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Identities with Multiplicative (Generalized) (α, β)-derivations in Semiprime Rings

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Abstract: Let R be an associative ring and α, β be automorphisms on R. A mapping $F : R \to R$ (not necessarily additive) is said to be a multiplicative (generalized)-(α, β)-derivation if $F(xy) = F(x)\alpha(y) + \beta(x)d(y)$ holds for all $x, y \in R$, where d is any mapping on R. Suppose that G and F are multiplicative (generalized)-(α, β)-derivations associated with the mappings g and d on R respectively. The main objective of this article is to study the following situations: (i) $G(xy) + F[x, y] \pm \alpha[x, y] = 0$; (ii) $G(xy) + F(x \circ y) \pm \alpha(x \circ y) = 0$; (iii) $G(xy) + F(x)F(y) \pm \alpha(xy) = 0$; (iv) $G(xy) + F(x)F(y) \pm \alpha[x, y] = 0$; (v) $G(xy) + F(x)F(y) \pm \alpha(x \circ y) = 0$; for all x, y in some non-zero subsets of a semiprime ring.

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1. Introduction

Throughout the paper, R will denote an associative ring with center $Z(R)$. Recall that a ring R is prime if for any $a, b \in R$, $aRb = 0$ implies that either $a = 0$ or $b = 0$ and is called semiprime if for any $a \in R$, $aRa = 0$ implies that $a = 0$. We shall write for any pair of elements $x, y \in R$, the commutator $[x, y] = xy - yx$ and skew commutator $x \circ y = xy + yx$.

An additive mapping $d : R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. Following [1], an additive mapping $H : R \to R$ is called a left (resp. right) multiplier (centralizer) of R if $H(xy) = H(x)y$ (resp. $H(xy) = xH(y)$) holds for all $x, y \in R$. If H is both left as well as right multiplier, then it is called a multiplier. The concept of a derivation was extended to generalized derivation in [6] by Bresar. An additive mapping $F : R \to R$ is said to be a generalized derivation if there exists a derivation $d : R \to R$ such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$. Obviously generalized derivation with $d = 0$ covers the concept of left multiplier.

Let us introduce the background of investigation about multiplicative generalized derivation. A mapping $d : R \to R$ which satisfies $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$ is called a multiplicative derivation of R. Ofcourse these mappings are not additive. To the best of my knowledge, the concept of multiplicative derivation appeared for the first time in the work of Daif [7]. Then the complete description of those maps was given by Goldman and Semrl in [4]. Further, Daif and Tammam-El-Sayiad [9] extended the notion of multiplicative derivation to multiplicative generalized derivation as follows: A mapping $F : R \to R$ (not necessarily additive) is called a multiplicative generalized derivation if it satisfies $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$, where d is a derivation on R. Obviously, every generalized derivation is a multiplicative generalized derivation

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on R. Chang [5] introduced the notion of a generalized (α, β)-derivation of a ring R and investigated some properties of such derivations. Let α, β be mappings of R into itself. An additive mapping $F : R \to R$ is called a generalized (α, β)-derivation of R, if there exists an (α, β) derivation d of R such that $F(xy) = F(x)\alpha(y) + \beta(x)d(y)$ for all $x, y \in R$. A mapping $F : R \to R$ is said to be a multiplicative (generalized)-(α, β)-derivation if there exists a map f on R such that $F(xy) = F(x)\alpha(y)+\beta(x)f(y)$ for all $x, y \in R$. Obviously every generalized (α, β)-derivation is a multiplicative (generalized)-(α, β)-derivation. Albas [3] introduced the notion of α-multipliers (centralizers) of R, i.e., an additive mapping $H : R \to R$ is called a left (resp. right) α-multiplier(centralizer) of R if $H(xy) = H(x)\alpha(y)$ (resp. $\alpha(x)H(y)$) holds for all $x, y \in R$, where α is an endomorphism of R. If H is both left as well as right α-multiplier then it is called an α-multiplier. Obviously every generalized (α, β)-derivation with $d = 0$ covers the concept of left α multiplier.

In 1992, Daif [8], proved a result that if R is a semiprime ring , I be a non-zero ideal of R and d is a derivation of R such that $d([x, y]) = \pm[x, y]$ for all $x, y \in I$, then $I \subseteq Z(R)$. Quadri [10] extended the result of Daif by replacing derivation d with a generalized derivation in a prime ring. Further, Dhara [2] proved the following result: Let R be a semiprime ring, I a non-zero ideal of R and F be a generalized derivation of R with associated derivation d satisfying $F[x, y] \pm [x, y] = 0$ or $F(x \circ y) \pm (x \circ y) = 0$ for all $x, y \in I$, then R must contain a non-zero central ideal, provided $d(I) \neq 0$. In case R is prime satisfying $F[x, y] \pm [x, y] \in Z(R)$ or $F(x \circ y) \pm (x \circ y) \in Z(R)$ for all $x, y \in I$, then R must be commutative provided $d(Z(R)) \neq 0$.

Recently, Shuliang [11] studied the following identities related on generalized (α, β) derivation on prime rings : (i) $F[x, y] \pm \alpha[x, y] \in Z(R)$, (ii) $F(x\circ y)\pm\alpha(x\circ y) \in Z(R)$, (iii) $F(xy)\pm\alpha(xy) \in Z(R)$, (iv) $F[x, y] = 0$, (v) $F(x\circ y) = 0$, for all x, y in some appropriate subset of prime ring R. In this line of investigation, in the present article our aim is to extend the above mentioned results studying the following identities involving multiplicative (generalized)-(α, β)-derivation. (i) $G(xy)+F[x, y]\pm\alpha[x, y] = 0$, (ii) $G(xy) + F(x \circ y) \pm \alpha(x \circ y) = 0$, (iii) $G(xy) + F(x)F(y) \pm \alpha(xy) = 0$, (iv) $G(xy) + F(x)F(y) \pm \alpha[x, y] = 0$, (v) $G(xy) + F(x)F(y) \pm \alpha(x \circ y) = 0$, for all x, y in some non-zero subsets of a semiprime ring.

1.1. Preliminary Result

Definition 1.1. *Let R be a ring, we need the following basic identities which will be used in the proof of our results. For any $x, y, z \in R$,*

$$\begin{aligned}
[xy, z] &= x[y, z] + [x, z]y \text{ and } [x, yz] = y[x, z] + [x, y]z. \\
x \circ (yz) &= (x \circ y)z - y[x, z] = y(x \circ z) + [x, y]z. \\
(xy) \circ z &= x(y \circ z) - [x, z]y = (x \circ z)y + x[y, z].
\end{aligned}$$

Lemma 1.2. *Let R be a semiprime ring, I a non-zero ideal of R and β an epimorphism of R. Let $d : R \to R$ be an additive mapping of R such that $\beta(I)d(I) \neq 0$. If $[R, \beta(x)]\beta(I)d(x) = \{0\}$, for all $x \in I$, then R contains a non zero central ideal.*

Proof. By our assumption, we have

$$[R, \beta(x)]\beta(I)d(x) = \{0\} \text{ for all } x \in I.$$

Since β is an epimorphism of R, we have

$$[R, \beta(x)]R\beta(I)d(x) = \{0\}.$$

Since a semiprime ring R contains collection of prime ideals $\mathcal{P} = \{P_\alpha | \alpha \in \wedge\}$ such that $\cap P_\alpha = \{0\}$. Thus for any P_α and $x \in I$ either $[R, \beta(x)] \subseteq P_\alpha$ or $\beta(I)d(x) \subseteq P_\alpha$. These two forms an additive subgroups of I whose union is I. Thus in any case, we have $[R, \beta(I)]\beta(I)d(I) \subseteq P_\alpha$. Therefore $[R, \beta(I)]\beta(I)d(I) \subseteq \cap P_\alpha$. This implies that $[R, \beta(I)]\beta(I)d(I) = \{0\}$. Thus we have $[R, \beta(RIR)]\beta(RI)d(I) = \{0\}$. Since β is an epimorphism of R, we get $[R, R\beta(I)R]R\beta(I)d(I) = \{0\}$. This implies that $[R, R\beta(I)d(I)R]R\beta(I)d(I)R = \{0\}$. We can write this as $[R, J]RJ = \{0\}$, where $J = R\beta(I)d(I)R$ is a nonzero ideal of R, since we have $\beta(I)d(I) \neq \{0\}$. This implies that $[R, J]R[R, J] = \{0\}$. By semiprimeness of R, we conclude that $[R, J] = \{0\}$. Therefore, J is commutative. Thus $J \subseteq Z(R)$. □

2. Main Section

Theorem 2.1. *Let R be a semiprime ring, I a non-zero ideal of R, α and β two epimorphisms of R. Suppose that G and F are two multiplicative (generalized)-(α, β)-derivations on R associated with the mappings g and d on R respectively, where d is an additive map. If $G(xy) + F[x, y] \pm \alpha[x, y] = 0$ for all $x, y \in I$ and $\beta(I)d(I) \neq 0$, then R contains a non-zero central ideal.*

Proof. By hypothesis,

$$G(xy) + F[x, y] \pm \alpha[x, y] = 0 \text{ for all } x, y \in I. \tag{1}$$

Replacing y by yx, we obtain that

$$G(xy)\alpha(x) + \beta(xy)g(x) + F[x, y]\alpha(x) + \beta[x, y]d(x) \pm \alpha[x, y]\alpha(x) = 0.$$

Using (1), it gives

$$\beta(xy)g(x) + \beta[x, y]d(x) = 0 \text{ for all } x, y \in I. \tag{2}$$

Putting $y = ry$ in (2), we get

$$\beta(x)\beta(r)\beta(y)g(x) + \beta(r)\beta[x, y]d(x) + \beta[x, r]\beta(y)d(x) = 0 \text{ for all } x, y \in I \text{ and } r \in R. \tag{3}$$

Left multiplying (2) by $\beta(r)$ and subtracting from (3), we obtain

$$[\beta(x), \beta(r)]\beta(y)g(x) + [\beta(x), \beta(r)]\beta(y)d(x) = 0 \tag{4}$$

Replacing y by xy in (4), we obtain

$$[\beta(x), \beta(r)]\beta(x)\beta(y)g(x) + [\beta(x), \beta(r)]\beta(x)\beta(y)d(x) = 0. \tag{5}$$

Left multiplying (2) by $[\beta(x), \beta(r)]$, we have

$$[\beta(x), \beta(r)]\beta(x)\beta(y)g(x) + [\beta(x), \beta(r)]\beta(x)\beta(y)d(x) - [\beta(x), \beta(r)]\beta(y)\beta(x)d(x) = 0. \tag{6}$$

Using (5) in (6), we get

$$[\beta(x), \beta(r)]\beta(y)\beta(x)d(x) = 0 \text{ for all } x, y \in I \text{ and } r \in R.$$

Since β is an epimorphism of R, above relation yields that

$$[\beta(x), R]R\beta(I)\beta(I)d(x) = 0 \text{ for all } x \in I. \tag{7}$$

Now take a family $\{P_\alpha\}$ of prime ideals of R such that $\cap P_\alpha = \{0\}$. Thus for any P_α either $[R, \beta(x)] \subseteq P_\alpha$ or $\beta(I)\beta(I)d(x) \subseteq P_\alpha$. For each $x \in I$, these two forms an additive subgroup of R whose union is R, Thus in any case, we get $[R, \beta(I)]\beta(I)d(I) \subseteq P_\alpha$. Therefore we have $[R, \beta(I)]\beta(I)d(I) \subseteq \cap P_\alpha = \{0\}$. Thus we have $[R, \beta(I)]\beta(I)d(I) = \{0\}$. Now by using Lemma 1.2, we get the required result. □

Corollary 2.2. *Let R be a prime ring, I a non-zero ideal of R, α and β two epimorphisms of R such that $\beta(I) \neq 0$. Suppose that G and F are two multiplicative (generalized)-(α, β)-derivations associated with the mappings g and d on R respectively. If $G(xy) + F[x, y] \pm \alpha[x, y] = 0$ for all $x, y \in I$, then either R is commutative or $g(I) = 0$.*

Proof. By Theorem 2.1 and using primeness of R, we have either $d(I) = 0$ or R is commutative. If $d(I) = 0$ in (2), we get $\beta(x)\beta(y)g(x) = 0$, for all $x, y \in I$. Again using primeness of R, we get either $\beta(I) = 0$ or $g(I) = 0$. By hypothesis $\beta(I)$ is a non-zero ideal of R and since β is an epimorphism of R, we get $g(I) = 0$. □

Theorem 2.3. Let R be a semiprime ring, I a non-zero ideal of R and α be an epimorphism on R. Suppose that G and F are two multiplicative (generalized)-(α, β)-derivations on R associated with the mappings g and d on R respectively, where d is an additive map. If $G(xy) + F(x \circ y) \pm \alpha(x \circ y) = 0$ for all $x, y \in I$ and $\beta(I)d(I) \neq 0$, then R contains a non-zero central ideal.

Proof. By hypothesis

$$G(xy) + F(x \circ y) \pm \alpha(x \circ y) = 0 \text{ for all } x, y \in I. \tag{8}$$

Replacing y by yx in (8), we get

$$G(xy)\alpha(x) + \beta(xy)g(x) + F(x \circ y)\alpha(x) + \beta(x \circ y)d(x) \pm \alpha(x \circ y)\alpha(x) = 0.$$

Using (8), we obtain

$$\beta(xy)g(x) + \beta(x \circ y)d(x) = 0 \text{ for all } x, y \in I. \tag{9}$$

Replacing y by ry in (9), we get

$$\beta(x)\beta(r)\beta(y)g(x) + \beta(r)\beta(x \circ y)d(x) + [\beta(x), \beta(r)]\beta(y)d(x) = 0 \text{ for all } x, y \in I \text{ and } r \in R. \tag{10}$$

Left multiplying (9) by $\beta(r)$ and subtracting from (10), we obtain

$$[\beta(x), \beta(r)]\beta(y)g(x) + [\beta(x), \beta(r)]\beta(y)d(x) = 0. \tag{11}$$

which is same as (4) of Theorem 2.1. Then by same argument, we obtain our conclusion. □

Theorem 2.4. *Let R be a semiprime ring, I a non-zero ideal of R, α and β two epimorphisms of R. Suppose that G and F are two multiplicative (generalized)-(α, β)-derivations associated with the mappings g and d on R respectively. If $G(xy) + F(x)F(y) \pm \alpha(xy) = 0$ for all $x, y \in I$, then F and G are left α- multipliers on I.*

Proof. We begin with the hypothesis,

$$G(xy) + F(x)F(y) \pm \alpha(xy) = 0. \tag{12}$$

Replacing y by yz, we get

$$G(xy)\alpha(z) + \beta(xy)g(z) + F(x)(F(y)\alpha(z) + \beta(y)d(z)) \pm \alpha(xy)\alpha(z) = 0.$$
$$(G(xy) + F(x)F(y) \pm \alpha(xy))\alpha(z) + \beta(xy)g(z) + F(x)\beta(y)d(z) = 0.$$

Using (12), we get

$$\beta(xy)g(z) + F(x)\beta(y)d(z) = 0. \tag{13}$$

Replacing y by ry, we obtain

$$\beta(x)r\beta(y)g(z) + F(x)r\beta(y)d(z) = 0. \tag{14}$$

Substituting xr for x in (13), we get that

$$\beta(x)r\beta(y)g(z) + F(x)r\beta(y)d(z) + \beta(x)d(r)\beta(y)d(z) = 0. \tag{15}$$

Subtracting (14) from (15), we obtain

$$\beta(x)d(r)\beta(y)d(z) = 0 \text{ for all } x, y, z \in I \text{ and } r \in R.$$

In particular for $r = z$, we have $\beta(x)d(z)\beta(y)d(z) = 0$. This implies that $\beta(x)d(z)R\beta(y)d(z) = 0$. Replacing y by x, $\beta(x)d(z)R\beta(x)d(z) = 0$, for all $x, z \in I$. By semiprimeness of R, we have $\beta(I)d(I) = 0$. Since we have $F(xy) = F(x)\alpha(y)+\beta(x)d(y)$, using $\beta(I)d(I) = 0$, we obtain $F(xy) = F(x)\alpha(y)$. Using $\beta(I)d(I) = 0$ in (13), we get $\beta(x)\beta(y)g(z) = 0$. Replacing y by ry, we get $\beta(x)r\beta(y)g(z) = 0$. Substituting $g(z)r$ for r in the last expression, we have $\beta(x)g(z)R\beta(y)g(z) = 0$. In particular, $\beta(x)g(z)R\beta(x)g(z) = 0$ for all $x, z \in I$. By semiprimeness of R, we have $\beta(I)g(I) = 0$. Thus we obtain $G(xy) = G(x)\alpha(y)$. □

Theorem 2.5. Let R be a semiprime ring, I a non-zero ideal of R and α an epimorphism of R. Suppose that G and F are two multiplicative (generalized)-(α, α)-derivations associated with the mappings g and d on R respectively. If $G(xy) + F(x)F(y) \pm \alpha[x, y] = 0$ for all $x, y \in I$, then $\alpha(I)[d(z), \alpha(z)] = 0$ and $\alpha(I)[g(z), \alpha(z)] = 0$, for all $z \in I$.

Proof. We begin with the hypothesis,

$$G(xy) + F(x)F(y) - \alpha[x, y] = 0 \text{ for all } x, y \in I. \tag{16}$$

Substituting yz in place of y in (16), we obtain

$$G(xyz) + F(x)F(yz) - \alpha[x, yz] = 0 \quad \text{for all } x, y, z \in I.$$
$$G(xy)\alpha(z) + \alpha(xy)g(z) + F(x)(F(y)\alpha(z) + \alpha(y)d(z)) - \alpha(y[x, z] + [x, y]z) = 0,$$
$$(G(xy) + F(x)F(y) - \alpha[x, y])\alpha(z) + \alpha(xy)g(z) + F(x)\alpha(y)d(z) - \alpha(y)\alpha[x, z] = 0.$$

Using (16), it gives

$$\alpha(xy)g(z) + F(x)\alpha(y)d(z) - \alpha(y)\alpha[x, z] = 0. \tag{17}$$

Substituting ry instead of y for $r \in R$ in (17), we get

$$\alpha(x)r\alpha(y)g(z) + F(x)r\alpha(y)d(z) - r\alpha(y)\alpha[x, z] = 0. \tag{18}$$

Now replacing x with xr in (17), we get

$$\begin{aligned} \alpha(x)r\alpha(y)g(z) + F(xr)\alpha(y)d(z) - \alpha(y)\alpha[xr, z] = 0, \\ \alpha(x)r\alpha(y)g(z) + (F(x)r + \alpha(x)d(r))\alpha(y)d(z) - \alpha(y)\alpha(x[r, z] + [x, z]r) = 0, \\ \alpha(x)r\alpha(y)g(z) + F(x)r\alpha(y)d(z) + \alpha(x)d(r)\alpha(y)d(z) - \alpha(y)\alpha(x)\alpha[r, z] - \alpha(y)\alpha[x, z]r = 0. \end{aligned} \tag{19}$$

Subtracting (18) from (19), we obtain

$$\alpha(x)d(r)\alpha(y)d(z) - \alpha(y)\alpha(x)\alpha[r, z] - [\alpha(y)\alpha[x, z], r] = 0. \tag{20}$$

Putting $y = ry$ in (20), we have

$$\alpha(x)d(r)\alpha(r)\alpha(y)d(z) - \alpha(r)\alpha(y)\alpha(x)\alpha[r, z] - \alpha(r)[\alpha(y)\alpha[x, z], r] = 0. \tag{21}$$

Left multiplying (20) by $\alpha(r)$ and then subtracting from (21), we obtain

$$[\alpha(x)d(r), \alpha(r)]\alpha(y)d(z) = 0. \tag{22}$$

In particular for $r = z$, we have $[\alpha(x)d(z), \alpha(z)]\alpha(y)d(z) = 0$. Replacing y by ry and since α is an epimorphism we get $[\alpha(x)d(z), \alpha(z)]R\alpha(y)d(z) = 0$. This implies that $[\alpha(x)d(z), \alpha(z)]R[\alpha(y)d(z), \alpha(z)] = 0$. Replacing y by x and using semiprimeness, we obtain

$$[\alpha(x)d(z), \alpha(z)] = 0 \text{ for all } x, z \in I.$$

Thus, we get

$$\alpha(x)[d(z), \alpha(z)] + [\alpha(x), \alpha(z)]d(z) = 0. \tag{23}$$

Replacing x by rx in (23), we get

$$r\alpha(x)[d(z), \alpha(z)] + r[\alpha(x), \alpha(z)]d(z) + [r, \alpha(z)]\alpha(x)d(z) = 0 \text{ for all } x, z \in I \text{ and } r \in R.$$

Using (23), we have

$$[r, \alpha(z)]\alpha(x)d(z) = 0 \text{ for all } x, z \in I \text{ and } r \in R.$$

This implies that

$$[r, \alpha(z)]\alpha(x)[d(z), \alpha(z)] = 0 \text{ for all } x, z \in I.$$

Putting $r = d(z)$ and using semiprimeness, we get

$$\alpha(I)[d(z), \alpha(z)] = 0 \text{ for all } z \in I.$$

Replacing y by yz in (17), we get

$$\alpha(xy)\alpha(z)g(z) + F(x)\alpha(y)\alpha(z)d(z) - \alpha(y)\alpha(z)\alpha[x, z] = 0. \tag{24}$$

Right multiplying (17) by $\alpha(z)$ and subtracting from (24), we obtain

$$\alpha(xy)[\alpha(z), g(z)] + F(x)\alpha(y)[\alpha(z), d(z)] - \alpha(y)[\alpha(z), \alpha[x, z]] = 0. \tag{25}$$

Using $\alpha(I)[d(z), \alpha(z)] = 0$ in (25), we get

$$\alpha(xy)[\alpha(z), g(z)] - \alpha(y)[\alpha(z), \alpha[x, z]] = 0. \tag{26}$$

Replacing y by ty in (26), and since α is an epimorphism of R, we get

$$\alpha(x)t\alpha(y)[\alpha(z), g(z)] - t\alpha(y)[\alpha(z), \alpha[x, z]] = 0 \text{ for all } x, z \in I \text{ and } t \in R \tag{27}$$

Left multiplying (26) by t and subtracting from (27), we obtain

$$[t, \alpha(x)]\alpha(y)[g(z), \alpha(z)] = 0. \tag{28}$$

In particular for $x = z$ and putting $t = g(z)$, we get

$$[g(z), \alpha(z)]\alpha(y)[g(z), \alpha(z)] = 0. \tag{29}$$

Replacing y by ry and semiprimeness of R yields that

$$\alpha(I)[g(z), \alpha(z)] = 0 \text{ for all } z \in I. \tag{30}$$

By using similar argument, we arrive at the same conclusion for $G(xy) + F(x)F(y) + \alpha[x, y] = 0$ for all $x, y \in I$. □

Corollary 2.6. *Let R be a prime ring, I a non-zero ideal of R and α be an epimorphism of R such that $\alpha(I) \neq 0$. Suppose that G and F are two multiplicative (generalized)-(α, α)-derivations associated with the mappings g and d on R respectively. If $G(xy) + F(x)F(y) \pm \alpha[x, y] = 0$ for all $x, y \in I$, then either R is commutative or $g(I) = 0$.*

Proof. By Theorem 2.5 and using primeness of R, we have either $d(I) = 0$ or R is commutative. If $d(I) = 0$ then using (17) we get that $g(I) = 0$. □

Using the same techniques with necessary variations, we can prove the following:

Theorem 2.7. *Let R be a semiprime ring, I a non-zero ideal of R and α be an epimorphism of R. Suppose that G and F are two multiplicative (generalized)-(α, α)-derivations associated with the mappings g and d on R respectively. If $G(xy) + F(x)F(y) \pm \alpha(x \circ y) = 0$ for all $x, y \in I$, then $\alpha(I)[d(z), \alpha(z)] = 0$ and $\alpha(I)[g(z), \alpha(z)] = 0$, for all $z \in I$.*

3. Example

The following example demonstrates that Corollary 2.2 and Corollary 2.6 do not hold for arbitrary rings.

Example 3.1. *Consider S be a set of integers. Let*

$$R = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} \mid x, y, z \in S \right\}.$$

Define maps F, d, $\alpha : R \to R$ as

$$F \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & x^2 z \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ d \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \alpha \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -x & -y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix}.$$

Then F is a multiplicative (generalized)-(α, α)-derivation on R associated with mapping d on R. Again define mappings $G, g :$ $R \to R$ such that $G \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, g \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & x & y^2 \\ 0 & 0 & -z \\ 0 & 0 & 0 \end{pmatrix}$. Also G is a multiplicative (generalized)-(α, α)-derivation on R associated with mapping g on R. Suppose that $I = \left\{ \begin{pmatrix} 0 & 0 & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mid y \in S \right\}$. Here we see that I is an ideal of R. For all $x, y \in I$ (i) $G(xy) + F(x)F(y) \pm \alpha[x, y] = 0$ and (ii) $G(xy) + F[x, y] \pm \alpha[x, y] = 0$, however R is neither commutative nor $g(I) = 0$.

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