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On Soft Multi Games

Sreelekshmi C. Warrier[1,*]

1 Department of Mathematics, St.Thomas College, Palai, Kerala, India and MACFAST.

Abstract: Molodstov introduced the concept of soft set theory for modelling uncertainty and vagueness.In this paper, after introduce soft multi game and their basic operations. Then we extended some theories based on Nash equilibrium using soft multiset approach. Finally we give an application of decision making problem between three persons using soft multi Nash equilibrium method.

Keywords: Soft set, Soft multiset, Game theory, Soft game, Soft multi game.

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1. Introduction

One of most appropriate mathematical theory for modelling vagueness and uncertainty in decision making problems,the theory of soft set developed by Molodstov in 1999 [1]. The work on soft set theory is progressing rapidly in many fields. Maji and Roy showed that an application of soft set theory in decision making problems [4]. The traditional soft set is a mapping from parameter set into a crisp subset of universal set. In 1944 John Von Neuman, was introduced Game theory for modelling and designing automated decision making process in interactive environment [12]. In recent years, many interesting applications of game theory have been expanded by using the ideas of fuzzy sets. The important concept of game theory is the fundamentals of Nash equilibrium. Alireza and Nassar extended the Nash equilibrium set to fuzzy set in games with payoffs as fuzzy numbers [16].

The notions of soft games and fuzzy soft games established by Çağman and Deli [6, 8]. In soft game, the payoff functions are set valued functions, thus the solutions of such games obtained by using the operations of sets [6]. The solution of fuzzy soft games are obtained by using the operations of both soft and fuzzy sets [8]. But, there exists situations where we need to model decision making with elements from multiple universes. Hence to solve problems with multiset of universes introduced soft multisets as a generalization of Molodtsov's [1] soft set and defined its basic operations such as complement, union and intersection. The definition and basic operations of soft multiset was introduced by Alkhazaleh and Saleh [9]. Furthermore Babitha and Sunil defined some operations on soft multiset [10]. Recently Ibrahim and Balami applied the concept of soft multiset in decision making problems [13].

In this paper we propose soft multi game and then improve some theories of Nash equilibrium using the concept of soft multiset. The remaining of this paper is organized as follows. Section 1.1 contains preliminaries, Section 2 contains definitions and some related results about two person soft multi game and Section 3 define n-person soft multi game with an application. We conclude in Section 4.

* *E-mail: sreelekshmicwarrier@gmail.com*

1.1. Preliminaries

Definition 1.1 ([2]). *Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A, B \subseteq E$. A soft set F_A over U is a set defined by a function f_A representing a mapping F_A from E to$P(U)$ such that $f_A = \emptyset$ if $x \notin A$. Here f_A is called approximate function of the soft set F_A. A soft set over U can be represented by the set of ordered pairs $F_A = \{(x, f_A(x)); x \in E, f_A(x) \in P(U)\}$*

Example 1.2 ([2]). *Suppose that$U$=$\{u_1, u_2, u_3, u_4, u_5\}$ is the universe contains four cars under consideration in an agent and E =$\{x_1, x_2, x_3, x_4, x_5\}$ is the set of parameters, where x_1= modern, x_2= cheap x_3= large x_4= small, and x_5= beautiful. A customer to select a car from the agent, can construct a soft setF_A that describes the characteristic of cars according to own requests. Assume that*

$$f_A(x_1) = \{u_1, u_2\},$$

$$f_A(x_2) = \{u_1, u_3\},$$

$$f_A(x_3) = \{u_2, u_4\},$$

$$f_A(x_4) = \{u_1, u_5\},$$

$$f_A(x_5) = \{u_1, u_3, u_5\}$$

then the soft set F_A is written by

$$F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_3\}), (x_3, \{u_2, u_4\}), (x_4, \{u_1, u_5\}), (x_5, \{u_1, u_3, u_5\})\}.$$

Definition 1.3 ([9]). *Let $\{U_i; i \in I\}$ be an collection of universes such that $\cap_{i\in I} U_i = \emptyset$, and $\{E_i = E_{U_i} : i \in I\}$ be a collection of parameters. Let$E = \Pi E_i, U = \Pi P(U_i)$ and $A \subset E$. A soft multiset (F_A, E) over U is called soft multiset over U, where F_A is mapping from A to U.*

Definition 1.4 ([6]). *Let X, Y be the set of strategies. A choice behaviour of player is called an action. The elements of $X \times Y$ are called action pairs. That is $X \times Y$ is the set of available actions.*

Definition 1.5 ([6]). *Let X and Y be a set strategies of Player 1 and 2 respectively ,U be a set alternatives and f_{sk} is a function from $X \times Y$ to $P(U)$ be a soft pay off function for player$k(k = 1, 2)$. Then for each player k, a two person soft game (tps-game) is defined by a soft set over U as*

$$S_k = \{((x, y), f_{sk}(x, y)) : (x, y) \in X \times Y\}$$

The tps-game is played as follows: at certain time player 1select strategy $x_i \in X$, simultaneously Player 2 chooses a strategy $y_j \in Y$, so each player receives the soft pay off $f_{sk}(x_i, y_j)$.

Definition 1.6 ([6]). *Let S_k be a tps-game with its soft pay off function f_{sk} for $k = 1, 2$. If the following properties hold*

(a). $f_{s_1}(x^*, y^*) \supseteq f_{s_1}(x, y^*)$ for each $x \in X$.

(b). $f_{s_2}(x^*, y^*) \supseteq f_{s_2}(x^*, y)$ for each $y \in Y$.

then, $(x^*, y^*) \in (X \times Y)$ is called a soft Nash equilibrium of a tps-game.

Some other results and definitions related to game theory, soft set and soft multiset theories are found in [2, 6, 9, 12, 13, 15, 16].

2. Soft Multi Game

In multi-stage games, the same players are played sequentially and their total payoff can be evaluated from the finite sequences of such games.In this games, payoff values of the players do not directly affect payoff values of each stages. In the following section, proposed a new multi stage game model based on soft multiset theory called soft multi-game.

Definition 2.1. *Let U be the universal set, U_1 and U_2 the set of alternatives such that $U = U_1 \times U_2, U_1 \cap U_2 = \emptyset$ and X and Y be the strategies of player K, where $K = 1, 2$ and $E = X \times Y$, then a set valued function $f_{sm} : E \to U$ is called soft multi pay off function. Then for each player k, a two person soft multi game(tpsm-game) is defined by a soft multiset over U as $S_{mk} = \{((x, y), f_{smk}(x, y)) : (x, y) \in X \times Y\}$. The tpsm-game is played as follows : Player 1 choose strategy $x_i \in X$, simultaneously Player 2 choose a strategy $y_j \in Y$, at the same time player 1 receives the soft multi payoff $f_{sm1}(x_i, y_j)$ and player 2 receives the soft multi pay off $f_{sm2}(x_i, y_j)$. Then soft multi game for player k, can be represented as following table:*

S_{mk}	y_1	y_2	$\cdots$	y_n
x_1	$f_{smk}(x_1, y_1)$	$f_{smk}(x_1, y_2)$	$\cdots$	$f_{smk}(x_1, y_n)$
x_2	$f_{smk}(x_2, y_1)$	$f_{smk}(x_2, y_2)$	$\cdots$	$f_{smk}(x_2, y_n)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_m	$f_{smk}(x_m, y_1)$	$f_{smk}(x_m, y_2)$	$\cdots$	$f_{smk}(x_m, y_n)$

Example 2.2. *Suppose there are two universe U_1, U_2 which represents two set of alternatives of stage 1 and 2 such that $U_1 = \{u_1, u_2, u_3, u_4, u_5\}$ and $U_2 = \{v_1, v_2, v_3, v_4, v_5\}$. If $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3\}$ be set of strategies player 1 and player 2 respectively, where $E = X \times Y$ such that $f_{sm} : X \times Y \to U$. If player 1 construct a tpsm -game as follows*

$$S_{m1} = \{((x_1, y_1), (\{u_1, u_2, u_4\}, \{v_2, v_3\})), ((x_1, y_2), (\{u_2, u_4, u_5\}, \{v_1, v_2, v_3\})), ((x_2, y_2), (\{u_1, u_2\}, \{v_3, v_4\})),$$
$$((x_2, y_3), (\{u_3, u_4, u_5\}, \{v_1, v_5\})), ((x_3, y_1), (\{u_1, u_2, u_4, u_5\}, \{v_1, v_2, v_3\})), ((x_3, y_3), (\{u_3, u_4, u_5\}, \{v_4, v_5\}))\}.$$

If player 1 select the strategy x_3 and player 2 select the strategy y_1,then soft multi game will be a set $(\{u_1, u_2, u_4, u_5\}, \{v_1, v_2, v_3\}))\} \Rightarrow f_{sm1}(x_3, y_1) = (\{u_1, u_2, u_4, u_5\}, \{v_1, v_2, v_3\}))\}$. Similarly construct tpsm-game for player 2. If player 2 construct a tpsm -game as follows

$$S_{m2} = \{((x_1, y_1), (\{u_1, u_3, u_4\}, \{v_1, v_3\})), ((x_1, y_2), (\{u_2, u_4, u_5\}, \{v_1, v_2, v_3\})), ((x_2, y_2), (\{u_1, u_2\}, \{v_3, v_4\})),$$
$$((x_2, y_3), (\{u_3, u_4, u_5\}, \{v_1, v_5\})), ((x_3, y_1), (\{u_1, u_2, u_4\}, \{v_1, v_3\})), ((x_3, y_3), (\{u_2, u_4, u_5\}, \{v_1, v_5\}))\}.$$

If player 1 select the strategy x_3 and player 2 select the strategy y_1,then soft multi game will be a set $(\{u_1, u_2, u_4\}, \{v_1, v_3\}))\} \Rightarrow f_{sm2}(x_3, y_1) = (\{u_1, u_2, u_4\}, \{v_1, v_3\}))\}$.

Definition 2.3. *Let S_{mk} and S_{ml} be two person soft multi game. A two person soft multi game S_{mk} is a sub game of S_{ml}, if $f_{smk}(x, y) \subseteq f_{sml}(x, y)$ for all $x \in X, y \in Y$, where $f_{sm} : X \times Y \to U$.*

Definition 2.4. *Let S_{mk} and S_{ml} be two person soft multi game,then intersection of S_{mk} and S_{ml} can be denoted as $S_{mk} \cap S_{ml}$ and defined by $f_{smk}(x, y) = f_{sm1}(x_i, y_j) \cap f_{sm2}(x_i, y_j), where (x_i, y_j) \in X \times Y$.*

Definition 2.5. *Let S_{mk} and S_{ml} be two person soft multi game,then union of S_{mk} and S_{ml} can be denoted as $S_{mk} \cup S_{ml}$ and defined by $f_{smk}(x, y) = f_{sm1}(x_i, y_j) \cup f_{sm2}(x_i, y_j), where (x_i, y_j) \in X \times Y$.*

Example 2.6. *Consider Example 3.2, the intersection and union of S_{mk} and S_{ml} is* $f_{sm1}(x_3,y_1)\cap f_{sm2}(x_3,y_1) = (\{u_1,u_2,u_4\},\{v_1,v_3\}) \Rightarrow f_{sm1}(x_3,y_1)\cup f_{sm2}(x_3,y_1) = (\{u_1,u_2,u_4,u_5\},\{v_1,v_2,v_3\})$.

Definition 2.7. *Let $S_{mk} = \{((x,y), f_{smk}(x,y)) : (x,y) \in (X\times Y)$ be tpsm-game. Then an action $(x^*,y^*) \in X\times Y$ is called an optimal action if $f_{smk}(x^*,y^*) \supseteq f_{smk}(x,y)$, for all $(x,y)\in X\times Y$.*

Definition 2.8. *Let f_{smk} be soft multi pay off function of tpsm-game S_{mk}. If the following properties :*

(1). $\cup_{i=1}^{m} f_{smk}(x_i,y_j) = f_{smk}(x,y)$.

(2). $\cap_{j=1}^{n} f_{smk}(x_i,y_j) = f_{smk}(x,y)$,

then $f_{smk}(x,y)$ is called a saddle point of Player K in tpsm-game.

Example 2.9. *Let $U_1 = \{u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8,u_9,u_{10}\}$ and $U_2 = \{v_1,v_2,v_3,v_4,v_5,v_6,v_7\}$ are the set of alternatives of stage 1 and 2 respectively and P(U) be the power set of U. Suppose $X = \{x_1,x_2,x_3\}$ be set of strategies of player 1 and $Y = \{y_1,y_2,y_3\}$ be set of strategies of player 2. Then a tpsm-game for player 1 is defined as:*

$$\begin{aligned} S_1 = \{&((x_1,y_1),\{u_1,u_3,u_5,u_7\},\{v_2,v_3,v_5,v_6\}),((x_1,y_2),\{u_3,u_5,u_{10}\},\{v_6,v_7\}),((x_1,y_3),\{u_5,u_6,u_{10}\}\{v_2,v_5,v_6\}),\\ &((x_2,y_1),\{u_7\},\{v_4,v_5,v_6,v_7\}\}),((x_2,y_2),\{u_5,u_{10}\},\{v_6\}),((x_2,y_3),\{u_7,u_9,u_{10}\}\{v_5,v_6,v_7\}),((x_3,y_1),\\ &\{u_1,u_3,u_4,u_5,u_{10}\}\{v_2,v_6,v_7\}),((x_3,y_2),\{u_3,u_5,u_{10}\}\{v6,v_7\}),((x_3,y_3),\{u_1,u_3,u_5,u_6,u_{10}\}\{v5,v_6,v_7\}). \end{aligned}$$

Clearly,

$$\begin{aligned} \cup_{=1}^{3} f_{s1}(x_i,y_1) &= \{(\{u_1,u_3,u_4,u_5,u_7,u_{10}\}\{v_1,v_2,v_3,v_4,v_5,v_6,v_7\})\}\\ \cup_{i=1}^{3} f_{s1}(x_i,y_2) &= \{(\{u_3,u_5,u_{10}\}\{v_2,v_3,v_5,v_6,v_7\})\}\\ \cup_{i=1}^{3} f_{s1}(x_i,y_3) &= \{(\{u_1,u_3,u_5,u_6,u_7,u_9,u_{10}\})\}\{v_2,v_5,v_6,v_7\})\}\\ \cap_{j=1}^{3} f_{s1}(x_1,y_j) &= \{(\{u_5\}\{v_6\})\}\\ \cap_{j=1}^{3} f_{s1}(x_2,y_j) &= \{(\{(u_{10}\}\{v_6\})\}\\ \cap_{j=1}^{3} f_{s1}(x_3,y_j) &= \{(\{u_3,u_5,u_{10}\}\{v_6,v_7\})\} \end{aligned}$$

Therefore $\{(\{u_3,u_5,u_{10}\},\{v_6,v_7\})\}$ is a soft multi saddle point of the tpsm-game. So the value of the tpsm-game is $\{(\{u_3,u_5,u_{10}\}\{v_6,v_7\})\}$.

Definition 2.10. *Let f_{smk} be soft multi pay off function of tpsm-game S_{mk},then Max-min value of the tpsm-game be denoted as Max- min of $f_{smk}(x,y) = \bigcup_{x\in X}(\bigcap_{y\in Y} f_{smk}(x,y))$ and Min- max value of the tpsm-game be denoted as Min max of $f_{smk}(x,y) = \bigcap_{y\in Y}(\bigcup_{x\in X} f_{smk}(x,y))$.*

Theorem 2.11. *If Max-min and Min -max value exists in tpsm-game then Max- min value $\subseteq$ min max value.*

Proof. Consider a tpsm-game $S_{smk} = \{((x,y), f_{smk}(x,y)) : (x,y) \in (X\times Y)$. Let $X = \{x_1,x_2,...x_m\}$ and $Y = \{y_1,y_2,y_3,...y_n\}$ are strategies for players 1 and 2 respectively. We select $x_i^* \in X$ and $y_j^* \in Y$. Then

$$\begin{aligned} Max-min\ value &= \bigcup_{x\in X}(\bigcap_{y\in Y} f_{smk}(x,y)) \subseteq \bigcap_{y\in Y} f_{X\times Y}(x^*,y) \subseteq f_{X\times Y}(x^*,y^*) \subseteq \bigcup_{x\in X} f_{X\times Y}(x,y^*) \subseteq \bigcap_{y\in Y}(\bigcup_{x\in X} f_{smk}(x,y))\\ &= Min-max\ value. \end{aligned}$$

□

Definition 2.12. *Let $S_{smk} = \{((x, y), f_{smk}(x, y)) : (x, y) \in (X \times Y)$ be tpsm-game with soft multi pay off function $f_{smk}, fork = 1, 2$ if*

(1). $f_{sm1}(x^*, y^*) \supseteq f_{sm1}(x, y^*)$ for each $x \in X$

(2). $f_{sm2}(x^*, y^*) \supseteq f_{sm2}(x^*, y)$ for each $y \in Y$

then $(x^*, y^*) \in (X \times Y)$ be soft multi Nash equilibrium of tpsm-game.

Definition 2.13. *Let $S_{smk} = \{((x, y), f_{smk}(x, y), \succeq) : (x, y) \in (X \times Y)$ is strictly competitive if for any (x, y) and $(x^1, y^1) \in (X, Y)$ we have $(x, y) \succeq_1 (x^1, y^1)$ iff $(x^1, y^1) \succeq_2 (x, y)$.*

Definition 2.14. *Let $S_{smk} = \{((x, y), f_{smk}(x, y), \succeq) : (x, y) \in (X \times Y)$ be strictly competitive soft strategic tpsm-game. Then action $x^* \in X$ is Maxi-minimizer for player 1*

$$\bigcap_{y \in Y} f_{sm1}(x^*, y) \supseteq \bigcap_{y \in Y} f_{sm1}(x, y), \quad \forall \ \ x \in X$$

Similarly action $y^* \in Y$ is Maxi-minimizer for player 2

$$\bigcap_{x \in X} f_{sm2}(x^*, y) \supseteq \bigcap_{x \in X} f_{sm2}(x, y), \quad \forall \ \ y \in Y.$$

Theorem 2.15. *Let $S_{smk} = \{((x, y), f_{smk}(x, y), \succeq) : (x, y) \in (X \times Y)$ be strictly competitive soft strategic tpsm-game.*

(1). If (x^*, y^*) is a Nash equilibrium of S_{mk} then x^* is a Maxi-minimizer for player 1 and y^* is a Maxi-minimizer for player 2.

(2). If (x^*, y^*) is a Nash equilibrium of S_{mk} then $\bigcup_{x \in X}(\bigcap_{y \in Y} f_{sm1}(x, y)) = \bigcap_{y \in Y}(\bigcup_{x \in X} f_{sm1}(x, y)) = f_{sm1}(x^*, y^*)$.

Proof. Let $S_{smk} = \{((x, y), f_{smk}(x, y), \succeq) : (x, y) \in (X \times Y)$ be strictly competitive soft strategic tpsm-game.Let (x^*, y^*) is a Nash equilibrium of S_{mk}. Then $f_{sm1}(x^*, y^*) \supseteq \bigcap_{y \in Y} f_{sm1}(x^*, y) \supseteq \bigcap_{y \in Y} f_{sm1}(x, y) \supseteq \bigcup_{x \in X}(\bigcap_{y \in Y} f_{sm1}(x, y))$ that is

$$f_{sm1}(x^*, y^*) \supseteq \bigcup_{x \in X}(\bigcap_{y \in Y} f_{sm1}(x, y)) \tag{1}$$

Also $f_{sm1}(x^*, y^*) \subseteq \bigcap_{y \in Y} f_{sm1}(x^*, y) \subseteq \bigcap_{y \in Y} f_{sm1}(x, y) \subseteq \bigcup_{x \in X}(\bigcap_{y \in Y} f_{sm1}(x, y))$ that is

$$f_{sm1}(x^*, y^*) \subseteq \bigcup_{x \in X}(\bigcap_{y \in Y} f_{sm1}(x, y)) \tag{2}$$

From Equations (1) and (2), we get $f_{sm1}(x^*, y^*) = \bigcup_{x \in X}(\bigcap_{y \in Y} f_{sm1}(x, y)) \subseteq \bigcap_{y \in Y}(\bigcup_{x \in X} f_{sm1}(x, y))$ and since (x^*, y^*) is a Nash equilibrium of S_{mk} then $f_{sm1}(x^*, y^* \supseteq \bigcap_{y \in Y}(\bigcup_{x \in X} f_{sm1}(x, y))$. Hence $f_{sm1}(x^*, y^*) = \bigcup_{x \in X}(\bigcap_{y \in Y} f_{sm1}(x, y)) = \bigcap_{y \in Y}(\bigcup_{x \in X} f_{sm1}(x, y)$. Therefore $f_{sm1}(x^*, y^*) = \bigcup_{x \in X}(\bigcap_{y \in Y} f_{sm1}(x, y)) = \bigcap_{y \in Y}(\bigcup_{x \in X} f_{sm1}(x, y))$ and x^* is a Maxi-minimizer for player 1 and y^* is a Maxi-minimizer for player 2 of strictly competitive soft strategic tpsm-game. □

3. n-Person Soft Multi Game

In many applications the soft multi game can be often played between more than two players. Therefore, we extended tpsm-game to npsm-games.

Definition 3.1. *Let U be the set of universes,U_1 and U_2 be the set of alternatives such that $u = U_1 \times U_2, U_1 \cap u_2 = \Phi$ and X_k be the set of strategies of Player k, $(k = 1, 2, 3, ...n)$. Then , for each Player k, an n-person soft multi game (npsm-game) is defined by a soft multi set over U as $S^n_{mk} = \{((x_1, x_2, x_3, ...x_n), fs^n_{mk}(x_1, x_2, x_3, ...x_n)) \in X_1 \times X_2 \times X_3 ... \times X_n\}$.*

Definition 3.2. *Let $S^n_{mk} = \{((x_1, x_2, x_3, ...x_n), fs^n_{mk}(x_1, x_2, x_3, ...x_n)) \in X_1 \times X_2 \times X_3 ... \times X_n\}$ be an n-person soft multi game(npsm-game). If for each player $k, (k = 1, 2, ...n)$ the satisfies the following property:*
$f^n_{Smk}(x^*_1, x^*_2, ...x^*_{k-1}, x^*_k, x^*_{k+1}, ...x^*_n) \supseteq f^n_{Smk}(x^*_1, x^*_2, ...x^*_{k-1}, x^*, x^*_{k+1}, ...x^*_n)$, for each $x \in X_k$, then $(x^*_1, x^*_2, ...x^*_n) \in S^n_{mk}$ is called a soft multi Nash equilibrium of an n-person soft multi game.

3.1. Application

In the following section ,we discussed a financial problem that are solved by using soft multi Nash equilibrium method.
There are three Auto mobile companies: Maruti, Hyundai, Toyota, as Player 1, Player 2 and Player 3 respectively in a soft multi game, who competitively want to increase the sale of their products (ie., cars). To do this they give promotions and advertisements. Assume that 3 Auto mobile companies have a set of 3 different types of cars say;
$U_{Maruti} = \{m_1, m_2, m_3, m_4, m_5\}$, where for $i = 1, 2, 3, 4, 5$; the products m_i stands for '$Maruti_{Swift}$, $Maruthi_{Celerio}$, $Maruthi_{Alto}$, $Maruthi_{Ritz}$, $Maruthi_{Breeza}$'.
$U_{Hyundai} = \{h_1, h_2, h_3, h_4, h_5\}$, where for $j = 1, 2, 3, 4, 5$; the products h_i stands for '$Hyundai_{i10}$, $Hyundai_{Eon}$, $Hyundai_{i20}$, $Hyundai_{Creta}$, $Hyundai_{Verna}$' and
$U_{Tyto} = \{t_1, t_2, t_3, t_4, t_5\}$, where for $p = 1, 2, 3, 4, 5$; the products t_p stands for '$Tyoto_{Fortuner}$, $Tyoto_{Inova}$, $Tyoto_{Etioscross}$, $Tyoto_{PlaniumEtios}$, $Tyoto_{Landcruiser}$'.
Let $X_1 = \{x_{11}, x_{12}, x_{13}\}$ be the strategies of Player 1. The $\{x_{1i}, i = 1, 2, 3\}$ stands for promotions such as "Exchange offer", "festival offer ", "monsoon offer" respectively, $X_2 = \{x_{21}, x_{22}, x_{23}\}$ be the strategies of Player 2.The $\{x_{2j}, j = 1, 2, 3\}$ stands for media advertisements through "News paper", "TV" and "online " respectively. And $X_3 = \{x_{31}, x_{32}, x_{33}\}$ be the strategies of Player 3. The $\{x_{3k}, k = 1, 2, 3\}$ stands for marketing through "exhibition", "flex " and "broucher " respectively.
Then three person soft multi game of Player 1 is given as:

$$
\begin{aligned}
S^3_1 = \{&((x_{11}, x_{21}, x_{31}), (\{m_1, m_3, m_4, m_5\}, \{h_1, h_2, h_3, h_4\}, \{t_1, t_2, t_3, t_5\})), ((x_{12}, x_{22}, x_{32}), (\{m_3, m_4, m_5\},\\
&\{h_1, h_2, h_3\}, \{t_1, t_2, t_3\}))((x_{13}, x_{23}, x_{33}), (\{m_1, m_5\}, \{h_2, h_3, h_4\}, \{t_3, t_5\})), ((x_{12}, x_{21}, x_{33}), (\{m_1, m_3, m_5\},\\
&\{h_3, h_4, h_5\}, \{t_1, t_3, t_5\})), ((x_{13}, x_{22}, x_{33}), (\{m_1, m_2\}, \{h_2, h_4, h_5\}, \{t_2, t_3\}))((x_{12}, x_{21}, x_{31}), (\{m_1, m_3, m_4\},\\
&\{h_3, h_4\}, \{t_1, t_3, t_4\})), ((x_{11}, x_{22}, x_{31}), (\{m_3, m_4, m_5\}, \{h_2, h_4\}, \{t_2, t_3, t_5\}))((x_{13}, x_{21}, x_{31}), (\{m_1, m_3, m_4\},\\
&\{h_1, h_4\}, \{t_2, t_3\}))\}.
\end{aligned}
$$

The Player 2 constructs three person soft multi-game is given as:

$$
\begin{aligned}
S^3_2 = \{&((x_{11}, x_{21}, x_{31}), (\{m_1, m_3, m_4, m_5\}, \{h_2, h_3, h_4, h_5\}, \{t_1, t_2, t_4, t_5\})), ((x_{12}, x_{22}, x_{32}), (\{m_2, m_4, m_5\},\\
&\{h_2, h_3, h_5\}, \{t_2, t_3, t_5\}))((x_{13}, x_{23}, x_{33}), (\{m_2, m_4, m_5\}, \{h_1, h_3, h_4\}, \{t_1, t_5\})), ((x_{12}, x_{21}, x_{33}), (\{m_2, m_3, m_4\},\\
&\{h_3, h_4, h_5\}, \{t_1, t_3, t_5\})), ((x_{13}, x_{22}, x_{33}), (\{m_1, m_4\}, \{h_1, h_4, h_5\}, \{t_3, t_4\}))((x_{12}, x_{21}, x_{31}), (\{m_1, m_2\},\\
&\{h_2, h_3, h_4\}, \{t_2, t_3, t_4\})), ((x_{11}, x_{22}, x_{31}), (\{m_4, m_5\}, \{h_2, h_4\}, \{t_2, t_4\}))((x_{13}, x_{21}, x_{31}), (\{m_2, m_3, m_4\},\\
&\{h_1, h_3, h_5\}, \{t_3, t_4, t_5\}))\}, ((x_{11}, x_{23}, x_{31}), (\{m_4, m_5\}, \{h_2, h_4\}, \{t_2, t_4\}))
\end{aligned}
$$

The Player 3 constructs three person soft multi-game is given as:

$$S_3^3 = \{((x_{11}, x_{21}, x_{31}), (\{m_2, m_3, m_4, m_5\}, \{h_1, h_2, h_4, h_5\}, \{t_1, t_2, t_3, t_5\})), ((x_{12}, x_{22}, x_{32}), (\{m_3, m_4, m_5\},$$
$$\{h_1, h_2, h_3\}, \{t_1, t_2, t_3\}))((x_{13}, x_{23}, x_{33}), (\{m_2, m_5\}, \{h_2, h_3, h_4\}, \{t_3, t_5\})), ((x_{12}, x_{21}, x_{33}), (\{m_1, m_3, m_5\},$$
$$\{h_3, h_4, h_5\}, \{t_1, t_3, t_5\})), ((x_{13}, x_{22}, x_{33}), (\{m_1, m_2\}, \{h_2, h_4, h_5\}, \{t_2, t_3\}))((x_{12}, x_{21}, x_{31}), (\{m_1, m_2, m_4\},$$
$$\{h_3, h_4\}, \{t_1, t_3, t_4\})), ((x_{11}, x_{21}, x_{32}), (\{m_3, m_4, m_5\}, \{h_2, h_4\}, \{t_3, t_4, t_5\}))((x_{11}, x_{21}, x_{33}), (\{m_2, m_3, m_4\},$$
$$\{h_1, h_4\}, \{t_1, t_5\}))\}.$$

From the above sets, we get,

$$f_{S1}(x_{11}, x_{21}, x_{31}) \supseteq f_{S1}(x_{1i}, x_{21}, x_{31}) \quad for\ each \quad x_{1i} \in X_1.$$
$$f_{S2}(x_{11}, x_{21}, x_{31}) \supseteq f_{S2}(x_{11}, x_{2j}, x_{31}) \quad for\ each \quad x_{2j} \in X_2.$$
$$f_{S3}(x_{11}, x_{21}, x_{31}) \supseteq f_{S3}(x_{11}, x_{21}, x_{3k}) \quad for\ each \quad x_{2k} \in X_3.$$

then $(x_{11}, x_{21}, x_{31}) \in (X_1 \times X_2 \times X_3)$ is a soft multi Nash equilibrium. Therefore

$$f_{S1}(x_{11}, x_{21}, x_{31}) = \{(\{m_1, m_3, m_4, m_5\}, \{h_1, h_2, h_3, h_4\}, \{t_1, t_2, t_3, t_5\})\}$$
$$f_{S2}(x_{11}, x_{21}, x_{31}) = \{(\{m_1, m_3, m_4, m_5\}, \{h_2, h_3, h_4, h_5\}, \{t_1, t_2, t_4, t_5\})\}$$
$$f_{S3}(x_{11}, x_{21}, x_{31}) = \{(\{m_2, m_3, m_4, m_5\}, \{h_1, h_2, h_4, h_5\}, \{t_1, t_2, t_3, t_5\})\}$$

are the solutions of the three person soft-multi game for Player 1,Player 2 and Player 3 respectively.

4. Conclusion

A soft set is a mapping from parameter to the crisp subset of universe. In this work, we have defined two person soft multi game and their operations. We then extended two person soft multi game into n-person soft multi game. Finally, we provided a real world example demonstrating the successful application of our concept. This method is suitable for multi-universe problems that contain uncertainty and it would be ideal to extend this work to subsequent studies as well. The soft multi strategic games may be applied in to many fields and it has many applications in future to solve problems in Decision making problems,Computer science, etc.

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