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Totally ψgs-Continuous Functions in Topological Spaces

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Abstract: The aim of this paper is to introduce and study a new class of functions called totally ψgs-continuous functions and ψgs-totally continuous functions in topological spaces as a new generalization of ψgs-continuity.

Keywords: Totally continuous function, ψgs-continuous function, ψgs-closed set and ψgs-open set.

1. Introduction

Levine [8] introduced the idea of continuous functions in topological spaces. Jain [6] introduced the concept of totally continuous functions in topological spaces. Balachandran [1] introduced generalized continuous functions in topological spaces. Sundaram [9] introduced semi generalized continuous functions in topological spaces. Devi [3] introduced generalized semi continuous functions in topological spaces. Veera kumar [10] introduced and studied ψ-continuous functions in topological spaces. Gowsalya and Balamani [4, 5] introduced ψgs-closed sets and ψgs-continuous functions in topological spaces. In this paper we introduce a new type of totally continuous functions called totally ψgs-continuous functions and ψgs-totally continuous functions in topological spaces.

2. Preliminaries

Definition 2.1. *Let (X,τ) be a topological space. A Subset A of a topological spaces (X,τ) is called*

(1). Semi open [7] if $A \subseteq cl(int(A))$ and semi-closed if $int(cl(A)) \subseteq A$.

(2). generalized closed [8] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

(3). Semi generalized closed [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X,τ).

(4). ψ-closed [10] if $scl(A) \subseteq U$ whenever $\subseteq U$ and U is sg-open in (X,τ).

(5). ψgs-closed [4] if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X,τ).

(6). ψgs-clopen [4] if it is both ψgs-closed and ψgs-open in (X,τ).

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Result 2.2.

(1). Every closed (respectively open) subset in (X, τ) is ψgs-closed (respectively ψgs-open).

(2). Every clopen subset in (X, τ) is ψgs-clopen.

Definition 2.3. *Let (X, τ) and (Y, σ) be two topological spaces. A function $f : (X, \tau) \to (Y, \sigma)$ is called*

(1). Continuous [8] if $f^{-1}(V)$ is closed in (X, τ) for every closed set V of (Y, σ).

(2). Totally continuous [6] if $f^{-1}(V)$ is clopen in (X, τ) for every open set V of (Y, σ).

(3). g-continuous [1] if $f^{-1}(V)$ is g-closed in (X, τ) for every closed set V of (Y, σ).

(4). ψ-continuous [10] if $f^{-1}(V)$ is ψ-closed in (X, τ) for every closed set V of (Y, σ).

(5). ψgs-continuous [5] if $f^{-1}(V)$ is ψgs-closed in (X, τ) for every closed set V of (Y, σ).

3. Totally ψgs-Continuous Function

Definition 3.1. *Let (X, τ) and (Y, σ) be two topological spaces. A function $f : (X, \tau) \to (Y, \sigma)$ is called totally ψgs-continuous if $f^{-1}(V)$ is ψgs-clopen in (X, τ) for every open set V of (Y, σ).*

Example 3.2. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = b$, $f(b) = c$, $f(c) = a$. Then f is totally ψgs-continuous.*

Theorem 3.3. *A function $f : (X, \tau) \to (Y, \sigma)$ is totally ψgs-continuous if and only if the inverse image of every closed subset of (Y, σ) is a ψgs-clopen subset of (X, τ).*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a totally ψgs-continuous function. Let V be any closed set in (Y, σ). Then $Y - V$ is open in (Y, σ). Since f is totally ψgs-continuous, $f^{-1}(Y - V) = X - f^{-1}(V)$ is ψgs-clopen in (X, τ) which implies that $f^{-1}(V)$ is ψgs-clopen in (X, τ).

Conversely, assume that U is any open set in (Y, σ). Then $Y - U$ is closed in (Y, σ). By assumption, $f^{-1}(Y - U) = X - f^{-1}(U)$ is ψgs-clopen in (X, τ) which implies that $f^{-1}(U)$ is ψgs-clopen in (X, τ). Hence f is totally ψgs-continuous. □

Proposition 3.4. *Every totally continuous function is a totally ψgs-continuous function but not conversely.*

Proof. Let V be any open set in (Y, σ). Since f is totally continuous, $f^{-1}(V)$ is clopen in (X, τ). By Result 2.2, $f^{-1}(V)$ is ψgs-clopen in (X, τ). Hence f is totally ψgs-continuous. □

Example 3.5. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = a$, $f(b) = c$, $f(c) = b$. Then f is totally ψgs-continuous but not totally continuous, since for the open set $\{a, b\}$ in (Y, σ), $f^{-1}(\{a, b\}) = \{a, c\}$ is ψgs-clopen in (X, τ) but not clopen subset in (X, τ).*

Proposition 3.6. *Every totally ψgs-continuous function is a ψgs-continuous function but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a totally ψgs-continuous function. Let V be any open set in (Y, σ). Since f is totally ψgs-continuous, $f^{-1}(V)$ is ψgs-clopen in (X, τ) which implies that $f^{-1}(V)$ is ψgs-open in (X, τ). Hence f is ψgs-continuous. □

Example 3.7. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is ψgs-continuous but not totally ψgs-continuous, since for the open set $\{a\}$ in (Y, σ), $f^{-1}(\{a\}) = \{a\}$ is ψgs-open but not ψgs-closed in (X, τ).*

Proposition 3.8. *Every continuous function is independent from totally ψgs-continuous function.*

Example 3.9. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is continuous but not totally ψgs-continuous, since for the open set $\{a, b\}$ in (Y, σ), $f^{-1}(\{a, b\}) = \{a, b\}$ is open but not ψgs-clopen in (X, τ).*

Example 3.10. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = c$, $f(b) = b$, $f(c) = a$. Then f is totally ψgs-continuous but not continuous, since for the open set $\{a\}$ in (Y, σ), $f^{-1}(\{a\}) = \{c\}$ is ψgs-clopen but not open in (X, τ).*

Proposition 3.11. *If $f : (X, \tau) \to (Y, \sigma)$ is totally ψgs-continuous and X is ψgs-connected, then Y is an indiscrete space.*

Proof. Suppose Y is not indiscrete space. Let V be a non-empty open subset of Y. Since f is totally ψgs-continuous, $f^{-1}(V)$ is non-empty ψgs-clopen subset of X. Then $X = f^{-1}(V) \cup (f^{-1}(V))^C$. Thus X is union of two non-empty disjoint ψgs-open sets which is a contradiction to the fact that X is ψgs-connected. Therefore Y must be indiscrete space. □

Theorem 3.12. *Let $f : (X, \tau) \to (Y, \sigma)$ be any function from discrete space (X, τ) into a topological space (Y, σ). Then f is totally ψgs-continuous function if and only if*

(1). f is a continuous function.

(2). f is a ψgs-continuous function.

Proof.

(1). Assume that f is a totally ψgs-continuous function. Let V be any open set in (Y, σ). Then $f^{-1}(V)$ is ψgs-clopen in (X, τ). Since (X, τ) is discrete space, every subset of (X, τ) is open and closed in (X, τ) which implies that $f^{-1}(V)$ is open in (X, τ). Therefore f is continuous.

Conversely, assume that f is continuous. Let V be any open set in (Y, σ). Then $f^{-1}(V)$ is open in (X, τ). Since (X, τ) is discrete space, every subset of (X, τ) is clopen in (X, τ) which implies that $f^{-1}(V)$ is ψgs-clopen in (X, τ). Therefore f is totally ψgs-continuous.

(2). Assume that f is a totally ψgs-continuous function. Let V be any open set in (Y, σ). Then $f^{-1}(V)$ is ψgs-clopen in (X, τ). Since (X, τ) is discrete space, $f^{-1}(V)$ is open in (X, τ). By Result 2.2, $f^{-1}(V)$ is ψgs-open in (X, τ). Therefore f is ψgs-continuous.

Conversely, assume that f is ψgs-continuous. Let V be any open set in (Y, σ). Then $f^{-1}(V)$ is ψgs-open in (X, τ). Since (X, τ) is discrete space, $f^{-1}(V)$ is clopen in (X, τ). By Result 2.2, $f^{-1}(V)$ is ψgs-clopen in (X, τ). Therefore f is totally ψgs-continuous. □

Proposition 3.13. *The composition of two totally ψgs-continuous functions need not be a totally ψgs-continuous function as seen from the following example.*

Example 3.14. *Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$, $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$ and $\eta = \{\phi, \{a, b\}, Z\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function and $g : (Y, \sigma) \to (Z, \eta)$ be the identity function. Then f and g are totally ψgs-continuous but their composition $g \circ f : (X, \tau) \to (Z, \eta)$ is not totally ψgs-continuous, since $\{a, b\}$ is open in (Z, η), where as $(g \circ f)^{-1}(\{a, b\}) = \{a, b\}$ is not ψgs-clopen in (X, τ).*

Proposition 3.15. *If $f : (X, \tau) \to (Y, \sigma)$ is a totally ψgs-continuous function and $g : (Y, \sigma) \to (Z, \eta)$ is a continuous function, then $g \circ f : (X, \tau) \to (Z, \eta)$ is a totally ψgs-continuous function.*

Proof. Let V be any open set in (Z,η). Since g is continuous, $g^{-1}(V)$ is open in (Y,σ). Since f is totally ψgs-continuous, $(g\circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs-clopen in (X,τ). Hence $g\circ f$ is a totally ψgs-continuous function. □

Proposition 3.16. *Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ be two functions, then $g\circ f:(X,\tau)\to(Z,\eta)$ is a totally ψgs-continuous function if*

(1). f is a totally ψgs-continuous function and g is a totally continuous function.

(2). f and g are totally continuous functions.

Proof.

(1). Let V be any open set in (Z,η). Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y,σ) which implies $g^{-1}(V)$ is open in (Y,σ). Since f is totally ψgs-continuous, $(g\circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs-clopen in (X,τ). Hence $g\circ f$ is a totally ψgs-continuous function.

(2). Let V be any open set in (Z,η). Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y,σ) which implies $g^{-1}(V)$ is open in (Y,σ). Since f is totally continuous, $(g\circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By Result 2.2, $(g\circ f)^{-1}(V)$ is ψgs-clopen in (X,τ). Hence $g\circ f$ is a totally ψgs-continuous function. □

4. ψgs-Totally Continuous Function

Definition 4.1. *A function $f:(X,\tau)\to(Y,\sigma)$ is called ψgs-totally continuous if $f^{-1}(V)$ is clopen in (X,τ) for every ψgs-open set V of (Y,σ).*

Example 4.2. *Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{b,c\},X\}$ and $\sigma = \{\phi,\{a,b\},Y\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be a function defined by $f(a) = f(b) = f(c) = a$. Then f is ψgs-totally continuous.*

Theorem 4.3. *A function $f:(X,\tau)\to(Y,\sigma)$ is ψgs-totally continuous if and only if the inverse image of every ψgs-closed subset of (Y,σ) is a clopen subset of (X,τ).*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ be a ψgs-totally continuous function. Let V be any ψgs-closed set in (Y,σ). Then $Y-V$ is ψgs-open in (Y,σ). Since f is ψgs-totally continuous, $f^{-1}(Y-V) = X - f^{-1}(V)$ is clopen in (X,τ) which implies that $f^{-1}(V)$ is clopen in (X,τ).
Conversely, assume that U is any ψgs-open set in (Y,σ). Then $Y-U$ is ψgs-closed in (Y,σ). By assumption, $f^{-1}(Y-U) = X - f^{-1}(U)$ is clopen in (X,τ) which implies that $f^{-1}(U)$ is clopen in (X,τ). Hence f is ψgs-totally continuous. □

Proposition 4.4. *If $f:(X,\tau)\to(Y,\sigma)$ is a ψgs-totally continuous function, then*

(1). f is a totally continuous function.

(2). f is a totally ψgs-continuous function.

Proof.

(1). Let V be any open set in (Y,σ). By Result 2.2, V is ψgs-open in (Y,σ). Since f is ψgs-totally continuous, $f^{-1}(V)$ is clopen in (X,τ). Therefore f is totally continuous.

(2). Let V be any open set in (Y,σ). By Result 2.2, V is ψgs-open in (Y,σ). Since f is ψgs-totally continuous, $f^{-1}(V)$ is clopen in (X,τ). By Result 2.2, $f^{-1}(V)$ is ψgs-clopen in (X,τ). Therefore f is totally ψgs-continuous. □

The converse of Proposition 4.4 (1) need not be true in general as seen from the following example.

Example 4.5. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then f is totally continuous but not ψgs-totally continuous, since for the ψgs-open set $\{a, b\}$ in (Y, σ), $f^{-1}(\{a, b\}) = \{b, c\}$ is not clopen in (X, τ).*

The converse of Proposition 4.4 (2) need not be true in general as seen from the following example.

Example 4.6. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = a$, $f(b) = c$, $f(c) = b$. Then f is totally ψgs-continuous but not ψgs-totally continuous, since for the ψgs-open set $\{a\}$ in (Y, σ), $f^{-1}(\{a\}) = \{a\}$ is not clopen in (X, τ).*

Proposition 4.7. *If $f : (X, \tau) \to (Y, \sigma)$ is a ψgs-totally continuous function, then*

(1). f is a continuous function.

(2). f is a ψgs-continuous function.

Proof.

(1). Let V be any open set in (Y, σ). By Result 2.2, V is ψgs-open in (Y, σ). Since f is ψgs-totally continuous, $f^{-1}(V)$ is clopen in (X, τ) which implies that $f^{-1}(V)$ is open in (X, τ). Therefore f is continuous.

(2). Let V be any open set in (Y, σ). By Result 2.2, V is ψgs-open in (Y, σ). Since f is ψgs-totally continuous, $f^{-1}(V)$ is clopen in (X, τ) which implies that $f^{-1}(V)$ is open in (X, τ). By Result 2.2, $f^{-1}(V)$ is ψgs-open in (X, τ). Therefore f is ψgs-continuous. □

The converse of Proposition 4.7 (1) need not be true in general as seen from the following example.

Example 4.8. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = a$, $f(b) = c$, $f(c) = b$. Then f is continuous but not ψgs-totally continuous, since for the ψgs-open set $\{a, c\}$ in (Y, σ), $f^{-1}(\{a, c\}) = \{a, b\}$ is not clopen in (X, τ).*

The converse of Proposition 4.7 (2) need not be true in general as seen from the following example.

Example 4.9. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a$, $f(b) = c$, $f(c) = b$. Then f is ψgs-continuous but not ψgs-totally continuous, since for the ψgs-open set $\{b\}$ in (Y, σ), $f^{-1}(\{b\}) = \{c\}$ is not clopen in (X, τ).*

Theorem 4.10. *Let $f : (X, \tau) \to (Y, \sigma)$ be any function from discrete space (X, τ) into a topological space (Y, σ). If f is a ψgs-totally continuous function, then*

(1). f is a continuous function.

(2). f is a ψgs-continuous function.

Proof.

(1). Let V be any open set in (Y, σ). By Result 2.2, V is ψgs-open in (Y, σ). Since f is ψgs-totally continuous, $f^{-1}(V)$ is clopen in (X, τ) which implies that $f^{-1}(V)$ is open in (X, τ). Therefore f is continuous.

(2). Let V be any open set in (Y, σ). By Result 2.2, V is ψgs-open in (Y, σ). Since f is ψgs-totally continuous, $f^{-1}(V)$ is clopen in (X, τ) which implies that $f^{-1}(V)$ is open in (X, τ). By Result 2.2, $f^{-1}(V)$ is ψgs-open in (X, τ). Therefore f is ψgs-continuous. □

Proposition 4.11. *Let $f : (X, \tau) \to (Y, \sigma)$ be a ψgs-totally continuous function and $g : (Y, \sigma) \to (Z, \eta)$ be any function. Then $g \circ f : (X, \tau) \to (Z, \eta)$ is a totally ψgs-continuous function if*

(1). g is a totally ψgs-continuous function.

(2). g is a totally continuous function.

Proof.

(1). Let V be any open set in (Z, η). Since g is totally ψgs-continuous, $g^{-1}(V)$ is ψgs-clopen in (Y, σ) which implies that $g^{-1}(V)$ is ψgs-open in (Y, σ). Since f is ψgs-totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X, τ). By Result 2.2, $(g \circ f)^{-1}(V)$ is ψgs-clopen in (X, τ). Therefore $g \circ f$ is totally ψgs-continuous.

(2). Let V be any open set in (Z, η). Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y, σ) which implies that $g^{-1}(V)$ is open in (Y, σ). By Result 2.2, $g^{-1}(V)$ is ψgs-open in (Y, σ). Since f is ψgs-totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X, τ). By Result 2.2, $(g \circ f)^{-1}(V)$ is ψgs-clopen in (X, τ). Therefore $g \circ f$ is totally ψgs-continuous. □

Proposition 4.12. *Let $f : (X, \tau) \to (Y, \sigma)$ be a ψgs-totally continuous function and $g : (Y, \sigma) \to (Z, \eta)$ be any function. Then $g \circ f : (X, \tau) \to (Z, \eta)$ is a totally ψgs-continuous function if*

(1). g is a continuous function.

(2). g is a ψgs-continuous function.

Proof.

(1). Let V be any open set in (Z, η). Since g is continuous, $g^{-1}(V)$ is open in (Y, σ). By Result 2.2, $g^{-1}(V)$ is ψgs-open in (Y, σ). Since f is ψgs-totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X, τ). By Result 2.2, $(g \circ f)^{-1}(V)$ is ψgs-clopen in (X, τ). Therefore $g \circ f$ is totally ψgs-continuous.

(2). Let V be any open set in (Z, η). Since g is ψgs-continuous, $g^{-1}(V)$ is ψgs-open in (Y, σ). Since f is ψgs-totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X, τ). By Result 2.2, $(g \circ f)^{-1}(V)$ is ψgs-clopen in (X, τ). Therefore $g \circ f$ is totally ψgs-continuous. □

Proposition 4.13. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be any two functions. If g is a ψgs-totally continuous function and if*

(1). f is a totally continuous function, then $g \circ f : (X, \tau) \to (Z, \eta)$ is a ψgs-totally continuous function.

(2). f is a continuous function, then $g \circ f : (X, \tau) \to (Z, \eta)$ is a continuous function.

(3). f is a ψgs-continuous function, then $g \circ f : (X, \tau) \to (Z, \eta)$ is a ψgs-continuous function.

Proof.

(1). Let V be any ψgs-open set in (Z, η). Since g is ψgs-totally continuous, $g^{-1}(V)$ is clopen in (Y, σ) which implies that $g^{-1}(V)$ is open in (Y, σ). Since f is totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X, τ). Therefore $g \circ f$ is ψgs-totally continuous.

(2). Let V be any open set in (Z, η). By Result 2.2, V is ψgs-open in (Z, η). Since g is ψgs-totally continuous, $g^{-1}(V)$ is clopen in (Y, σ) which implies that $g^{-1}(V)$ is open in (Y, σ). Since f is continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is open in (X, τ). Therefore $g \circ f$ is continuous.

(3). Let V be any open set in (Z, η). By Result 2.2, V is ψgs-open in (Z, η). Since g is ψgs-totally continuous, $g^{-1}(V)$ is clopen in (Y, σ) which implies that $g^{-1}(V)$ is open in (Y, σ). Since f is ψgs-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs-open in (X, τ). Therefore $g \circ f$ is ψgs-continuous. □

Proposition 4.14. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be two ψgs-totally continuous function, then $g \circ f : (X, \tau) \to (Z, \eta)$ is also a ψgs-totally continuous function.*

Proof. Let V be any ψgs-open set in (Z, η). Since g is ψgs-totally continuous, $g^{-1}(V)$ is clopen in (Y, σ). By Result 2.2, $g^{-1}(V)$ is ψgs-clopen in (Y, σ) which implies that $g^{-1}(V)$ is ψgs-open in (Y, σ). Since f is ψgs-totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X, τ). Therefore $g \circ f$ is ψgs-totally continuous. □

References

[1] K. Balachandran, P. Sundaram and H. Maki, *On generalized continuous maps in topological spaces*, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 12(1991), 5-13.

[2] P. Bhattacharyya and B. K. Lahiri, *Semi-generalized closed sets in topology*, Indian J. Math., 29(1987), 376-382.

[3] R. Devi, H. Maki and K. Balachandran, *Semi-generalized homeomorphisms and generalized semi-homeomorphism in topological spaces*, Indian J. Pure. Appl. Math., 26(3)(1995), 271-284.

[4] S. Gowsalya and N. Balamani, *On ψgs-closed sets in topological spaces*, International Journal of Advance Foundation and Research in Computer, 3(4)(2016), 52-61.

[5] S. Gowsalya and N. Balamani, *ψ generalized semi closed sets in topological spaces*, M.Sc. Thesis, Avinashilingam Institute for Home Science and Higher Education for Women University, Coimbatore, India, (2016).

[6] R. C. Jain, *The Role of Regular Open sets in General Topology*, Ph.D. Thesis, Meerut University Institute of a Duanced Studies, Meerut, India, (1980).

[7] N. Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer, Math. Monthly, 70(1963), 36-41.

[8] N. Levine, *Generalized closed sets in topology*, Rend. Circ. Math. Palermo, 19(2)(1970), 89 -96.

[9] P. Sundram, H. Maki and K. Balachandran, *Semi-generalized continuous maps and semi-$T_{1/2}$ spaces*, Bull. Fukuoda Univ. Ed. Part III, 40(1991), 33-40.

[10] M. K. R. S. Veerakumar, *Between semi-closed sets and semi-pre closed sets*, Rend. Istit. Mat. Univ. Trieste, (ITALY), XXXXII(2000), 25-41.