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Positive Pre A^*-algebra Function in terms of DNF

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Abstract: In this paper, positive Pre A^*-algebra function is defined and to support this two theorems have been proved.

Keywords: Positive Pre A^*-algebra function, prime implicants, absorption,monotone function.

1. Introduction

In 1994, P.KoteswaraRao [1] first introduced the concept of A^*-algebra $\left(A, \wedge, \vee, *, (-)^{\sim}\, (-)_{\pi}\,, 0, 1, 2\right)$ In 2000, J.Venkateswara Rao [2] introduced the concept Pre A*-algebra $(A, \wedge, \vee, (-)^{\sim})$ analogous to C-algebra as a reduct of A^*- algebra, K.Srinivasa Rao [3] describes the concept of Pre A^*-Algebra as a Poset and established necessary conditions for a poset to become a lattice with respect to meet and as well as join. J. Venkateswara Rao [4] analyze the properties of Pre A^*-function. He defined implicants of Pre A^*-algebra function [5]. In this paper, Positive Pre A^*-algebra function is defined with two theorems.

2. Pre A^*-algebra

In this section, we concentrate on the algebraic structure of Pre A^*-algebra and C-algebra. We recalled some fundamental results which are also used in the later text.

Definition 2.1 ([3]). *An algebra $(A, \wedge, \vee, (-)^{\sim})$ where A is non-empty set with $1, \wedge, \vee$ are binary operations and $(-)^{\sim}$ is a unary operation satisfying*

(a). $x^{\sim\sim} = x, \forall\ x \in A$

(b). $x \wedge x = x, \forall\ x \in A$

(c). $x \wedge y = y \wedge x, \forall\ x, y \in A$

(d). $(x \wedge y)^{\sim} = x^{\sim} \vee y^{\sim}, \forall\ x, y \in A$

(e). $x \wedge (y \wedge z) = (x \wedge y) \wedge z, \forall\ x, y, z \in A$

(f). $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z), \forall\ x, y, z \in A$

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(g). $x \wedge y = x \wedge (x^{\sim} \vee y), \forall\ x, y \in A$

is called Pre A^**-algebra.*

Example 2.2 ([3]). $2 = \{0, 1, 2\}$ *with operations* $\wedge, \vee, (-)^{\sim}$ *defined below is a Pre* A^**-algebra.*

$\wedge$	0	1	2
0	0	0	2
1	0	1	2
2	2	2	2

$\vee$	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

x	$x^{\sim}$
0	1
1	0
2	2

Lemma 2.3 ([3]). *Every Pre* A^**-algebra with 1 satisfies the following laws*

$$x \vee 1 = x \vee x^{\sim}$$

$$x \wedge 0 = x \wedge x^{\sim}$$

Lemma 2.4 ([3]). *Every Pre* A^**-algebra with 1 satisfies the following laws.*

$$x \wedge (x^{\sim} \vee x) = x \vee (x^{\sim} \wedge x = x$$

$$(x \vee x^{\sim}) \wedge y = (x \wedge y) \vee (x^{\sim} \wedge y)$$

$$(x \vee y) \wedge z = (x \wedge z) \vee (x^{\sim} \wedge y \wedge z)$$

Definition 2.5 ([3]). *Let A be a Pre* A^**-algebra. An element* $x \in A$ *is called central element of A if* $x \vee x^{\sim} = 1$ *and the set* $\{x \in A / x \vee x^{\sim} = 1\}$ *of all central elements of A is called the centre of A and it is denoted by* $B(A)$*.*

Theorem 2.6 ([3]). *Let A be a Pre* A^**-algebra with 1, then* $B(A)$ *is a Boolean algebra with the induced operations* $\wedge, \vee, (-)^{\sim}$*.*

Definition 2.7 ([6]). *Let* $(A, \wedge, \vee, \sim)$ *be a Pre* A^**-algebra. Expressions involving members of A and the operations* $\wedge, \vee, \sim$ *are called Pre* A^**-algebra expressions (or) polynomials.*

Definition 2.8 ([6]). *A Pre* A^**-algebra function is said to be in disjunctive normal form in n variables* $\alpha_1, \alpha_2, \ldots, \alpha_n$ *if it can be written as join of terms of the type* $f_1(\alpha_1) \wedge f_2(\alpha_2) \wedge \cdots \wedge f_n(\alpha_n)$ *where* $f_i(\alpha_i) = \alpha_i$ *or* $\alpha_i^{\sim}$ $\forall\ i = 1$ *to n and no two terms are same.* $f_1(\alpha_1) \wedge f_2(\alpha_2) \wedge \cdots \wedge f_n(\alpha_n)$ *are called minterms or minimal polynomials.*

Theorem 2.9 ([6]). *Every Pre* A^**-algebra function can be put in DNF.*

Definition 2.10 ([6]). *A Pre* A^**-algebra function is said to be in conjunctive normal form in n variables* $\alpha_1, \alpha_2, \ldots, \alpha_n$ *if it can be written as meet of terms of the type* $f_1(\alpha_1) \vee f_2(\alpha_2) \vee \cdots \vee f_n(\alpha_n)$ *where* $f_i(\alpha_i) = \alpha_i$ *or* $\alpha_i^{\sim}$ $\forall\ i = 1$ *to n and no two terms are same.* $f_1(\alpha_1) \vee f_2(\alpha_2) \vee \cdots \vee f_n(\alpha_n)$ *are called maxterms or maximal polynomials*

Theorem 2.11 ([6]). *Each Pre* A^**-algebra function can be put into Canonical form in one and only one way.*

Definition 2.12. *Given two pre* A^**-algebra function f and g on* A^n*, then f implies g if* $f(x) = 2 \Rightarrow g(x) = 2 \quad \forall \quad x \in A^n$ *then* $f \leq g$ *[f is aminorant of g or g is a majorant of f].*

Definition 2.13 ([6]). *Let f be a pre* A^* *algebra function and c be an elementary conjunction. Then c is an implicant of f if c implies f.*

Note 2.14. *Suppose $f = (x \wedge y) \vee (x \wedge \bar{y} \wedge z)$ is a pre A^* algebra function. Then $x \wedge y, x \wedge \bar{y} \wedge z, x \wedge z$ are implicant of f.*

Definition 2.15 ([4])**.** *If φ is a DNF representant of the pre A^* algebra function, then every term of φ is an implicant of f.*

Note 2.16. *Simplification of pre A^* algebra function to replace long implicants by short ones in DNF.*

3. Positive Pre A^*-algebra Function

Definition 3.1. *Let f be a pre A^* algebra function and C_1, C_2 be an implicant of f. Then C_1 absorbs C_2 if $C_1 \vee C_2 = C_1$ that is $C_2 \leq C_1$.*

Definition 3.2. *Let f be a pre A^* algebra function and c_1 be an implicant of f. Then c_1 is a prime implicant of f, if c_1 is not absorbs by any other implicant of f i.e., $c_1 = c_2$.*

Note 3.3. $f = (x \wedge y) \vee (x \wedge \bar{y} \wedge z)$*. Here xy, xz are prime implicant of f and $x \wedge \vec{y} \wedge z$ is not prime. Since $x \wedge \vec{y} \wedge z \leq x \wedge z$.*

Definition 3.4 (Positive Function)**.** *Let f be a pre A^* algebra function on A^n and let $k \in \{1, 2, 3, \dots\}$. Then f is positive (respectively negative) in the variable x_k. If $f_{|x_k=0} \leq f_{|x_k=1} \leq f_{|x_k=2}$ (respectively $f_{|x_k=0} \geq f_{|x_k=1} \geq f_{|x_k=2}$) f is monotone in x_k of f is either positive or negative in x_k; f is positive in x_k, if the value of x_k is from 0 to 1, 2 to 2. It will not be from 2 to 2, 1 to 0.*

Definition 3.5. *A pre A^* algebra function is positive if it is positive in each of its variables.*

Note 3.6. *Let $f(x, y, z) = (\bar{x}_1 \wedge \bar{x}_2) \vee x_3$; f is negative in x_1 and x_2 and positive in x_3. f is monotone but it is neither positive or negative.*

Theorem 3.7. *Let f be a pre A^* algebra function on A^n and let g be the function defined by $g(x_1, x_2, \dots, x_n) = f(\bar{x}_1, x_2, \dots, x_n); \forall\ (x_1, x_2, \dots, x_n) \in A^n$. Then g is positive in the variable x_1 if and only if f is negative in x_1.*

Proof. Let $X = (x_1, x_2, \dots, x_n)$ be an element in A^n. Since g is positive in the variable x_1; $g_{|x_1=0} \leq g_{|x_1=1} \leq g_{|x_1=2}$.
If $x_1 = 0$, then

$$g(0, x_2, \dots, x_n) = f(\bar{0}, x_2, \dots, x_n)$$
$$g(0, x_2, \dots, x_n) = f(1, x_2, \dots, x_n)$$

If $x_1 = 2$, then

$$g(2, x_2, \dots, x_n) = f(\bar{2}, x_2, \dots, x_n)$$
$$g(2, x_2, \dots, x_n) = f(2, x_2, \dots, x_n)$$

Here f goes from 0 to 1, 2 to 2. Therefore $f_{|x_1=2} \leq f_{|x_1=1} \leq f_{|x_1=0}$. Therefore f is negative.
Conversely, if f is negative. Then $f_{|x_1=2} \leq f_{|x_1=1} \leq f_{|x_1=0}$ i.e., $f(\bar{x}_1, x_2, \dots, x_n) = g(x_1, x_2, \dots, x_n)$
If $x_1 = 2$, then

$$f(\bar{2}, x_2, \dots, x_n) = g(2, x_2, \dots, x_n)$$
$$f(2, x_2, \dots, x_n) = g(2, x_2, \dots, x_n)$$

If $x_1 = 1$, then

$$f(\bar{1}, x_2, \ldots, x_n) = g(1, x_2, \ldots, x_n)$$

$$f(0, x_2, \ldots, x_n) = g(1, x_2, \ldots, x_n)$$

Hence f value from 2 to 2, 1 to 0. But g value is from 2 to 2, 1 to 1. That is $g_{|x_1=0} \leq g_{|x=1} \leq g_{|x_1=2}$. Therefore g is positive. □

Definition 3.8. *For two points $X = (x_1, x_2, \ldots, x_n)$ and $Y = (y_1, y_2, \ldots, y_n)$ in A^n we can write $X \leq Y$ if $x_i \leq y_i$ $\forall$ $i = 1, 2, \ldots, n$.*

Theorem 3.9. *A pre A^* algebra function f on A^n is positive if and only if $f(X) \leq f(Y)$ $\forall$ $X, Y \in A^n$ such that $X \leq Y$.*

Proof. Given that f is a positive function on pre A^* algebra function on A^n i.e., $f_{|x_1=0} \leq f_{|x=1} \leq f_{|x_1=2}$ i.e., f changes value from 0 to 1, 2 to 2. Let us take two points in A^n i.e., $X, Y \in A^n$ i.e., $X = (x_1, x_2, \ldots, x_n)$ and $Y = (y_1, y_2, \ldots, y_n)$. Let us take this is an increasing function $X \leq Y$ i.e., $x_i \leq y_i$ $\forall$ $i = 1, 2, \ldots, n$. Therefore f is a positive function i.e., it is an increasing function i.e., it changes value from 0 to 1, 2 to 2 for X and Y.

$$f_{|x_k=0}(X) \leq f_{|x_k=1}(X) \leq f_{|x_k=2}(X) \leq f_{|y_k=0}(Y) \leq f_{|y_k=1}(Y) \leq f_{|y_k=2}(Y)$$

This implies that $f(X) \leq f(Y)$.
Conversely, if $f(X) \leq f(Y)$ for all $X, Y \in A^n$ such that $X \leq Y$. We shall prove of by induction. If $x_1 \leq y_1$ then $f(x_1) \leq f(y_1)$

$$f_{|x_1=0}(X) \leq f_{|x_1=1}(X) \leq f_{|x_1=2}(X) \leq f_{|y_1=0}(Y) \leq f_{|y_1=1}(Y) \leq f_{|y_1=2}(Y)$$

Hence at x_1 the function changes value from 0 to 1, 2 to 2. Let us assume that this is true for k

$$f_{|x_k=0}(X) \leq f_{|x_k=1}(X) \leq f_{|x_k=2}(X) \leq f_{|y_k=0}(Y) \leq f_{|y_k=1}(Y) \leq f_{|y_k=2}(Y)$$

Hence f is an increasing function. It changes value from 0 to 1, 2 to 2 for x_{k+1} upto x_k it is an increasing function i.e., values are from 0 to 1, 2 to 2.

$$x_k \leq y_k \Rightarrow x_{k+1} \leq y_{k+1}, \therefore \; x_k \leq x_{k+1}$$

We can write this as

$$f_{|x_{k+1}=0}(X) \leq f_{|x_{k+1}=1}(X) \leq f_{|x_{k+1}=2}(X) \leq f_{|y_{k+1}=0}(Y) \leq f_{|y_{k+1}=1}(Y) \leq f_{|y_{k+1}=2}(Y)$$

at $(k+1)^{th}$ values f values is increasing and it changes value from 0 to 1, 2 to 2. Therefore it is true for $k+1$. Therefore it is true for all value. Therefore f is positive on A^n. □

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