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Computing Square Revan Index and its Polynomial of Certain Benzenoid Systems

V. R. Kulli[1,*]

1 Department of Mathematics, Gulbarga University, Gulbarga, Karnataka, India.

Abstract: We propose the square Revan index and square Revan polynomial of a graph. In this paper, exact formulas for the square Revan index and square Revan polynomial of certain benzenoids of chemical importance like triangular benzenoids, bonzenoid rhombus, benzenoid hourglass and jagged rectanagle benzenoid systems.

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Keywords: Square Revan index, square Revan polynomial, benzenoid.

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1. Introduction

We are concerned with finite, connected, undirected graphs having no loops and multiple edges. Let $V(G)$ and $E(G)$ denote the vertex set and edge set of a graph G respectively. Let $d_G(v)$ denote the degree of a vertex v in G. Let $\Delta(G)(\delta(G))$ denote the maximum (minimum) degree among the vertices of G. The Revan vertex degree of v in G is defined as $r_G(v) = \Delta(G) + \delta(G) - d_G(v)$. Any undefined term here may be found in [1]. A topological index of a graph is a graph invariant which is a numeric value and applicable in Chemistry, see [2, 3]. Kulli proposed the square ve-degree index, defined as [4]

$$Q_{ve}(G) = \sum_{uv\in E(G)} [d_{ve}(u) - d_{ve}(v)]^2 .$$

In recent years, some novel variants of square indices were introduced and studied such as square reverse index [5], square KV index [6]. We now propose the square Revan index, defined as

$$QR(G) = \sum_{uv\in E(G)} [r_G(u) - r_G(v)]^2 . \tag{1}$$

Very recently, some noval variants of Revan indices [7] were introduced and studied such as hyper Revan indices [8], sum connectivity Revan index [9], product connectivity index [10], multiplicative Revan indices [11], multiplicative connectivity Revan indices [12, 13]. Considering the square Revan index, we introduce the square Revan polynomial of G, defined as

$$QR(G,x) = \sum_{uv\in E(G)} x^{[r_G(u)-r_G(v)]^2} . \tag{2}$$

Recently, some polynomials were studied in [14–20]. In this paper, the square Revan index and square Revan polynomial of certain benzenoid systems are determined. A study of benzenoids has received much attention in Mathematical and Chemical literature, see [20, 21].

* *E-mail: vrkulli@gmail.com*

2. Triangular Benzenoids

The family of triangular benzenoids is denoted by T_p, where p is the number of hexagons in the base graph. Then $|V(T_p)| = p^2 + 4p + 1$ and $|E(T_p)| = \frac{3}{2}p(p+3)$. The graph T_p is shown in Figure 1. From Figure 1, we see that $\Delta(T_p) = 3$ and $\delta(T_p) = 2$. Thus $r_{T_p}(u) = \Delta(T_p) + \delta(T_p) - d_{T_p}(u) = 5 - d_{T_p}(u)$.

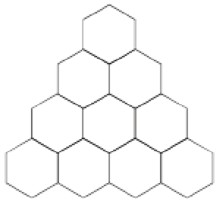

Figure 1. **Graph T_4**

By calculation in T_p, there are three types of edges based on the degree of end vertices of each edge as given in Table 1.

$d_{T_p}(u), d_{T_p}(v) \backslash uv \in E(T_p)$	$(2,2)$	$(2,3)$	$(3,3)$
Number of edges	6	$6p-6$	$\frac{3}{2}p(p-1)$

Table 1. **Edge partition of T_p**

Thus, in T_p, there are three types of Revan edges as given in Table 2.

$d_{T_p}(u), d_{T_p}(v) \backslash uv \in E(T_p)$	$(3,3)$	$(3,2)$	$(2,2)$
Number of edges	6	$6p-6$	$\frac{3}{2}p(p-1)$

Table 2. **Revan edge partition of T_p**

In the following theorem, we determine the square Revan index of T_p.

Theorem 2.1. *The square Revan index of a triangular benzenoid T_p is $QR(T_p) = 6p - 6$.*

Proof. By using equation (1) and Table 2, the square Revan index of T_p is

$$\begin{aligned} QR(T_p) &= \sum_{uv \in E(G)} [r_G(u) - r_G(v)]^2 \\ &= (3-3)^2 6 + (3-2)^2 (6p-6) + (2-2)^2 \frac{3}{2} p(p-1) \\ &= 6p - 6. \end{aligned}$$

□

In the following theorem, we compute the square Revan polynomial of T_p.

Theorem 2.2. *The square Revan polynomial of triangular benzenoid T_p is $QR(T_p, x) = (6p-6)x^1 + \frac{3}{2}\left(p^2 - p + 4\right)x^0$.*

Proof. By using equation (2) and Table 2, the square Revan polynomial of T_p is

$$\begin{aligned} QR(T_p, x) &= \sum_{uv \in E(G)} x^{[r_G(u) - r_G(v)]^2} \\ &= 6x^{(3-3)^2} + (6p-6)x^{(3-2)^2} + \frac{3}{2}p(p-1)x^{(2-2)^2} \\ &= (6p-6)x^1 + \frac{3}{2}\left(p^2 - p + 4\right)x^0. \end{aligned}$$

□

3. Benzenoid Rhombus

The family benzenoid rhombus is symbolized by R_P and R_p is obtained from the copies of a triangular benzenoid T_p by identifying hexagons in one of their base rows. Then $|V(R_p)| = 2p^2+4p$ and $|E(R_p)| = 3p^2+4p-1$. The graph R_4 is depicted in Figure 2. From Figure 2, one can see that $\Delta(T_p) = 3$ and $\delta(T_p) = 2$. Thus $r_{R_p}(u) = \Delta(R_p)+\delta(R_p)-d_{R_p}(u) = 5-d_{R_p}(u)$.

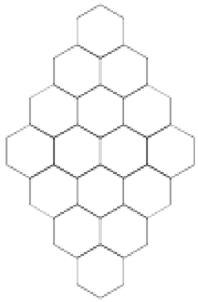

Figure 2. **Graph** R_4

By calculation in R_p, there are three types of edges based on the degree of and vertices of each edge as given in Table 3.

$d_{R_p}(u), d_{R_p}(v) \backslash uv \in E(R_p)$	$(2,2)$	$(2,3)$	$(3,3)$
Number of edges	6	$8p-8$	$3p^2-4p+1$

Table 3. **Revan edge partition** R_p

Therefore, in R_p, there are three types of Revan edges as given in Table 4.

$r_{R_p}(u), r_{R_p}(v) \backslash uv \in E(R_p)$	$(3,3)$	$(3,2)$	$(2,2)$
Number of edges	6	$8p-8$	$3p^2-4p+1$

Table 4. **Revan edge partition** R_p

In the following theorem, we compute the square Revan index of R_p.

Theorem 3.1. *The square Revan index of a benzenoid rhombus R_p is $QR(R_p) = 8p-8$.*

Proof. From equation (1) and Table 4, the square Revan index of R_p is

$$QR(R_p) = \sum_{uv\in E(G)} [r_G(u) - r_G(v)]^2$$
$$= (3-3)^2 6 + (3-2)^2 (8p-8) + (2-2)^2 \left(3p^2-4p+1\right)$$
$$= 8p-8$$

□

In the following theorem, we compute the square Revan polynomial of R_p.

Theorem 3.2. *The square Revan polynomial of a benzenoid rhombus R_p is $QR(R_p, x) = (8p-8)x^1 + \left(3p^2-4p+7\right)x^0$.*

Proof. From equation (2) and Table 4, the square Revan polynomial of R_p is

$$QR(R_p, x) = \sum_{uv\in E(G)} x^{[r_G(u)-r_G(v)]^2}$$

$$= 6x^{(3-3)^2} + (8p-8)\,x^{(3-2)^2} + \left(3p^2-4p+1\right)x^{(2-2)^2}$$
$$= (8p-8)\,x^1 + \left(3p^2-4p+7\right)x^0.$$

□

4. Benzenoid Hourglass

We consider the graph of benzonoid hourglass. The family of benzenoid hourglass is denoted by X_p and it is obtained from two copies of a triangular benzenoid T_p by overlapping hexagons. Then $|V(X_p)| = 2\left(p^2+4p-2\right)$ and $|E(X_p)| = 3p^2+9p-4$. The graph X_p is presented in Figure 3. From Figure 3, we see that $\Delta(X_p) = 3$ and $\delta(X_p) = 2$. Thus $r_{X_p}(u) = \Delta(X_p) + \delta(X_p) - d_{X_p}(u) = 5 - d_{X_p}(u)$.

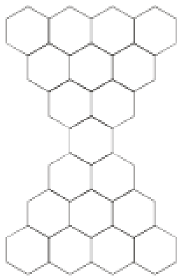

Figure 3. **Graph** X_p

Let $G = X_p$. In G, there are three types of edges as given in Table 5.

$d_G(u), d_G(v) \backslash uv \in E(G)$	$(2,2)$	$(2,3)$	$(3,3)$
Number of edges	8	$12p-16$	$3p^2-3p+4$

Table 5. **Edge partition of** X_p

Thus we obtain that G has three types of Revan edges as given in Table 6.

$d_G(u), d_G(v) \backslash uv \in E(G)$	$(3,3)$	$(3,2)$	$(3,3)$
Number of edges	8	$12p-16$	$3p^2-3p+4$

Table 6. **Revan edge partition of** X_p

In the following theorem, we compute the square Revan index of X_p.

Theorem 4.1. *The square Revan index of a benzenoid hourglass X_p is $QR(X_p) = 12p-16$.*

Proof. By using equation (1) and Table 6, the square Revan index of X_p is

$$QR(X_p) = \sum_{uv \in E(G)} \left[r_G(u) - r_G(v)\right]^2$$
$$= (3-3)^2\,8 + (3-2)^2\,(12p-16) + (2-2)^2\left(3p^2-3p+4\right)$$
$$= 12p-16.$$

□

In the following theorem, we compute the square Revan polynomial of X_p.

Theorem 4.2. *The square Revan polynomial of a benzenoid hourglass X_p is $QR(X_p, x) = (12p - 16)\, x^1 + \left(3p^2 - 3p + 16\right) x^0$.*

Proof. From equation (2) and Table 6, the square Revan polynomial of X_p is

$$\begin{aligned} QR(X_p, x) &= \sum_{uv \in E(G)} x^{[r_G(u) - r_G(v)]^2} \\ &= 8x^{(3-3)^2} + (12p - 16)\, x^{(3-2)^2} + \left(3p^2 - 3p + 4\right) x^{(2-2)^2} \\ &= (12p - 16)\, x^1 + \left(3p^2 - 3p + 12\right) x^0. \end{aligned}$$

□

5. Jagged Rectangle Benzenoid Systems

We consider the family of a jagged rectangle benzenoid system, and it is denoted by $B_{m,n}$, $m, n \geq N$. Three graphs of $B_{m,n}$ are depicted in Figure 4. The by calculation, we obtain that $|V(B_{m,n})| = 4mn+4m+2n-2$ and $|E(B_{m,n})| = 6mn+5m+n-4$.

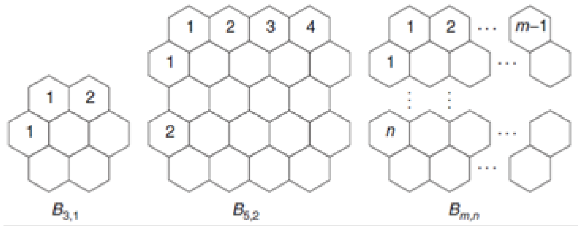

Figure 4. **Graphs of $B_{m,n}$**

Let $G = B_{m,n}$. From Figure 4, we see that $\Delta(G) = 3$ and $\delta(G) = 2$. Then $r_G(u) = \Delta(G) + \delta(G) - d_G(u) = 5 - d_G(u)$. A graph G has three types of edges as given in Table 7.

$d_G(u), d_G(v) \backslash uv \in E(G)$	$(2,2)$	$(2,3)$	$(3,3)$
Number of edges	$2n+4$	$4m+4n-4$	$6mn+m-5n-4$

Table 7. **Edge partition of $B_{m,n}$**

Therefore, we obtain that G has 3 types of Revan edges as given in Table 8.

$d_G(u), d_G(v) \backslash uv \in E(G)$	$(3,3)$	$(3,2)$	$(2,2)$
Number of edges	$2n+4$	$4m+4n-4$	$6mn+m-5n-4$

Table 8. **Revan edge partition of $B_{m,n}$**

In the following theorem, we determine the square Revan index of $B_{m,n}$.

Theorem 5.1. *The square Revan index of a jagged rectangle benzenoid system $B_{m,n}$ is $QR(B_{m,n}) = 4m + 4n - 4$.*

Proof. From equation (1) and Table 4, the square Revan index of $B_{m,n}$ is

$$\begin{aligned} QR(B_{m,n}) &= \sum_{uv \in E(G)} [r_G(u) - r_G(v)]^2 \\ &= (3-3)^2 (2n+4) + (3-2)^2 (4m+4n-4) + (2-2)^2 (6mn+m-5n-4) \\ &= 4m + 4n - 4. \end{aligned}$$

□

In the following theorem, we determine the square Revan polynomial of $B_{m,n}$.

Theorem 5.2. *The square Revan polynomial of a jagged rectangle benzenoid system* $B_{m,n}$ *is* $QR(B_{m,n}, x) = (4m + 4n - 4)\, x^1 + (6mn + m - 3n)\, x^0$.

Proof. By using equation (2) and Table 4, the square Revan polynomial of $B_{m,n}$ is

$$\begin{aligned}
QR(B_{m,n}, x) &= \sum_{uv \in E(G)} x^{[r_G(u) - r_G(v)]^2} \\
&= (2n+4)\, x^{(3-3)^2} + (4m+4n-4)\, x^{(3-2)^2} + (6mn+m-5n-4)\, x^{(2-2)^2} \\
&= (4m+4n-4)\, x^1 + (6mn+m-3n)\, x^0 \\
&= \left(1 + \frac{2\sqrt{2}}{3} + \frac{6\sqrt{6}}{5} + \frac{12\sqrt{3}}{7}\right) 2^{n+2} + \left(1 - \frac{12\sqrt{6}}{5} - \frac{24\sqrt{3}}{7}\right).
\end{aligned}$$

□

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