted by* $\langle \tau_e \rangle$. *In other words a fuzzy soft sub graph induced by* τ_e *is the maximal fuzzy soft sub graph that has a fuzzy soft vertex set* τ_e.

Definition 2.12 ([9]). *A path of length 'n' in a fuzzy soft graph is a sequence of distinct vertices* $x_1, x_2, x_3, \ldots, x_n$ *such that* $\forall\ \ e \in A$ *and* $\mu_e(x_{i-1}, x_i) > 0$ *and* $\forall\ \ i = 1,2,3,\ldots,n$.

Definition 2.13 ([10]). *The fuzzy soft cardinality of a fuzzy soft subset* S *of* V *is* $\sum_{e \in A} \left(\sum_{x_i \in S} \rho_e(x_i) \right)$.

Definition 2.14 ([9]). *Let* $G_{A,V}$ *be a fuzzy soft graph and let* x_i *and* x_j *be two vertices of* $G_{A,V}$. *If* $\mu_e(x_i,x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)$ *for each parameter* $e \in A$, *then we say that* x_i *dominates* x_j *in* $G_{A,V}$. *A subset* S *of* V *is called a fuzzy soft dominating set if for every* $x_j \in V - S$, *there exist a vertex* $x_i \in S$ *such that* x_i *dominates* x_j.

Definition 2.15 ([9]). *A fuzzy soft dominating set* S *of a fuzzy soft graph* $G_{A,V}$ *is said to be a minimal fuzzy soft dominating set if for each parameter* $e \in A$, *deletion of an element from* S *is not a fuzzy soft dominating set.*

Definition 2.16 ([9]). *The minimum cardinality of all minimal fuzzy soft dominating set is called the fuzzy soft domination number and is denoted by* $\gamma_{fs}(G_{A,V})$.

3. Secure Fuzzy Soft (SFS) Domination

Definition 3.1. *Let $G_{A,V}$ be a fuzzy soft graph and $S \subseteq V$ be a fuzzy soft dominating set, then S is said to be a secure fuzzy soft dominating set if for each vertex $x_i \in V - S$ is adjacent to a vertex $x_j \in S$ such that $(S - \{x_j\}) \cup \{x_i\}$ is a dominating set for all $e \in A$ and the minimum fuzzy soft cardinality taken over all minimal secure fuzzy soft dominating set is called secure fuzzy soft domination number and is denoted by $\gamma_{sefs}(G_{A,V})$.*

Definition 3.2. *A secure fuzzy soft dominating set $S \subseteq V$ of a fuzzy soft graph $G_{A,V}$ is said to be a total secure fuzzy soft dominating set if $\langle S \rangle$ has no isolated vertices and the minimum fuzzy soft cardinality taken over all minimal total secure fuzzy soft dominating set is called the total secure fuzzy soft domination number and is denoted by $\gamma_{tsefs}(G_{A,V})$.*

Theorem 3.3. *Every minimal fuzzy soft dominating set in a complete fuzzy soft graph is a secure fuzzy soft dominating set. Further more it is not a total secure fuzzy soft dominating set.*

Proof. Suppose that $G_{A,V}$ is a complete fuzzy soft graph and S is a minimal fuzzy soft dominating set. Since all edges are effective, the minimal fuzzy soft dominating set contains a single vertex, say $S = \{x_i\}$. Again since $G_{A,V}$ is complete, every vertex $x_j \in V - S$ is adjacent to $x_i \in S$. So $(S - \{x_i\}) \cup \{x_j\}$ is a dominating set. Hence S is a secure fuzzy soft dominating set.
Also we know that any minimal secure fuzzy soft dominating set S in a complete fuzzy soft graph contains a single vertex. So by definition of total secure fuzzy soft dominating set, $S$is not a total secure fuzzy soft dominating set. □

Remark 3.4. *Every secure dominating set is a dominating set. But the converse need not be true.*

Theorem 3.5. *In any complete fuzzy soft graph, $\gamma_{sefs}(G_{A,V}) = \gamma_{fs}(G_{A,V})$.*

Proof. Let $G_{A,V}$ be a complete fuzzy soft graph and S is a minimal fuzzy soft dominating set. Then S contains a single vertex so $\gamma_{fs}(G_{A,V}) = \left\{ \sum_{e \in A} \rho_e(x_i) \ ; \ x_i \in V \right\}$ and by above theorem single vertex set is a secure fuzzy soft dominating set and so the minimum fuzzy soft cardinality of the single vertex is the secure fuzzy soft domination number and so $\gamma_{sefs}(G_{A,V}) = \left\{ \sum_{e \in A} \rho_e(x_i) \ ; \ x_i \in V \right\}$. Hence $\gamma_{sefs}(G_{A,V}) = \gamma_{fs}(G_{A,V})$. □

Definition 3.6. *A fuzzy soft dominating set $S \subseteq V$ of a fuzzy soft graph $G_{A,V}$ is said to be a fuzzy soft 2-dominating set if for every vertex of $V - S$ has atleast two neighbours in S. The minimum fuzzy soft cardinality taken over all minimal fuzzy soft 2-dominating set is called the fuzzy soft 2-domination number and is denoted by $\gamma_{fs-2}(G_{A,V})$.*

Definition 3.7. *A fuzzy soft dominating set $S \subseteq V$ of a fuzzy soft graph $G_{A,V}$ is said to be a total fuzzy soft 2-dominating set if $\langle S \rangle$ has no isolated vertices and for S is a fuzzy soft 2-dominating set. The minimum fuzzy soft cardinality taken over all minimal total fuzzy soft 2-dominating set is called the total fuzzy soft 2- domination number and is denoted by $\gamma_{tfs-2}(G_{A,V})$.*

Theorem 3.8. *Any minimal fuzzy soft dominating set in a complete fuzzy soft graph is not a fuzzy soft 2-dominating set. Further more it is not a total fuzzy soft 2-dominating set.*

Proof. Suppose that S is a minimal fuzzy soft dominating set in a complete fuzzy soft graph $G_{A,V}$. Then S contains a single vertex of minimum fuzzy soft cardinality. But fuzzy soft 2-dominating set should contains atleast 2 vertices. Thus S is not a fuzzy soft 2-dominating set. Since S is not a fuzzy soft 2-dominating set, it is not a total fuzzy soft 2-dominating set. □

Remark 3.9. *Every total fuzzy soft 2-dominating set is a fuzzy soft 2-dominating set.*

4. Secure Fuzzy Soft 2-Domination (SFS-2)

Definition 4.1. *Let $G_{A,V}$ be a fuzzy soft graph. A fuzzy soft 2-dominating set $S \subseteq V$ is said to be a secure fuzzy soft 2-dominating set if for every vertex $x_i \in V - S$ is adjacent to a vertex $x_j \in S$ such that $(S - \{x_j\}) \cup \{x_i\}$ is a fuzzy soft 2-dominating set for all $e \in A$ and the minimum fuzzy soft cardinality taken over all minimal secure fuzzy soft 2-dominating set is called secure fuzzy soft 2-domination number and is denoted by $\gamma_{sefs-2}(G_{A,V})$.*

Definition 4.2. *Let $G_{A,V}$ be a fuzzy soft graph. A secure fuzzy soft 2-dominating set $S \subseteq V$ is said to be a total secure fuzzy soft 2-dominating set if $\langle S \rangle$ has no isolated vertices. The minimum fuzzy soft cardinality taken over all minimal total secure fuzzy soft 2-dominating set is called total secure fuzzy soft 2-domination number and is denoted by $\gamma_{tsefs-2}(G_{A,V})$.*

Theorem 4.3. *Every secure fuzzy soft 2-dominating set of a fuzzy soft graph $G_{A,V}$ is a secure fuzzy soft dominating set.*

Proof. Let S be a secure fuzzy soft 2-dominating set of a fuzzy soft graph $G_{A,V}$. Then every vertex $x_i \in V - S$ is adjacent to a vertex $x_j \in S$ such that $(S - \{x_j\}) \cup \{x_i\}$ is a fuzzy soft 2-dominating set for all $e \in A$ and since every fuzzy soft 2-dominating set is a fuzzy soft dominating set, we get $G_{A,V}$ is a secure fuzzy soft dominating set. □

Remark 4.4. *If $G_{A,V}$ is any fuzzy soft graph other than the complete fuzzy soft graph, then every secure fuzzy soft dominating set is a dominating set and the converse need not be true.*

Theorem 4.5. *Let $G_{A,V}$ be a complete fuzzy soft graph and S is any secure fuzzy soft dominating set, then $V - S$ is also a secure dominating set.*

Proof. Let $G_{A,V}$ be a complete fuzzy soft graph and S is any secure fuzzy soft dominating set. To prove $V - S$ is also a secure dominating set. Suppose on the contrary that $V - S$ is not a secure dominating set. Then for each $x_i \in S$.

Case (1): there does not $x_j \in V - S$ such that x_i and x_j are adjacent. But it is not possible, since $G_{A,V}$ be a complete fuzzy soft graph.

Case (2): there exist $x_j \in V - S$ such that x_i and x_j are adjacent. But $((V - S) - \{x_j\}) \cup \{x_i\}$ is not a dominating set, which is also not possible, since any single vertex set is a dominating set. Hence $V - S$ is a secure dominating set. □

Theorem 4.6. *If S is a fuzzy soft 2-dominating set off a fuzzy soft path in a fuzzy soft graph $G_{A,V}$, then S is not a secure fuzzy soft 2-dominating set.*

Proof. Suppose S be a 2-dominating set in a fuzzy soft path. Then S contains two pendent vertices x_i and x_j. Now for some $x_t \in V - S$ is adjacent tox_i or x_j. Then $(S - \{x_i\}) \cup \{x_t\}$ or $(S - \{x_j\}) \cup \{x_t\}$ is not a fuzzy soft 2-dominating set. There fore S is not a secure fuzzy soft 2-dominating set. □

Theorem 4.7. *In a fuzzy soft graph $\gamma_{fs}(G_{A,V}) \leq \gamma_{fs-2}(G_{A,V}) \leq \gamma_{sefs-2}(G_{A,V})$.*

Proof. We know that not every fuzzy soft dominating set is a fuzzy soft 2-dominating set. So we get $\gamma_{fs}(G_{A,V}) \leq \gamma_{fs-2}(G_{A,V})$. Also know that every secure fuzzy soft 2-dominating set is a secure fuzzy soft dominating set. Thus every minimal secure fuzzy soft 2-dominating set is a secure fuzzy soft set dominating set. So $\gamma_{fs-2}(G_{A,V}) \leq \gamma_{sefs-2}(G_{A,V})$. Combining above two relations we get $\gamma_{fs}(G_{A,V}) \leq \gamma_{fs-2}(G_{A,V}) \leq \gamma_{sefs-2}(G_{A,V})$. □

5. Conclusion

In this paper we defined new concepts such as secure fuzzy soft dominating set, total secure fuzzy soft dominating set, fuzzy soft 2-dominating set, total fuzzy soft 2-dominating set, secure fuzzy soft 2-dominating set, total secure fuzzy soft 2-dominating set are very useful for solving wide range of problems.

References

[1] Anas Al-Masarwah and Majdoleen Abu Qamar, *Some New Concepts of Fuzzy Soft Graphs*, fuzzy Inf. Engg., 8(2016), 427-438.

[2] A. Nagoor Gani and V. T. Chandra Sekaran, Domination in Fuzzy Graph, Advances in fuzzy Sets and Systems, 1((1)(2006), 17-26.

[3] A. Somasundaram and S. Somasundaram, *Domination of Fuzzy Graphs-I*, Elsevier Science, 19(1988), 787-791.

[4] E. J. Cockayne and S. T. Hedetniemi, *Towards a theory of domination in graphs*, Networks, (1977), 247-261.

[5] M. G. Karunambigai, S. Sivasankar and K. Palanivel, *Secure domination in fuzzy graphs and intuitionistic fuzzy graphs*, Annals of Fuzzy Mathematics and Informatics, 14(4)(2017), 419-431.

[6] Muhammad Akram and Saira Nawaz, *On Fuzzy Soft Graphs*, Italian Journal of Pure and Applied Mathematics, 34(2015), 497-514.

[7] Sumit Mohinta and T. K. Samanta, *An Introduction to Fuzzy Soft Graph*, Mathematica Morarica, 19(2)(2015), 35-48.

[8] T. K. Mathew Varkey and Rani Rajeevan, *Perfect vertex domination in fuzzy soft graphs*, Advances in fuzzy Mathematics, 12(4)(2017), 793-799.

[9] T. K. Mathew Varkey and Rani Rajeevan, *On FSC, FSS and FSPS domination*, International Journal of Mathematical Archive, 9(4)(2018), 59-63.

[10] T. W. Haynes, Stephen Hedetniemi and Peter Slater, *Fundamentals of Domination in Graphs*, Marcek Dekker, New York, (1998).