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Vague $\widehat{g}$ Closed Set in Topological Spaces

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Abstract: This paper aims to study the concepts of a new class of vague $\widehat{g}$-closed sets in vague topological space along with the basic properties of vague $\widehat{g}$-closed sets and the relationships of the set with some pre-existing sets in vague topological space.

Keywords: Vague set (VS), Vague topology (VT), Vague $\widehat{g}$closed set ($V\widehat{g}CS$).

1. Introduction

The elementary objects of a topological space are closed sets. Norman Levine [7] initiated generalized closed (briefly g-closed) sets in 1970. This concept has been studied extensively in recent years by many topologists because generalized closed sets are not only natural generalizations of closed sets, they also suggest several new properties of topological spaces. The concept of fuzzy sets was introduced by Zadeh [14] in 1965. The proposal of fuzzy set handles uncertainty and vagueness. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. The theory of fuzzy topology was introduced by C.L. Chang [5] in 1967. Several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. In 1986, the concept of intuitionistic fuzzy sets was introduced by Atanassov [2] as a notion of fuzzy sets. The theory of vague sets was first initiated by Gau and Buehrer [6] as an extension of fuzzy set theory and vague sets are regarded as a special case of context-dependent fuzzy sets. We introduce the basic properties of vague $\widehat{g}$-closed sets together with the relationships of these sets with some other sets.

2. Preliminaries

Definition 2.1 ([3]). *A vague set A in the universe of discourse X is characterized by two membership functions given by:*

(1). A true membership function $T_A : X \to [0,1]$ and

(2). A false membership function $F_A : X \to [0,1]$,

where $T_A(x)$ is lower bound on the grade of membership of x derived from the evidence for x, $F_A(x)$ is a lower bound on the negation of x derived from the evidence against x and $T_A(x) + F_A(x) \leq 1$. Thus the grade of membership of x in the vague set A is bounded by a subinterval $[T_A(x), 1 - F_A(x)]$ of $[0,1]$. This indicates that if the actual grade of membership

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$\mu(x)$, then $T_A(x) \leq \mu(x) \leq F_A(x)$ The vague set A is written as, $A = \{\langle x, [T_A(x), 1 - F_A(x)]\rangle / x \in X\}$, where the interval $[T_A(x), 1 - F_A(x)]$ is called the vague value of x in A and is denoted by $V_A(x)$.

Definition 2.2 ([3]). *Let A ad B be vague sets of the form $A = \{\langle x, [T_A(x), 1 - F_A(x)]\rangle / x \in X\}$ and $B = \{\langle x, [T_B(x), 1 - F_B(x)]\rangle / x \in X\}$ Then*

(a). $A \subseteq B$ if and only if $T_A(x) = T_B(x)$ and $1 - F_A(x) \leq 1 - F_B(x)$ for all $x \in X$

(b). $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c). $A^C = \{\langle x, F_A(x), 1 - T_A(x)/x \in X\rangle\}$

(d). $A \cap B = \{\langle x, \min(T_A(x), T_B(x)), \min(1 - F_A(x), 1 - F_B(x))\rangle / x \in X\}$.

(e). $A \cup B = \{\langle x, \max(T_A(x), T_B(x)), \max(1 - F_A(x), 1 - F_B(x))\rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, T_A, 1 - F_A\rangle$ instead of

$$A = \{\langle x, T_A(x), 1 - F_A(x)_A\rangle / x \in X\}.$$

Definition 2.3. *Let (X, τ) be a topological space. A subset A of X is called*

(a). a semi closed set if $int(cl(A) \subseteq A$.

(b). a pre-closed set if $cl(int(A)) \subseteq A$.

(c). a α-closed set if $cl(int(cl(A))) \subseteq A$.

(d). a regular closed set if $A = cl(int(A))$.

Definition 2.4. *Let (X, τ) be a topological space. A subset A of X is called*

(a). a generalised closed set (briefly g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(b). a generalised semi-closed set (briefly gs-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(c). a generalized pre closed (briefly gp-closed) if $pcl \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(d). a α-generalized closed (briefly αg-closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(e). a generalized α closed (briefly gα-closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X.

(f). a regular generalised closed set (briefly rg closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

3. Preliminaries - Vague Topological Space

Definition 3.1. *A vague topology (VT in short) on X is a family τ of vague sets (VS in short) in X satisfying the following axioms.*

(a). $0, 1 \in \tau$.

(b). $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$.

(c). $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called a vague topological space (VTS in short) and any VS in τ is known as a vague open set (VOS in short) in X. The complement A^c of a VOS in a VTS (X, τ) is called a vague closed set (VCS in short) in X.

Definition 3.2. *Let (X, τ) be a VTS and $A = \langle x, T_A, 1 - F_A \rangle$ be a VS in X. Then the vague interior and a vague closure are defined by*

$$V\ int\,(A) = \bigcup \{G \backslash G \text{ is an VOS in } X \text{ and } G \subseteq A\}$$
$$V\ cl\,(A) = \bigcap \{K \backslash K \text{ is an VCS in } X \text{ and } A \subseteq K\}$$

Note that for any VS A in (X, τ), we have $Vcl\,(A^c) = (V\ int\,(A))^C$ and $V\ int\,(A^c) = (V\ cl\,(A))^C$.

Definition 3.3. *A Vague set A of (X, τ) is said to be*

(a). a vague semi closed set (VSCS in short) if $Vint(Vcl(A)) \subseteq A$.

(b). a vague semi open set (VSOS in short) if $A \subseteq Vcl(Vint(A))$.

(c). a vague pre-closed set (VPCS in short) if $Vcl(Vint(A)) \subseteq A$.

(d). a vague pre-open set (VPOS in short) if $A \subseteq Vint(Vcl(A))$.

(e). a vague α-closed set (VαCS in short) if $Vcl(Vint(Vcl(A))) \subseteq A$.

(f). a vague α-open set (VαOS in short) if $A \subseteq Vint(Vcl(Vint(A)))$.

(g). a vague regular open set (VROS in short) if $A = Vint(Vcl(A))$.

(h). a vague regular closed set (VRCS in short) if $A = Vcl(Vint(A))$.

Definition 3.4. *A vague set A of (X, τ) is said to be a vague generalized closed set (VGCS) if $Vcl(A) \subseteq U$ whenever $A \subseteq U$ and U is VOS in X.*

Definition 3.5. *A vague set A of (X, τ) is said to be a vague generalized semi closed set (VGSCS) if $Vscl\,(A) \subseteq U$ whenever $A \subseteq U$ and U is VOS in X.*

Definition 3.6. *A vague set A of (X, τ) is said to be vague alpha generalized closed set (VαGCS) if $V\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is VOS in X.*

Definition 3.7. *A vague set A of (X, τ) is said to be a vague generalized pre-closed set (VGPCS) if $Vpcl(A) \subseteq U$ whenever $A \subseteq U$ and U is VOS in X.*

Definition 3.8. *A vague set A of (X, τ) is said to be a vague regular generalised closed set (VRGCS) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is VOS in X.*

Definition 3.9. *A vague set A of (X, τ) is said to be a vague generalized α closed (VGαCS) if $\alpha cl\,(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X.*

4. Vague $\widehat{g}$-Closed Sets

Definition 4.1. *A vague set A of (X, τ) is said to be a vague $\widehat{g}$-closed sets ($V\widehat{G}CS$ in short) if $Vcl(A) \subseteq U$ whenever $A \subseteq U$ and U is vague semi open set in X.*

Theorem 4.2. *Every vague closed set is a vague $\widehat{g}$ closed set but not conversely.*

Proof. Let A be a vague closed set and let $A \subseteq U$, where U is vague semi open in X then $Vcl(A) \subseteq U$. Therefore A is a vague $\widehat{g}$ closed set. ◻

Example 4.3. *Let $X = \{a, b\}$, $\tau = \{0, G, 1\}$ be a VT on X, $G = \{\langle x, [0.1, 0.4], [0.2, 0.3]\rangle\}$, then the vague set $A = \{\langle x, [0.2, 0.8], [0.3, 0.6]\rangle\}$ is a $V\widehat{g}CS$ but not VCS.*

Theorem 4.4. *Every $\widehat{g}$ closed set is a vague gs closed set but not conversely.*

Proof. Let A be vague $\widehat{g}$ closed set and let $A \subseteq U$, where U is vague open in X, vague open implies vague semi open in X, then $Vscl(A) \subseteq Vcl(A) \subseteq U$. Therefore A is VGSCS. ◻

Example 4.5. *Let $X = \{a, b\}$, $\tau = \{0, G, 1\}$ be a VT on X, $G = \{\langle x, [0.2, 0.5], [0.1, 0.4]\rangle\}$ then the vague set $A = \{\langle x, [0.2, 0.7], [0.3, 0.6]\rangle\}$ is a VGSCS but not $V\widehat{g}CS$.*

Theorem 4.6. *Every $\widehat{g}$ closed set is a vague g closed set but not conversely.*

Proof. Let A be vague $\widehat{g}$ closed set and let $A \subseteq U$, where U is vague open in X, vague open implies vague semi open in X, then $Vcl(A) \subseteq U$. Therefore A is VGCS. ◻

Example 4.7. *Let $X = \{a, b\}$, $\tau = \{0, G, 1\}$ be a VT on X, $G = \{\langle x, [0.3, 0.4], [0.2, 0.5]\rangle\}$ then the vague set $A = \{\langle x, [0.2, 0.6], [0.3, 0.6]\rangle\}$ is a VGCS but not $V\widehat{g}CS$.*

Theorem 4.8. *Every $\widehat{g}$ closed set is a vague rg closed set but not conversely.*

Proof. Let A be vague $\widehat{g}$ closed set and let $A \subseteq U$, where U is vague regular open in X, vague regular open implies vague semi open in X, then $Vcl(A) \subseteq U$. Therefore A is VRGCS. ◻

Example 4.9. *Let $X = \{a, b\}$, $\tau = \{0, G, 1\}$ be a VT on X, $G = \{\langle x, [0.5, 0.9], [0.4, 0.8]\rangle\}$ then the vague set $A = \{\langle x, [0.5, 0.8], [0.4, 0.7]\rangle\}$ is a VRGCS but not $V\widehat{g}CS$.*

Theorem 4.10. *Every $\widehat{g}$ closed set is a vague gp closed set but not conversely.*

Proof. Let A be vague $\widehat{g}$ closed set and let $A \subseteq U$, where U is vague open in X, vague open implies vague semi open in X, then $Vpcl(A) \subseteq Vcl(A) \subseteq U$. Therefore A is VGPCS. ◻

Example 4.11. *Let $X = \{a, b\}$, $\tau = \{0, G, 1\}$ be a VT on X, $G = \{\langle x, [0.3, 0.6], [0.3, 0.5]\rangle\}$ then the vague set $A = \{\langle x, [0.5, 0.6], [0.3, 0.5]\rangle\}$ is a VGPCS but not $V\widehat{g}CS$. Since $Vcl(A) = 1 \not\subset G$.*

Theorem 4.12. *Every $\widehat{g}$ closed set is a vague $g\alpha$ closed set but not conversely.*

Proof. Let A be vague $\widehat{g}$ closed set and let $A \subseteq U$, where U is vague α open in X, vague α open implies vague semi open in X, then $V\alpha\ cl(A) \subseteq Vcl(A) \subseteq U$. Therefore A is VG$\alpha$CS. ◻

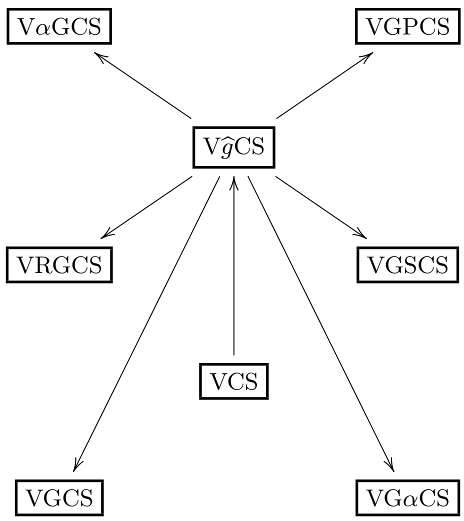
Example 4.13. *Let $X = \{a, b\}$, $\tau = \{0, G, 1\}$ be a VT on X, $G = \{\langle x, [0.5, 0.9], [0.4, 0.8]\rangle\}$ then the vague set $A = \{\langle x, [0.5, 0.8], [0.4, 0.7]\rangle\}$ is a $VG\alpha CS$ but not $V\widehat{g}CS$.*

Theorem 4.14. *Every $\widehat{g}$ closed set is a vague αg closed set but not conversely.*

Proof. Let A be vague $\widehat{g}$ closed set and let $A \subseteq U$, where U is vague open in X, vague open implies vague semi open in X, then $V\alpha\ cl(A) \subseteq Vcl\,(A) \subseteq U$. Therefore A is VαGCS. □

Example 4.15. *Let $X = \{a, b\}$, $\tau = \{0, G, 1\}$ is a VT on X, $G = \{\langle x, [0.5, 0.9], [0.4, 0.8]\rangle\}$ then the vague set $A = \{\langle x,\ [0.5, 0.8], [0.4, 0.7]\rangle\}$ is a $V\alpha GCS$ but not $V\widehat{g}CS$.*

Summing up the above theorems, we have the following implication:



5. Basic Properties of Vague $\widehat{g}$ Closed Sets

Theorem 5.1. *Union of any two vague $\widehat{g}$ closed is again vague $\widehat{g}$ closed.*

Proof. $Vcl\,(A \cup B) = Vcl\,(A) \cup Vcl\,(B) \subseteq U$ whenever $A \cup B \subseteq U$ and U is vague semi open set in X. □

Theorem 5.2. *Intersection of any two vague $\widehat{g}$ closed is again vague $\widehat{g}$ closed.*

Proof. $Vcl\,(A \cap B) = Vcl\,(A) \cap Vcl\,(B) \subseteq U$ whenever $A \cap B \subseteq U$ and U is vague semi open set in X. □

Example 5.3. *Let $X = \{a, b\}$ and $\tau = \{0, G, 1\}$ be a VT on X, where $G = \{\langle x, [0.1, 0.7], [0.3, 0.8]\rangle\}$, then $A = \{\langle x, [0.2, 0.8], [0.3, 0.6]\rangle\}$ and $B = \{\langle x, [0.2, 0.6], [0.3, 0.7]\rangle\}$.*

Proof. $A \cup B = \{\langle x,\ [0.2, 0.8], [0.3, 0.7]\rangle\}$ and $A \cap B = \{\langle x, [0.2, 0.6], [0.3, 0.6]\rangle\}$. The above two theorems are true from this example. □

Theorem 5.4. *If A is a vague $\widehat{g}$ closed set in a space (X, τ) and $(A \subseteq B \subseteq Vcl\,(A))$, then B is also a vague $\widehat{g}$ closed set.*

Proof. Let A is a vague $\widehat{g}$ closed set, Since $B \subseteq Vcl(A)$ then $Vcl\,(B) \subseteq Vcl\,(Vcl\,(A)) \subseteq U$. Hence B is also a vague $\widehat{g}$ closed set. □

Theorem 5.5. *Let A be a vague $\widehat{g}$ closed set of a topological space (X, τ) then*

(1). $Vsint\,(A)$ is $V\widehat{g}$ closed.

(2). $Vpcl\,(A)$ is $V\widehat{g}$ closed.

(3). If A is Vague regular open, then $Vpint\,(A)$ and $Vscl\,(A)$ are also $V\widehat{g}$ closed sets.

Proof. For a subset A of (X, τ), $Vscl\,(A) = A \cup Vint\,(Vcl\,(A))$ and $Vpcl\,(A) = A \cup Vcl\,(Vint\,(A))$. Moreover $Vsint\,(A) = A \cap Vcl\,(Vint\,(A))$ and $Vpint\,(A) = A \cap Vint\,(Vcl\,(A))$.

(1). Since $Vcl\,(VintA)$ is a vague closed set implies $Vcl\,(VintA)$ is a vague $\widehat{g}$ closed set. The intersection of two $V\widehat{g}CS$ is also a $V\widehat{g}CS$. Therefore $Vsint(A)$ is a $V\widehat{g}CS$.

(2). Since $Vpcl\,(A) = A \cup Vcl\,(Vint\,(A))$. The union of two $V\widehat{g}CS$ is also a $V\widehat{g}CS$. Therefore $Vpcl(A)$ is a $V\widehat{g}CS$.

(3). Since A is a vague regular open set then $A = int\,(cl\,(A))$, $scl\,(A) = A \cup int\,(cl\,(A)) = A$, where A is vague $\widehat{g}$ closed, therefore $scl\,(A)$ is a vague $\widehat{g}$ closed set.

Similarly, $pint\,(A) = A \cap int\,(cl\,(A)) = A$, where A is vague $\widehat{g}$ closed, therefore p $int\,(A)$ is a vague $\widehat{g}$ closed set. □

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