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Radiation Effect on MHD Flow of Nanofluids Over an Exponentially Stretching Sheet

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Abstract: The problem of MHD radiation flow over an exponentially sheet through a porous medium is considered. The consequences of various parameters will be analyzed in Copper (Cu) and Silver (Ag) nanofluid. The non-linear governing equations of flow of thermal fields are converted to ordinary differential equation using similarity transformations and their numerical solution is obtained using MATLAB software "bvp4c" under the related boundary conditions. The interesting outcomes for variant physical parameters are exhibited through plots and numerical tables.

Keywords: Boundary layer, exponentially stretching sheet, MHD, nanofluid, porous medium, thermal radiation.

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1. Introduction

Any fluid having elements with dimension less than 100nm is admitted as nanofluid. The base fluid or dispersing medium can be present as aqueous or non-aqueous. Nanoparticles have been made of various materials such as, oxide, ceramics, nitride ceramics etc. Convective heat transfer in nanofluid is a matter of consideration in science and engineering. Modern heat transfer industries depend upon high performance heat transfer accessories. Maxwell [1] was the first to present the idea of improving heat and mass transfer performance of fluid with the insertion of solid particles. Many authors [6–9] did numerical analysis on natural convective heat transfer in nanofluids. The application of boundary layer flows and heat transfer past a stretching surface has earned an enormous popularity owing to its broad applications in industry and technology, for example, in metallurgical process such as annealing and tinning of copper wires, glass blowing, crystal growing, manufacturing of plastic and rubber sheets etc. In aspects of these functions Sakiadis [14, 15] explored the boundary layer flow of a viscous fluid done with a moving solid surface. Despite, entire research is confined to linear stretching of the sheet. It is worth saying that the stretching is not compulsorily linear. Ibrahim [16] interpreted the radiation effect on viscous flow of a nanofluid and heat transfer over a nonlinearly stretching sheet and he analyzed that addition in thermal parameter and nonlinearly stretching sheets parameter leads to an increase in heat transfer rates. The consequences of variant parameters governing the flow of a viscous fluid past a nonlinearly stretching sheet was analyzed by Vajravelu [17], Cortell [18, 19] and Afzal [22]. The problem of flow over a quadratic stretching sheet was studied by Kumaran and Ramaniah [23]. Despite all these studies, not too many analyses focused on an exponentially stretching sheet. Nadeem and Lee [24] has taken into consideration the problem of boundary layer flow of nanofluid over an exponentially stretching surface and he analyzed

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that the boundary layer thickness reduces with increase in thermopheresis parameter. Bidin and Nazar [25], Nadeem [26], Magyari and Keller [27], Sanjayanand and Khan [?], Sajid and Hyat [?], Partha [?] and Elbashbeshy [?] analyzed heat transfer temperament past an exponentially stretching sheet. The main objective of the present chapter is to analyze the consequences of variant parameters on MHD boundary layer flow over an exponentially stretching sheet through a porous media in Copper (Cu) and Silver (Ag) nanofluid.

2. Formulation of the Problem

We consider the two-dimensional steady flow of Copper (Cu) and Silver (Ag) nanofluid past an exponentially stretching sheet. Let the x-axis is taken along the stretching surface in the direction of motion and y-axis is normal to it. The plate is stretched along the x-direction with a velocity $U_w = U_\infty e^{\frac{x}{l}}$ defined at $y = 0$. A variable magnetic field $B(x) = B_0 e^{\frac{x}{2l}}$ is applied normal to the sheet, B_0 being a constant. The thermo-physical properties of regular fluid and nanoparticles are given in Table 1.

Physical properties	Regular fluid(water)	Copper (Cu)	Silver (Ag)
c_P (J/kg K)	4179	385	235
ρ (kg/m^3)	997.1	8933	10500
k (W/mK)	0.613	400	429
$\beta \times 10^{-5}(1/K)$	21	1.67	1.89

Table 1: Thermo-physical properties of regular fluid and nanoparticles

The continuity, momentum, energy and concentration equations governing such type of flow can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} - \frac{\upsilon}{K}u - \frac{\sigma B^2(x)}{\rho_{nf}}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho_{cp})_{nf}}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D_B\frac{\partial^2 c}{\partial y^2} - \gamma_0(C - C_8) \tag{4}$$

q_r is the radiative heat flux, C is the nanoparticle fluid concentration, ν_{nf} is the kinematic viscosity, $\mu_{nf} = \frac{\mu_f}{(1-\emptyset)^{2.5}}$ is the dynamic viscosity of the nanofluid, $\rho_{nf} = (1-\emptyset)\rho_f + \emptyset\rho_s$ is the density of the nanofluid, $\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$ is the thermal diffusivity with k_{nf} is the thermal conductivity of the fluid, where

$$k_{nf} = k_f\frac{(k_s + 2k_f) - 2\phi(k_s - k_f)}{(k_s + 2k_f) + \phi(k_s - k_f)},$$

c_p is the heat capacity at constant pressure and

$$(\rho Cp)_{nf} = (\rho Cp)_f(1 - \phi) + (\rho Cp)_s\phi$$

The corresponding boundary conditions are:

$$\begin{aligned} &u = U_w(x,t), \quad v = 0, \quad T = T_w(x,t), \quad C = C_w(x,t) \quad at \quad y = 0 \\ &u \to U(x), \quad T \to T_\infty, \quad C \to C_\infty \quad at \quad y \to \infty. \end{aligned} \tag{5}$$

The sheet of the temperature is

$$T_w = T_8 + T_0 e^{\frac{x}{2l}} \tag{6}$$

where T_0 is the reference temperature, T_w is the surface temperature and T_∞ is the temperature of the fluid outside the boundary layer. The wall surface concentration $C_w(x,t)$ is given by the expression

$$C_w = C_\infty + C_0 e^{\frac{x}{2l}} \tag{7}$$

c_w is the wall surface concentration and C_∞ is the concentration of the fluid outside the boundary layer and

$$\gamma_0(x) = \gamma e^{\frac{x}{l}} \tag{8}$$

where $\gamma_0(x)$ is the variable reaction rate, L is the reference length and γ is a constant. The radiative heat flux under rossel and approximation [32] has the form:

$$q_r = -\frac{4\sigma}{3k_1}\frac{\partial T^4}{\partial y} \tag{9}$$

where k_1 and σ are the mean absorption coefficient and the Stefan-Boltzman constant. We assume that the temperature difference within the flow is sufficiently small such that T^4 can be expressed as a linear function of temperature. Hence expanding T^4 in Taylor series about T_∞ and neglecting higher order terms, we get

$$T^4 \cong 4{T_8}^3 - 3{T_8}^4 \tag{10}$$

Using Equations (5) and (6), Equation (3) reduces to:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma {T_8}^3 \partial^2 T}{3k_1 \left(\rho c_p\right)_{nf} \partial y^2} \tag{11}$$

Now introducing the following similarity transformations

$$\eta = \sqrt{\frac{u_0}{2\upsilon l}} e^{\frac{x}{2l}} y, \quad u = u_0 e^{\frac{x}{l}} f'(\eta), \quad v = -\sqrt{\frac{\upsilon u_0}{2l}} e^{\frac{x}{2l}} f(\eta) + \eta f'(\eta), \quad \theta(\eta) = \frac{T - T_8}{T_W - T_8}, \quad h(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{12}$$

Using Equations (11)-(13), the governing equations becomes

$$f''' - (1-\phi)^{2.5}\left\{1 - \phi + \phi\frac{\rho_s}{\rho_f}\right\}\left[2f'^2 - ff'' + Mf'\right] = 0 \tag{13}$$

$$\theta'' + \frac{1}{\left(1 + \frac{4}{3}Nr\right)} Pr \frac{k_f}{k_{nf}}\left\{1 - \phi + \phi\frac{(\rho Cp)_s}{(\rho Cp)_f}\right\}\left\{f\theta' - f'\theta\right\} = 0 \tag{14}$$

$$h'' + Le\left\{fh' - f'h - \zeta h\right\} = 0 \tag{15}$$

And the transformed boundary conditions are

$$\begin{aligned} &f'(0) = 1, \quad \theta(0) = 1, \quad h(0) = 1 \quad at \quad \eta = 0 \\ &f(\infty) = 0, \quad \theta(\infty) = 0, \quad h(\infty) = 0 \quad at \quad \eta = \infty \end{aligned} \tag{16}$$

Where $Pr = \frac{\nu_f}{\alpha_f}$ is the Prandtl number, $Nr = \frac{4\sigma {T_8}^3}{kk_1}$ is the parameter of radiation, $Le = \frac{\upsilon_f}{D_B}$ is the Lewis number, and $M = \frac{\sigma {B_0}^2}{u_0 \rho_{nf}} + \frac{\nu_{nf}}{k_0 u_0}$ is the combined magnetic and porosity parameter. ζ is the instantaneous reaction rate parameter. The physical quantities of interest are the skin friction coefficient, the local Nusselt number and Sherwood number which are defined as

$$C_f = \frac{\mu}{\rho_f e^{\frac{2x}{l}} U_0^2}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad Nu = -\frac{x}{(T_W - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0}, \quad Sh = -\frac{x}{(C_W - C_\infty)}\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{17}$$

With μ and k are the dynamic viscosity and thermal conductivity, respectively. Using non-dimensional variables, we have

$$\sqrt{2}Re_x^{1/2} C_f = f''(0), \quad \frac{Nu}{\sqrt{2}Re_x^{1/2}} = -\left(1 + \frac{4}{3}Nr\right)\frac{x}{2l}\theta'(0), \quad \frac{Sh}{\sqrt{2}Re_x^{1/2}} = -\sqrt{\frac{x}{2l}}h'(0) \tag{18}$$

3. Method of Solution

The sets of Equations (13)-(15) with boundary conditions 16 constitute a two-point boundary value problem. These equations are solved using "bvp4c" function of MATLAB software package. "bvp4c" is a finite difference code that implements Lobattollla formula and the collocation polynomials allocate a C^1-continuous solution that is fourth-order accurate uniformly in the interval of integration. Mesh selection and error control are based on residual of the continuous solution. The function "bvp4c" has three input variables namely: M-file enumerating an ordinary differential equation system of the design, M-file enumerating the boundary values, and an initial approximation of the result prepared with the MATLAB function "bvpinit". The variables are defined as:

$$y_1 = f, \quad y_2 = f', \quad y_3 = f'', \quad y_3' = f''' \tag{19}$$

$$y_4 = \theta, \quad y_5 = \theta', \quad y_5' = \theta'' \tag{20}$$

$$y_6 = \phi, \quad y_7 = \phi', \quad y_7' = \phi'' \tag{21}$$

Using (19), (20) and (21), the Equations (13)-(15) can be written as

$$y_3' - (1-\phi)^{2.5}\left\{1-\phi+\phi\frac{\rho_s}{\rho_f}\right\}\left(2{y_2}^2 - y_1y_3 + My_2\right) = 0 \tag{22}$$

$$y_5' + \frac{1}{\left(1+\frac{4}{3}Nr\right)}Pr\frac{k_f}{k_{nf}}\left\{1-\phi+\phi\frac{(\rho Cp)_s}{(\rho Cp)_f}\right\}\{y_1y_5 - \; y_2y_4\} = 0 \tag{23}$$

$$y_7' + Le\{y_2y_6 - y_1y_7 - \zeta y_7\} = 0 \tag{24}$$

Bvp4c implements a collocation method for the solution of BVPs subject to general nonlinear, two-point boundary condition. The approximate solution is a continuous function that is a cubic polynomial on each subinterval of a mesh. It satisfies the differential equations at both ends and the midpoint of each subinterval and its boundary conditions. The solver then estimates the error of the numerical solution on each subinterval. If the solution does not satisfy the tolerance criteria, the solver adapts the mesh and repeats the procedure.

4. Results and Discussion

In order to bring out prime characteristic of the flow over an exponentially stretching sheet in a porous media with Copper Cu and Silver Ag nanoparticles are depicted in Figures 1-7 for variant values of volume fraction ϕ, combined porosity parameter and magnetic parameter M, Prandtl number Pr, thermal radiation parameter Nr, Lewis number Le. Table 2 shows computational values of $-f''(0)$ and $-\theta'(0)$ and $-h'(0)$ for Copper and Silver nanoparticles with different values for ϕ.

ϕ	$-f''(0)$		$-\theta'(0)$		$-h'(0)$	
	Cu	Ag	Cu	Ag	Cu	Ag
0.0	1.3817	1.3817	0.8134	0.8134	1.4953	1.4953
0.1	1.5964	1.6648	0.7765	0.7681	1.4814	1.4759
0.2	1.6552	1.7528	0.7353	0.7178	1.4767	1.4691
0.3	1.6015	1.7093	0.6953	0.6666	1.4810	1.4724
0.4	1.4678	1.5742	0.6502	0.6142	1.4921	1.4832

Table 2: $Pr = 0.7$, $Nt = Nb = Nr = 0.5$, $Le = 2$, $M = 1$

Figure 1 depicts the variation in combined magnetic and porosity parameter M when $\phi = 0.1$. It is obvious from the figure that the velocity profile decreases with an increase in value of η, this exhibits a reduction of the thickness of the momentum boundary layer.

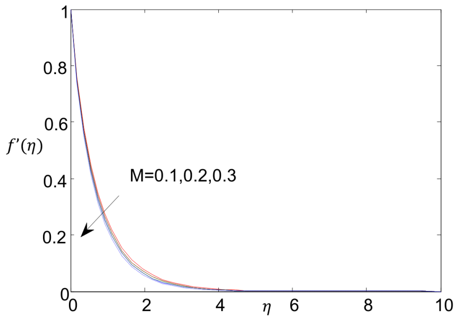

Figure 1: $M = 0.1, 0.2, 0.3$, $\phi = 0.1$ velocity profile for variation in combined magnetic and porosity parameter

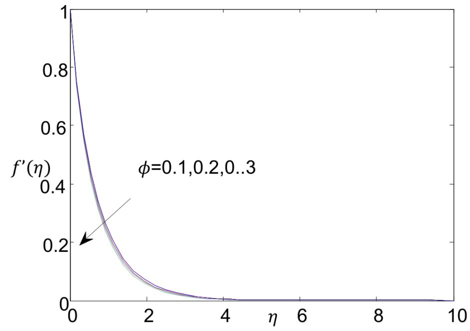

Figure 2: $\phi = 0.1, 0.2, 0.3$, $M = 0.2$ velocity profile for variation in solid volume fraction ϕ

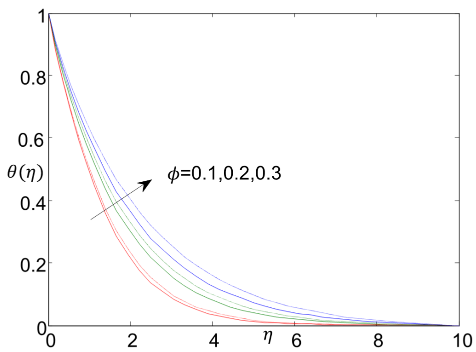

Figure 3: $\phi = 0.1, 0.2, 0.3$, $Pr = 3$, $Nr = 0.5 = Nt = Nb$, $Le = 2$ Temperature profile for variation in solid volume fraction ϕ

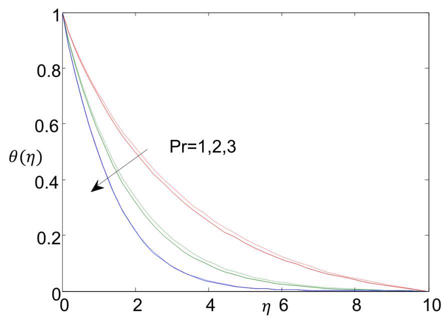

Figure 4: $Pr = 1, 2, 3$, $Nt = Nb = Nr = 0.5$, $Le = 2$, $\phi = 0.1$ Temperature profile for variation in Prandtl number Pr

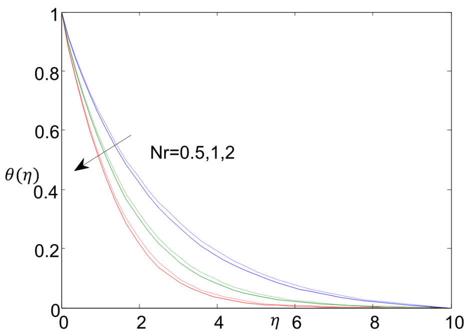

Figure 5: $Nr = 0.5, 1, 2$, $Nt = Nb = 0.5$, $Le = 2$, $Pr = 1$, $\phi = 0.1$. Temperature profile for variation in radiation parameter Nr

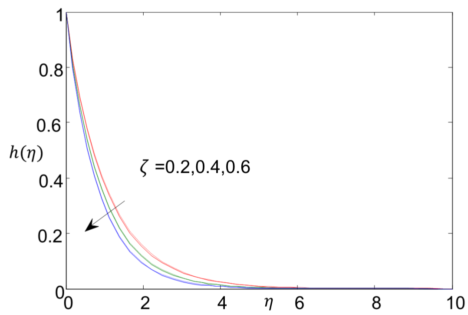

Figure 6: $\zeta = 0.2, 0.4, 0.6$, $Le = 2$. Concentration profile for variation in ζ

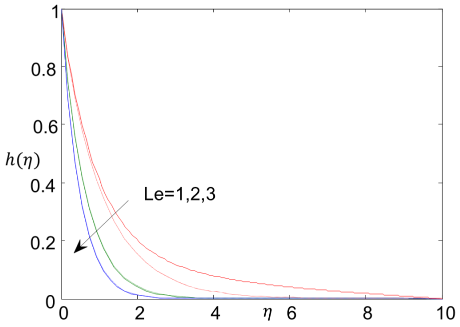

Figure 7: $Le = 1, 2, 3$, $\zeta = 0.2$. Concentration profile for variation in Lewis number Le

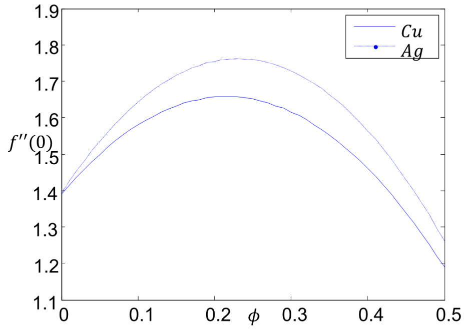

Figure 8: $Pr = 6.25$, $M = 0.2$, $Nr = 0.5$, $Le = 1, 1.2$. Skin friction for variation in ϕ

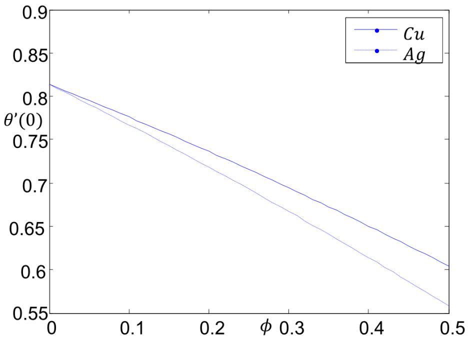

Figure 9: $Pr = 6.25$, $M = 0.2$, $Nr = 0.5$, $Le = 1, 1.2$. Nusselt number for variation in ϕ

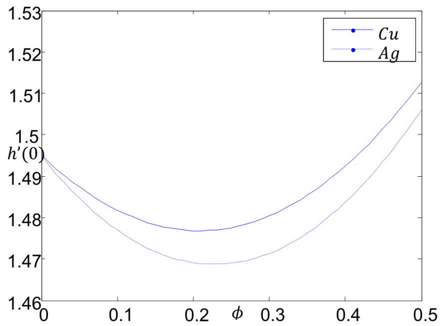

Figure 10: $Pr = 6.25$, $M = 0.2$, $Nr = 0.5$, $Le = 1, 1.2$. Sherwood number for variation in ϕ

Figure 2 reveals the variation in solid volume fraction ϕ, the figure itself expresss that the velocity profile decreases with an increase in value of η.This happens due to the presence of solid nano-particles which leads to further thinning of the velocity boundary layer thickness. Figure 3 exhibits for variation in solid volume fraction ϕ on temperature profile, from the figure it is obvious that the temperature profile declines with an increase in value of η and the thermal boundary layer thickness increases with the increase in value of ϕ. Figure 4 is plotted for variation in Prandtl number Pr. As an actual important thermo-physical property of a fluid, Prandtl number expresses the ratio of momentum diffusivity to thermal diffusivity in the regime.It is apparent from the figure that the thermal boundary layer thickness increases as Pr increases.

Figure 5 exhibits the changes that are seen in temperature profile owe to increase in values of thermal radiation parameter Nr. It is apparent from the figure 5 that the fluid temperature increases with the increasing Nr because the conduction effect of the nanofluid increases in the presence of thermal radiation Nr and the thermal boundary layer thickness is increased with the increasing Nr. Figure 6 is depicted the variation in chemical reaction parameter on concentration profiles. It is witnessed that the concentration boundary layer thickness reduces with increase in chemical reaction parameter γ. Figure 7 exhibits the variation in Lewis number Le. Lewis number signifies the relative contribution of thermal diffusion rate to species diffusion rate in the boundary layer regime and it is evident from the figure that the concentration profile decreases with increasing η and boundary layer for $h(\eta)$ is decreased. Figures 8-10 is for variation in $-f''(0)$, $-\theta'(0)$, $-h'(0)$ against ϕ for variant thermo-physical properties of water and nanoparticles.

5. Conclusion

In this chapter, the consequences of MHD fluid flow over a stretching sheet in presence of thermal radiation in porous media have been analyzed. The investigation is performed for variant mentioned parameters and some conclusions are summarized as follow:

(1). The thermal boundary layer thickness increases with increase in Prandtl number.

(2). Nusselt number decreases with increase in solid volume fraction.

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