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Some Properties of Intuitionistic Fuzzy Soft Graph

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Abstract: We present a novel framework for handling intuitionistic fuzzy soft information by combining the theory of intuitionistic fuzzy soft sets with graphs. We presented the notions of Intuitionistic Fuzzy Soft graph, Strong intuitionistic fuzzy soft graph, Complete intutionistic fuzzy soft graph in our previous paper. In this article, we introduce some new concepts of intuitionistic fuzzy soft graph, explain notion of ifs-complement of an intuitionistic fuzzy soft graph and ifs-ϕ-complement of an intuitionistic fuzzy soft graph with example. Finally, we investigate the properties of ifs-complement and ifs-ϕ-complement of an intuitionistic fuzzy soft graph.

Keywords: Intuitionistic Fuzzy Soft Graph, ifs-Complement of an Intuitionistic Fuzzy Soft Graph, ifs-ϕ-Complement of an Intuitionistic Fuzzy Soft Graph.

1. Introduction and Preliminaries

Molodtsov [6] introduced the concept of soft set that can be seen as a new mathematical theory for dealing with uncertainties. The soft set theory has been applied to many different fields with greatness. P. K. Maji [11] worked on theoretical study of soft sets in detail. The most appreciate theory to deal with uncertainties is the theory of fuzzy sets, developed by Zadeh [8] in 1965. But it has an inherent difficulty to set the membership function in each particular cases. The generalization of Zadeh's fuzzy set, called intuitionistic fuzzy set was introduced by Atanassov [4] which is characterized by a membership function and a non-membership function. The first definition of intuitionistic fuzzy graph was introduced by K. T. Atanassov [5] in 1999. M. G. Karunambigai and R. Parvathy [9] introduced intuitionistic fuzzy graph as a special case of Atanassov's intuitionistic fuzzy graph. In 2015, Sumit Mohinta and T. K. Samanta [13] introduced the concept of Fuzzy Soft Graph. We presented the definition of Intuitionistic Fuzzy Soft Graph [2]. We present a novel framework for handling intuitionistic fuzzy soft information by combining the theory of intuitionistic fuzzy soft sets with graphs. We presented the notions of Intuitionistic Fuzzy Soft graph, Strong intuitionistic fuzzy soft graph, Complete intutionistic fuzzy soft graph in our previous paper. In this article, we introduce some new concepts of intuitionistic fuzzy soft graph, explain notion of ifs-complement of an intuitionistic fuzzy soft graph and ifs-ϕ-complement of an intuitionistic fuzzy soft graph with example. Finally, we investigate the properties of ifs-complement and ifs-ϕ-complement of an intuitionistic fuzzy soft graph.

Definition 1.1. *A Fuzzy Set of a base set $V = \{v_1, v_2, \ldots, v_n\}$ (non-empty set) is specified by its membership function σ; where $\sigma : V \to [0,1]$ assigning to each $v_i \in V$, the degree or grade to which $V \in \sigma$.*

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Definition 1.2. *A Fuzzy graph $G = (\sigma, \mu)$ is a pair of function $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$, where for all $v_i, v_j \in V$ we have $\mu(v_i, v_j) \leq \sigma(v_i) \wedge \sigma(v_j)$ for each $(v_i, v_j) \in V \times V$. Here σ and μ are respectively called fuzzy vertex and fuzzy edge of the fuzzy graph $G = (\sigma, \mu)$.*

Definition 1.3. *An Intuitionistic Fuzzy Graph is defined as $G = (V, E, \mu, \gamma)$ where*

(1) $V = \{v_1, v_2, \ldots, v_n\}$ (non-empty set) such that $\mu_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V,\ i = 1, 2 \ldots n$

(2) $E \subset V \times V$ where $\mu_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are such that

(a) $\mu_2(v_i, v_j) \leq min\{\mu_1(v_i), \mu_1(v_j)\}$

(b) $\gamma_2(v_i, v_j) \leq max\{\gamma_1(v_i), \gamma_1(v_j)\}$ and

(c) $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$, $0 \leq \mu_2(v_i, v_j), \gamma_2(v_i, v_j), \pi(v_i, v_j) \leq 1$, where $\pi(v_i, v_j) = 1 - \mu_2(v_i, v_j) - \gamma_2(v_i, v_j)$ for every $(v_i, v_j) \in E,\ i, j = 1, 2, \ldots, n$

Let U be an initial universal set, P be a set of parameters, $\mathscr{P}(U)$ be the power set of U and $A \subseteq P$.

Definition 1.4. *A pair (F, A) is called a $softset$ over U if and only if F is a mapping of A into the set of all subsets of the set U.*

Definition 1.5. *A pair $(\bar{F}, A)$ is called fuzzy soft set over U , where F is a mapping given by $\bar{F} : A \to I^U$; I^U denotes the collection of all fuzzy subsets of U; $A \subseteq P$.*

Definition 1.6. *A pair $(\breve{F}, A)$ is called an intuitionistic fuzzy soft set over U, where $\breve{F}$ is a mapping given by $\breve{F} : A \to IF^U$; IF^U denotes the collection of all intuitionistic fuzzy subsets of U; $A \subseteq P$.*

2. Intuitionistic Fuzzy Soft Graph

Definition 2.1 ([2]). *Let $G = (V, E)$ be a simple graph, $V = \{v_1, v_2, \ldots, v_n\}$ (non-empty set), $E \subseteq V \times V$, P(parameter set) and $A \subseteq P$. Also let*

(i) μ_1 is a membership function defined by

$\mu_1 : A \to IF^U(V)$ ($IF^U(V)$ denotes collection of all intuitionistic fuzzy subsets in V)

$a \mapsto \mu_1(a) = \mu_{1a}$ (say), $a \in A$ and

$\mu_{1a} : V \to [0, 1]$

$v_i \mapsto \mu_{1a}(v_i)$

(A, μ_1) Intuitionistic fuzzy soft vertex of membership function and

$\gamma_1 : A \to IF^U(V)$ ($IF^U(V)$ denotes collection of all intuitionistic fuzzy subsets in V)

$a \mapsto \gamma_1(a) = \gamma_{1a}$ (say), $a \in A$ and

$\gamma_{1a} : V \to [0, 1]$

$v_i \mapsto \gamma_{1a}(v_i)$

(A, γ_1) Intuitionistic fuzzy soft vertex of non-membership function such that $0 \leq \mu_{1a}(v_i) + \gamma_{1a}(v_i) \leq 1$ for every $v_i \in V,\ i = 1, 2 \ldots n$ and $a \in A$.

(ii) μ_2 *is a membership function defined on* E *by*

$\mu_2 : A \to IF^U(V \times V)$ *($IF^U(V \times V)$ denotes collection of all intuitionistic fuzzy subsets in* E*)*

$a \mapsto \mu_2(a) = \mu_{2a}$ *(say),* $a \in A$ *and*

$\mu_{2a} : V \times V \to [0, 1]$

$(v_i, v_j) \mapsto \mu_{2a}(v_i, v_j)$

γ_2 *is the non membership function defined on* E *by*

$\gamma_2 : A \to IF^U(V \times V)$ *($IF^U(V \times V)$ denotes collection of all intuitionistic fuzzy subsets in* $V \times V$*)*

$a \mapsto \gamma_2(a) = \gamma_{2a}$ *(say),* $a \in A$ *and*

$\gamma_{2a} : V \times V \to [0, 1]$

$(v_i, v_j) \mapsto \gamma_{2a}(v_i, v_j)$

where (A, μ_2),(A, γ_2) *are Intuitionistic fuzzy soft edge of membership function and non- membership function satisfying*

(a) $\mu_{2a}(v_i, v_j) \leq min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\}$

(b) $\gamma_{2a}(v_i, v_j) \leq max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\}$ *and*

(c) $0 \leq \mu_{2a}(v_i, v_j) + \gamma_{2a}(v_i, v_j) \leq 1$, $0 \leq \mu_{2a}(v_i, v_j), \gamma_{2a}(v_i, v_j) \leq 1$, *forevery* $(v_i, v_j) \in E, i, j = 1, 2, \ldots, n$ *and* $a \in A$.

Then $G^*_{A,V,E} = (V, E, (A, \mu_1), (A, \gamma_1), (A, \mu_2), (A, \gamma_2))$ *is said to be the Intuitionistic Fuzzy Soft Graph (IFSG) and this IFSG is denoted by* $G^*_{A,V,E}$.

Note 2.2. *For every Intuitionistic fuzzy soft graph* $G^*_{A,V,E}$*, the degree of indeterminacy of the vertex* $v_i \in V$ *for the parameter* $a \in A$ *is* $\epsilon_{1a}(v_i) = 1 - \mu_{1a}(v_i) - \gamma_{1a}(v_i)$ *and the degree of indeterminacy of the edge* $(v_i, v_j) \in E$ *for the parameter* $a \in A$ *is* $\epsilon_{2a}(v_i, v_j) = 1 - \mu_{2a}(v_i, v_j) - \gamma_{2a}(v_i, v_j)$.

Example 2.3. *Consider a simple graph* $G = (V, E)$ *where* $V = \{v_1, v_2, v_3\}$ *and* $E = \{(v_1, v_2), (v_2, v_3), (v_1, v_3)\}$. *Let* $A = \{a_1, a_2, a_3\}$ *be the parameter set. Then the Intuitionistic Fuzzy Soft Graph (IFSG),* $G^*_{A,V,E} = (V, E, (A, \mu_1), (A, \gamma_1), (A, \mu_2), (A, \gamma_2))$ *is described in Table 1 and Figure 1.*

(a)

μ_{1a}	v_1	v_2	v_3
a_1	0.8	0.6	0
a_2	0.9	0.65	0.5
a_3	0.2	0.75	0.8

(b)

γ_{1a}	v_1	v_2	v_3
a_1	0.1	0.2	0
a_2	0.05	0.25	0.4
a_3	0.6	0.1	0.15

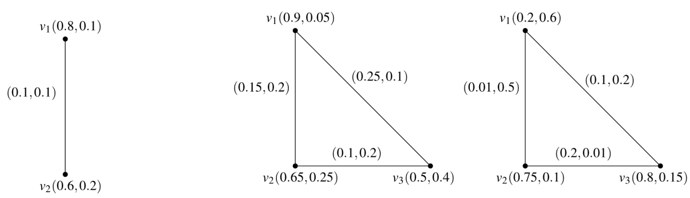
(c)

μ_{2a}	(v_1, v_2)	(v_1, v_3)	(v_2, v_3)
a_1	0.1	0	0
a_2	0.15	0.25	0.1
a_3	0.01	0.1	0.2

(d)

γ_{2a}	(v_1, v_2)	(v_1, v_3)	(v_2, v_3)
a_1	0.1	0	0
a_2	0.2	0.1	0.2
a_3	0.5	0. 2	0.01

Table 1:



1(a) IFSG corresponding to the parameter a_1.

1(b) IFSG corresponding to the parameter a_2.

1(c) IFSG corresponding to the parameter a_3.

Figure 1:

Definition 2.4 ([3]). *Ifs-order of an intuitionistic fuzzy soft graph* $G^*_{A,V,E} = (V, E, (A, \mu_1), (A, \gamma_1), (A, \mu_2), (A, \gamma_2))$ *is defined as*

$$O(G^*_{A,V,E}) = \left(\sum_{a \in A} \left(\sum_{v_i \in V} \mu_{1a}(v_i) \right), \sum_{a \in A} \left(\sum_{v_i \in V} \gamma_{1a}(v_i) \right) \right)$$

for every $v_i \in V$ *and for every* $a \in A$ *.*

Definition 2.5 ([3]). *Ifs-size of an intuitionistic fuzzy soft graph* $G^*_{A,V,E} = (V, E, (A, \mu_1), (A, \gamma_1), (A, \mu_2), (A, \gamma_2))$ *is defined as*

$$S(G^*_{A,V,E}) = \left(\sum_{a \in A} \left(\sum_{v_i, v_j \in V, v_i \neq v_j} \mu_{2a}(v_i, v_j) \right), \sum_{a \in A} \left(\sum_{v_i, v_j \in V, v_i \neq v_j} \gamma_{2a}(v_i, v_j) \right) \right)$$

for every $(v_i, v_j) \in E$ *and for every* $a \in A$ *.*

Example 2.6. *Ifs-order of the intuitionistic fuzzy soft graph* $G^*_{A,V,E}$ *of Example 2.3 is*

$O(G^*_{A,V,E}) = (((0.6+0.8)+(0.9+0.65+0.5)+(0.2+0.75+0.8)), ((0.1+0.2)+(0.05+0.25+0.4)+(0.6+0.1+0.15))) = (5.2, 1.85)$

Ifs-size of the intuitionistic fuzzy soft graph $G^*_{A,V,E}$ *of Example 2.3 is*

$S(G^*_{A,V,E}) = (((0.1) + (0.15 + 0.1 + 0.25) + (0.01 + 0.2 + 0.1)), ((0.1) + (0.2 + 0.2 + 0.1) + (0.5 + 0.01 + 0.2))) = (0.91, 1.31)$

3. Basic Definitions of an Intuitionistic Fuzzy Soft Graph and Some of its Properties

Definition 3.1. *An edge* $(v_i, v_j) \in E$ *in an Intuitionistic Fuzzy Soft Graph* $G^*_{A,V,E}$ *is said to be an ifs-effective edge for some parameter* $a \in A$ *if* $\mu_{2a}(v_i, v_j) = min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\}$ *and* $\gamma_{2a}(v_i, v_j) = \max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\}$ *for* $(v_i, v_j) \in E$ *and for* $a \in A$*.*

Definition 3.2. *A node in an IFSG* $G^*_{A,V,E}$ *is called an end node for some parameter* $a \in A$ *if it has atmost one strong neighbour in* $G^*_{A,V,E}$ *for* $a \in A$*.*

Definition 3.3. *An edge for some* $a \in A$ *in an IFSG joining a vertex to itself is called an ifs-loop.*

Definition 3.4. *In an IFSG* $G^*_{A,V,E}$*, if for every* $a \in A$*,there is more than one edge joining two vertices, then* $G^*_{A,V,E}$ *is called an intuitionistic fuzzy soft pseudo graph (ifs-pseudo graph) and these edges are called intuitionistic fuzzy soft multiple edges (ifs-multiple edges).*

Definition 3.5. *An IFSG is said to be Intuitionistic fuzzy soft simple Graph if it has neither ifs-loop nor ifs-multiple edges for every parameter* $a \in A$*.*

Definition 3.6. *In an IFSG* $G^*_{A,V,E}$*, if two vertices have an edge joining them for some* $a \in A$*, then they are called ifs-adjacent vertices for* $a \in A$ *and if two edges are incident on a common vertex for some* $a \in A$*, then they are called ifs-adjacent edges.*

Definition 3.7. *An IFSG* $H^*_{B,V',E'} = (V', E', (A, \mu'_1), (A, \gamma'_1), (A, \mu'_2), (A, \gamma'_2))$ *is said to be Intuitionistic fuzzy soft subgraph (IFSSG) of* $G^*_{A,V,E} = (V, E, (A, \mu_1), (A, \gamma_1), (A, \mu_2), (A, \gamma_2))$ *if* $B \subseteq A$*,* $V' \subseteq V$ *and* $E' \subseteq E$*. In other words, if* $B \subseteq A$ *,* $\mu'_{1a}(v_i) \leq \mu_{1a}(v_i)$*;* $\gamma'_{1a}(v_i) \geq \gamma_{1a}(v_i)$ *and* $\mu'_{2a}(v_i, v_j) \leq \mu_{2a}(v_i, v_j)$*;* $\gamma'_{2a}(v_i, v_j) \geq \gamma_{2a}(v_i, v_j)$ *for every* $v_i \in V'$*,* $(v_i, v_j) \in E'$ *and for every* $a \in B$*.*

Definition 3.8. *An Intuitionistic fuzzy soft subgraph* $H^*_{B,V',E'}$ *of an IFSG* $G^*_{A,V,E}$ *is said to be Spanning intuitionistic fuzzy Soft subgraph if* $\mu'_{1a}(v_i) = \mu_{1a}(v_i)$ *and* $\gamma'_{1a}(v_i) = \gamma_{1a}(v_i)$ *for every* $v_i \in V$ *and for every* $a \in A$*.*

Example 3.9. *Consider the Intuitionistic fuzzy soft graph in Example 2.3, here the IFSG* $H^*_{B,V',E'} = (V', E', (B, \mu'_1),(B, \gamma'_1),(B, \mu'_2),(B, \gamma'_2))$,*where* $V' = \{v_1, v_2, v_3\}$, $E' = \{(v_1, v_2), (v_2, v_3), (v_1, v_3)\}$ *and* $B = \{a_1, a_2\}$ *be the parameter set described in Table 2 and Figure 2 is an Intuitionistic fuzzy soft subgraph (IFSSG) of the IFSG* $G^*_{A,V,E} = (V, E, (A, \mu_1),(A, \gamma_1),(A, \mu_2),(A, \gamma_2))$ *described in Table 1 and Figure 1.*

(a)

μ_{1a}	v_1	v_2	v_3
a_1	0.2	0.4	0
a_2	0.7	0.65	0.2

(b)

γ_{1a}	v_1	v_2	v_3
a_1	0.2	0.2	0
a_2	0.1	0.3	0.6

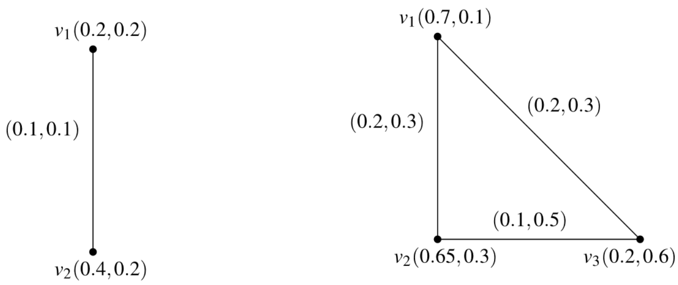
(c)

μ_{2a}	(v_1, v_2)	(v_1, v_3)	(v_2, v_3)
a_1	0.1	0	0
a_2	0.2	0.2	0.1

(d)

γ_{2a}	(v_1, v_2)	(v_1, v_3)	(v_2, v_3)
a_1	0.1	0	0
a_2	0.3	0.3	0.5

Table 2:



2(a) IFSSG corresponding to the parameter a_1.

2(b) IFSSG corresponding to the parameter a_2.

Figure 2:

Here $H^*_{B,V',E'} = (V', E', (B, \mu'_1),(B, \gamma'_1),(B, \mu'_2),(B, \gamma'_2))$ is an Intuitionistic fuzzy soft subgraph (IFSSG) of IFSG $G^*_{A,V,E} = (V, E, (A, \mu_1), (A, \gamma_1), (A, \mu_2), (A, \gamma_2))$ of Example 2.3.

Definition 3.10. *An intuitionistic fuzzy soft graph* $G^*_{A,V,E}$ *is said to be Semi-*μ_a*-strong Intuitionistic Fuzzy Soft Graph if* $\mu_{2a}(v_i, v_j) = min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\}$ *for every* $(v_i, v_j) \in E$ *and for every* $a \in A$.

Definition 3.11. *An intuitionistic fuzzy soft graph* $G^*_{A,V,E}$ *is said to be Semi-*γ_a*-strong Intuitionistic Fuzzy Soft Graph if* $\gamma_{2a}(v_i, v_j) = max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\}$ *for every* $(v_i, v_j) \in E$ *and for every* $a \in A$.

Definition 3.12. *An intuitionistic fuzzy soft graph* $G^*_{A,V,E}$ *is said to be Complete-*μ_a*-strong Intuitionistic Fuzzy Soft Graph if* $\mu_{2a}(v_i, v_j) = min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\}$ *and* $\gamma_{2a}(v_i, v_j) \leq max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\}$ *for every* $v_i, v_j \in V$ *and for every* $a \in A$.

Definition 3.13. *An intuitionistic fuzzy soft graph* $G^*_{A,V,E}$ *is said to be Complete-*γ_a*-strong Intuitionistic Fuzzy Soft Graph if* $\mu_{2a}(v_i, v_j) \leq min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\}$ *and* $\gamma_{2a}(v_i, v_j) = max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\}$ *for every* $v_i, v_j \in V$ *and for every* $a \in A$.

Definition 3.14 ([2]). *An intuitionistic fuzzy soft graph* $G^* = (V, E, (A, \mu_1), (A, \gamma_1), (A, \mu_2), (A, \gamma_2))$ *is said to be* **Strong Intuitionistic Fuzzy Soft Graph** *if* $\mu_{2a}(v_i, v_j) = min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\}$ *and* $\gamma_{2a}(v_i, v_j) = max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\}$ *for every* $(v_i, v_j) \in E$ *and for every* $a \in A$ *and is said to be* **Complete Intuitionistic Fuzzy Soft Graph** *if* $\mu_{2a}(v_i, v_j) = min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\}$ *and* $\gamma_{2a}(v_i, v_j) = max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\}$ *for every* $v_i, v_j \in V$ *and for every* $a \in A$.

Definition 3.15. *Let* $G = (V, E)$ *be a simple graph, P(Parameter set). Also let* $V_1, V_2 \subseteq V$, $E_1, E_2 \subseteq E$, $A, B \subseteq P$ *and* $G^*_{A,V_1,E_1} = (V_1, E_1, (A, \mu_1), (A, \gamma_1), (A, \mu_2), (A, \gamma_2))$ *and* $G^*_{B,V_2,E_2} = (V_2, E_2, (B, \mu'_1), (B, \gamma'_1), (B, \mu'_2), (A, \gamma'_2))$ *be* *two intuitionistic fuzzy soft graphs. An isomorphism between* G^*_{A,V_1,E_1} *and* G^*_{B,V_2,E_2} *denoted by* $G^*_{A,V_1,E_1} \cong G^*_{B,V_2,E_2}$ *if there exists a bijective mapping* $h : V_1 \to V_2$ *which satisfies* $\mu_{1a}(v_i) = \mu'_{1a}(h(v_i))$, $\gamma_{1a}(v_i) = \gamma'_{1a}(h(v_i))$ *and* $\mu_{2a}(v_i, v_j) = \mu'_{2a}(h(v_i), h(v_j))$, $\gamma_{2a}(v_i, v_j) = \gamma'_{2a}(h(v_i), h(v_j))$ *for every* $v_i, v_j \in V$ *and for every* $a \in A$.

4. Ifs-complement of an Intuitionistic Fuzzy Soft Graph and Some of its Properties

Definition 4.1. *Let* $G^*_{A,V,E} = (V, E, (A, \mu_1), (A, \gamma_1), (A, \mu_2), (A, \gamma_2))$ *be an IFSG. The* **ifs − complement** *of IFSG* $G^*_{A,V,E}$ *is defined as* $\overline{G}^*_{A,\overline{V},\overline{E}} = (\overline{V}, \overline{E}, (A, \overline{\mu}_1), (A, \overline{\gamma}_1), (A, \overline{\mu}_2), (A, \overline{\gamma}_2))$ *where*

(i). $\overline{\mu}_{1a}(v_i) = \mu_{1a}(v_i)$; $\overline{\gamma}_{1a}(v_i) = \gamma_{1a}(v_i)$ *for every* $v_i \in V$ *and for every* $a \in A$.

$$\text{(ii). } \overline{\mu}_{2a}(v_i, v_j) = \begin{cases} \min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\} - \mu_{2a}(v_i, v_j) & \text{if } \mu_{2a}(v_i, v_j) \neq 0,\ a \in A \\ \min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\} & \text{if } \mu_{2a}(v_i, v_j) = 0,\ a \in A \end{cases}$$

$$\overline{\gamma}_{2a}(v_i, v_j) = \begin{cases} \max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\} - \gamma_{2a}(v_i, v_j) & \text{if } \gamma_{2a}(v_i, v_j) \neq 0,\ a \in A \\ \max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\} & \text{if } \gamma_{2a}(v_i, v_j) = 0,\ a \in A \end{cases}$$

for every $v_i, v_j \in V$ *and for every* $a \in A$.

Theorem 4.2. *For any IFSG,* $G^*_{A,V,E}$, $\overline{\overline{G^*}}_{A,\overline{\overline{V}},\overline{\overline{E}}} \cong G^*_{A,V,E}$

Proof. Let $G^*_{A,V,E} = (V, E, (A, \mu_1), (A, \gamma_1), (A, \mu_2), (A, \gamma_2))$ be an Intuitionistic fuzzy soft graph. Then by Definition 4.1 $\overline{\overline{G^*}}_{A,\overline{\overline{V}},\overline{\overline{E}}} = (\overline{\overline{V}}, \overline{\overline{E}}, (A, \overline{\overline{\mu}}_1), (A, \overline{\overline{\gamma}}_1), (A, \overline{\overline{\mu}}_2), (A, \overline{\overline{\gamma}}_2))$, where

(i). $\overline{\overline{\mu}}_{1a}(v_i) = \overline{\mu}_{1a}(v_i) = \mu_{1a}(v_i)$ and $\overline{\overline{\gamma}}_{1a}(v_i) = \overline{\gamma}_{1a}(v_i) = \gamma_{1a}(v_i)$ for every $v_i \in V$ and for every $a \in A$.

$$\text{(ii). } \overline{\overline{\mu}}_{2a}(v_i, v_j) = \begin{cases} \min\{\overline{\mu}_{1a}(v_i), \overline{\mu}_{1a}(v_j)\} - \overline{\mu}_{2a}(v_i, v_j) & \text{if } \overline{\mu}_{2a}(v_i, v_j) \neq 0,\ a \in A \\ \min\{\overline{\mu}_{1a}(v_i), \overline{\mu}_{1a}(v_j)\} & \text{if } \overline{\mu}_{2a}(v_i, v_j) = 0,\ a \in A \end{cases}$$

$$\overline{\overline{\gamma}}_{2a}(v_i, v_j) = \begin{cases} \max\{\overline{\gamma}_{1a}(v_i), \overline{\gamma}_{1a}(v_j)\} - \overline{\gamma}_{2a}(v_i, v_j) & \text{if } \overline{\gamma}_{2a}(v_i, v_j) \neq 0,\ a \in A \\ \max\{\overline{\gamma}_{1a}(v_i), \overline{\gamma}_{1a}(v_j)\} & \text{if } \overline{\gamma}_{2a}(v_i, v_j) = 0,\ a \in A \end{cases}$$

$\overline{\mu}_{2a}(v_i, v_j) \neq 0,\ a \in A$ *then,*

$$\overline{\overline{\mu}}_{2a}(v_i, v_j) = \begin{cases} \min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\} - (min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\} - (\mu_{2a}(v_i, v_j)) & \text{if } \mu_{2a}(v_i, v_j) \neq 0,\ a \in A \\ 0 & \text{if } \mu_{2a}(v_i, v_j) = 0,\ a \in A \end{cases}$$

$$= \begin{cases} \mu_{2a}(v_i, v_j) & \text{if } (v_i, v_j) \in E \\ 0 & \text{if } (v_i, v_j) \notin E \end{cases}$$

That is, $\overline{\overline{\mu}}_{2a}(v_i, v_j) = \mu_{2a}(v_i, v_j), if\ \mu_{2a}(v_i, v_j) \neq 0$. Also if $\overline{\mu}_{2a}(v_i, v_j) = 0 \ \Rightarrow min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\} - (\mu_{2a}(v_i, v_j) = 0 \Rightarrow \mu_{2a}(v_i, v_j) = min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\} = \overline{\overline{\mu}}_{2a}(v_i, v_j)$. Thus $\overline{\overline{\mu}}_{2a}(v_i, v_j) = \mu_{2a}(v_i, v_j)$ for every $(v_i, v_j) \in E$ *and for every* $a \in A$. Similarly we can prove that $\overline{\overline{\gamma}}_{2a}(v_i, v_j) = \gamma_{2a}(v_i, v_j)$, *for every* $(v_i, v_j) \in E$ *and for every* $a \in A$. Thus $\overline{\overline{G^*}}_{A,V,E} \cong G^*_{A,V,E}$. □

Example 4.3. *Let* $G^*_{A,V,E} = (V, E, (A, \mu_1), (A, \gamma_1), (A, \mu_2), (A, \gamma_2))$ *be an IFSG described in Table 3 and Figure 3, where* $V = \{v_1, v_2, v_3, v_4, v_5\}$ *be the vertex set,* $E = \{(v_1, v_2), (v_1, v_3), (v_3, v_4), (v_3, v_5)\}$ *be the edge set and let* $A = \{a_1, a_2\}$ *be the parameter set. Then the ifs-complement of* $G^*_{A,V,E}$ *be represented as* $\overline{G}^*_{A,\overline{V},\overline{E}} = (\overline{V}, \overline{E}, (A, \overline{\mu}_1), (A, \overline{\gamma}_1), (A, \overline{\mu}_2), (A, \overline{\gamma}_2))$, *which is described in Table 4 and Figure 4.*

(a)

μ_{1a}	v_1	v_2	v_3	v_4	v_5
a_1	0.2	0.3	0.3	0	0
a_2	0.3	0.2	0.4	0.6	0.1

(b)

γ_{1a}	v_1	v_2	v_3	v_4	v_5
a_1	0.3	0.2	0.5	0	0
a_2	0.4	0.5	0.1	0.1	0.1

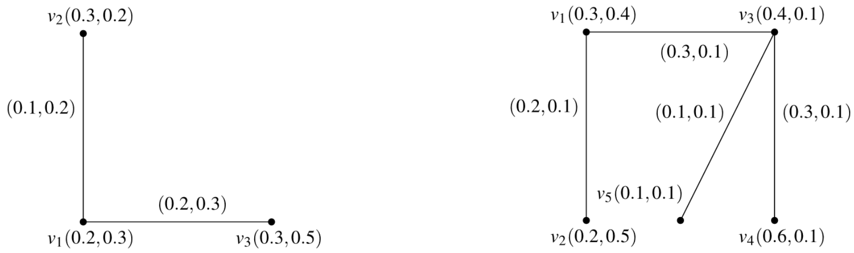
(c)

μ_{2a}	(v_1, v_2)	(v_1, v_3)	(v_3, v_4)	(v_3, v_5)
a_1	0.1	0.2	0	0
a_2	0.2	0.3	0.3	0.1

(d)

γ_{2a}	(v_1, v_2)	(v_1, v_3)	(v_3, v_4)	(v_3, v_5)
a_1	0.2	0.3	0	0
a_2	0.1	0.1	0.1	0.1

Table 3:



3(a) IFSG corresponding to the parameter a_1.

3(b) IFSG corresponding to the parameter a_2

Figure 3:

(a)

$\overline{\mu}_{1a}$	v_1	v_2	v_3	v_4	v_5
a_1	0.2	0.3	0.3	0	0
a_2	0.3	0.2	0.4	0.6	0.1

(b)

$\overline{\gamma}_{1a}$	v_1	v_2	v_3	v_4	v_5
a_1	0.3	0.2	0.5	0	0
a_2	0.4	0.5	0.1	0.1	0.1

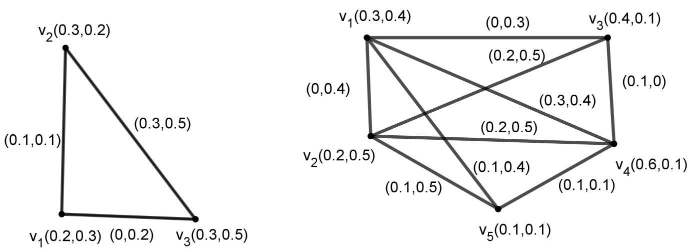
(c)

$\overline{\mu}_{2a}$	(v_1, v_2)	(v_1, v_3)	(v_1, v_4)	(v_1, v_5)	(v_2, v_3)	(v_2, v_4)	(v_2, v_5)	(v_3, v_4)	(v_4, v_5)
a_1	0.1	0	0	0	0.3	0	0	0	0
a_2	0	0	0.3	0.1	0.2	0.2	0.1	0.1	0.1

(d)

$\overline{\gamma}_{2a}$	(v_1, v_2)	(v_1, v_3)	(v_1, v_4)	(v_1, v_5)	(v_2, v_3)	(v_2, v_4)	(v_2, v_5)	(v_3, v_4)	(v_4, v_5)
a_1	0.1	0.2	0	0	0.5	0	0	0	0
a_2	0.4	0.3	0.4	0.4	0.5	0.5	0.5	0	0.1

Table 4:



ifs − complement of IFSG corresponding to the parameter a_1

ifs − complement of IFSG corresponding to the parameter a_2

Figure 4:

Proposition 4.4. *The ifs-complement* $\overline{G}^*_{A,\overline{V},\overline{E}}$ *of an IFSG* $G^*_{A,V,E}$ *satisfies the following properties.*

(i). $O(\overline{G}^*_{A,\overline{V},\overline{E}}) = O(G^*_{A,V,E})$

(ii). If the crisp graph corresponding to every parameter $a \in A$ is complete, then

(a). $\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} \mu_{2a}(v_i,v_j)\right) + \sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} \overline{\mu}_{2a}(v_i,v_j)\right) \leq 2\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} (min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})\right),$

for every $(v_i,v_j)\in E$ and for $a\in A$ and

(b). $\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} \gamma_{2a}(v_i,v_j)\right) + \sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} \overline{\gamma}_{2a}(v_i,v_j)\right) \leq 2\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} (max\{\gamma_{1a}(v_i),\gamma_{1a}(v_j)\})\right),$

for every $(v_i,v_j)\in E$ and for $a\in A$.

Proof.

(i). From the definition of $\overline{G}^*_{A,\overline{V},\overline{E}}$, $O(\overline{G}^*_{A,\overline{V},\overline{E}}) = O(G^*_{A,V,E})$.

(ii). (a). If the crisp subgraph corresponding to every parameter $a\in A$ is complete, then there exists an edge joining every pair of vertices. That is $\mu_{2a}(v_i,v_j)\neq 0$ or $\gamma_{2a}(v_i,v_j)\neq 0$ for every $(v_i,v_j)\in E$ and for every $a\in A$. Then

$$\mu_{2a}(v_i,v_j) \leq min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\},\ \textit{for every } (v_i,v_j)\in E \textit{ and for every } a\in A,$$

$$\overline{\mu}_{2a}(v_i,v_j) = min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\} - \mu_{2a}(v_i,v_j) \textit{ and}$$

$$\overline{\mu}_{2a}(v_i,v_j) \leq min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\} \textit{ for every } (v_i,v_j)\in E \textit{ and for every } a\in A,$$

$$\therefore\ \mu_{2a}(v_i,v_j)+\overline{\mu}_{2a}(v_i,v_j) \leq 2(min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\}) \textit{ for every } (v_i,v_j)\in E \textit{ and for every } a\in A,$$

Now,

$$\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} \mu_{2a}(v_i,v_j)\right) + \sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} \overline{\mu}_{2a}(v_i,v_j)\right) \leq 2\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} (min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})\right)$$

for every $(v_i,v_j)\in E$ and for every $a\in A$. Similarly we can prove that (ii)(b),

$$\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} \gamma_{2a}(v_i,v_j)\right) + \sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} \overline{\gamma}_{2a}(v_i,v_j)\right) \leq 2\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} (max\{\gamma_{1a}(v_i),\gamma_{1a}(v_j)\})\right),$$

for every $(v_i,v_j)\in E$ and for every $a\in A$. □

Example 4.5. *Consider Example 2.3,*

$$O(\overline{G}^*_{A,\overline{V},\overline{E}}) = (((0.8+0.6)+(0.9+0.65+0.5)+(0.2+0.75+0.8)),((0.2+0.1)+(0.05+0.25+0.4)+(0.6+0.1+0.15)))$$
$$= (5.2, 1.85)$$
$$= (O(G^*_{A,V,E})$$

Also,

$$\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} (\mu_{2a}(v_i,v_j)+\overline{\mu}_{2a}(v_i,v_j))\right) = ((0.1)+(0.15+0.1+0.25)+(0.01+0.2+0.1))$$

$$+((0.5)+(0.5+0.4+0.25)+(0.19+0.55+0.1)) = 3.4$$

$$2\left(\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} (min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})\right)\right) = 2((0.6)+(0.65+0.5+0.5)+(0.2+0.2+0.75)) = 6.8$$

$$\textit{Thus} \sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} (\mu_{2a}(v_i,v_j)+\overline{\mu}_{2a}(v_i,v_j))\right) \leq 2\left(\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} (min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})\right)\right)$$

Also,

$$\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(\gamma_{2a}(v_i,v_j)+\overline{\gamma}_{2a}(v_i,v_j))\right)=((0.1)+(0.2+0.2+0.1)+(0.5+0.01+0.2)+$$

$$((0.1)+(0.05+0.2+0.3)+(0.1+0.4+0.14))=2.6$$

$$2\left(\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(max\{\gamma_{1a}(v_i),\gamma_{1a}(v_j)\})\right)\right)=2(((0.2)+(0.25+0.4+0.4)+(0.6+0.6+0.15)))=5.2$$

$$Thus\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(\mu_{2a}(v_i,v_j)+\overline{\mu}_{2a}(v_i,v_j))\right)\leq 2\left(\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})\right)\right)$$

Note 4.6. *For an IFSG, the inequality (ii) of Proposition 4.4 is not generally true. The Example 4.7 illustrates the situation.*

Example 4.7. *Consider the IFSG and its ifs-complement in Example 4.3,*

$$\begin{aligned}O(G^*_{A,V,E})&=((0.2+0.3+0.3+0+0)+(0.3+0.2+0.4+0.6+0.1),(0.3+0.2+0.5+0+0)+(0.4+0.5+0.1+0.1+0.1))\\&=(2.4,2.2)\\&=O(\overline{G}^*_{A,\overline{V},\overline{E}}).\end{aligned}$$

Also,

$$\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(\mu_{2a}(v_i,v_j)+\overline{\mu}_{2a}(v_i,v_j))\right)=((0.1+0.2)+(0.2+0.3+0.3+0.1)+(0.1+0+0.3)$$

$$+(0+0+0.1+0.1+0.1+0.1+0.3+0.2+0.2))=2.7$$

$$2\left(\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})\right)\right)=2(((0.2+0.0.2)+(0.2+0.3+0.4+0.1)))=2.8$$

$$Thus\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(\mu_{2a}(v_i,v_j)+\overline{\mu}_{2a}(v_i,v_j))\right)\leq 2\left(\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})\right)\right)$$

$$But\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(\gamma_{2a}(v_i,v_j)+\overline{\gamma}_{2a}(v_i,v_j))\right)=((0.2+0.3)+(0.1+0.1+0.1+0.1)+$$

$$(0.1+0.2+0.5)+(0.4+0.5+0.1+0+0.3+0.4+0.5+0.4+0.5))=4.8$$

Also,

$$2\left(\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(max\{\gamma_{1a}(v_i),\gamma_{1a}(v_j)\})\right)\right)=2(((0.3+0.5)+(0.5+0.4+0.1+0.1)))=3.8$$

$$Thus\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(\gamma_{2a}(v_i,v_j)+\overline{\gamma}_{2a}(v_i,v_j))\right)\nleq 2\left(\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(max\{\gamma_{1a}(v_i),\gamma_{1a}(v_j)\})\right)\right)$$

Proposition 4.8.

(i) The ifs-complement of a semi-μ_a-strong IFSG is a semi-μ_a-strong IFSG.

(ii) The ifs-complement of a semi-γ_a-strong IFSG is a semi-γ_a-strong IFSG

Proof.

(i) Let $G^*_{A,V,E}$ be a semi-μ_a-strong IFSG and let $\overline{G}^*_{A,\overline{V},\overline{E}}$ be its ifs-complement. Since $G^*_{A,V,E}$ is a semi-μ_a-strong IFSG, $\mu_{2a}(v_i, v_j) = \{min(\mu_{1a}(v_i), \mu_{1a}(v_j)\}$ for every $(v_i, v_j) \in E$ and for every $a \in A$. Then for every $(v_i, v_j) \in \overline{E}$ and for every $a \in A$.

$$\overline{\mu}_{2a}(v_i, v_j) = \begin{cases} min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\} & \text{if } \mu_{2a}(v_i, v_j) = 0,\ (v_i, v_j) \notin E,\ a \in A \\ 0 & \text{if } \mu_{2a}(v_i, v_j) \neq 0,\ (v_i, v_j) \in E,\ a \in A \end{cases}$$

Thus $\overline{\mu}_{2a}(v_i, v_j) = \{min(\overline{\mu}_{1a}(v_i), \overline{\mu}_{1a}(v_j)\}$ for every $(v_i, v_j) \in \overline{E}$ and for every $a \in A \Rightarrow \overline{G}^*_{A,\overline{V},\overline{E}}$ is a semi-μ_a-strong IFSG.

(ii) Similarly, let $G^*_{A,V,E}$ be a semi-γ_a-strong IFSG and let $\overline{G}^*_{A,\overline{V},\overline{E}}$ be its ifs-complement. Since $G^*_{A,V,E}$ is a semi-γ_a-strong IFSG, $\gamma_{2a}(v_i, v_j) = \{max(\gamma_{1a}(v_i), \gamma_{1a}(v_j))\}$ for every $(v_i, v_j) \in E$ and for every $a \in A$. Then for every $(v_i, v_j) \in \overline{E}$ and for every $a \in A$.

$$\overline{\gamma}_{2a}(v_i, v_j) = \begin{cases} max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\} & \text{if } \gamma_{2a}(v_i, v_j) = 0,\ (v_i, v_j) \notin E,\ a \in A \\ 0 & \text{if } \gamma_{2a}(v_i, v_j) \neq 0,\ (v_i, v_j) \in E,\ a \in A \end{cases}$$

Thus $\overline{\gamma}_{2a}(v_i, v_j) = \{min(\overline{\gamma}_{1a}(v_i), \overline{\gamma}_{1a}(v_j)\}$ for every $(v_i, v_j) \in \overline{E}$ and for every $a \in A \Rightarrow \overline{G}^*_{A,\overline{V},\overline{E}}$ is a semi-γ_a-strong IFSG. □

Theorem 4.9. *If an Intuitionistic fuzzy soft graph be a strong intuitionistic fuzzy soft graph, then its ifs-complement is also a strong intuitionistic fuzzy soft graph.*

Proof. Let $G^*_{A,V,E}$ be a strong IFSG and let $\overline{G}^*_{A,\overline{V},\overline{E}}$ be its ifs-complement. Since $G^*_{A,V,E}$ is strong, $\mu_{2a}(v_i, v_j) = \{min(\mu_{1a}(v_i), \mu_{1a}(v_j)\}$ and $\gamma_{2a}(v_i, v_j) = \{max(\gamma_{1a}(v_i), \gamma_{1a}(v_j)\}$ for every $(v_i, v_j) \in E$ and for every $a \in A$. Then

(i) $\overline{\mu}_{1a}(v_i) = \mu_{1a}(v_i)$; $\overline{\gamma}_{1a}(v_i) = \gamma_{1a}(v_i)$ for every $v_i \in V$ and for every $a \in A$.

(ii) $$\overline{\mu}_{2a}(v_i, v_j) = \begin{cases} min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\} & \text{if } \mu_{2a}(v_i, v_j) = 0, (v_i, v_j) \notin E,\ a \in A \\ 0 & \text{if } \mu_{2a}(v_i, v_j) \neq 0,\ (v_i, v_j) \in E,\ a \in A \end{cases}$$

$$\overline{\gamma}_{2a}(v_i, v_j) = \begin{cases} max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\} & \text{if } \gamma_{2a}(v_i, v_j) = 0,\ (v_i, v_j) \notin E,\ a \in A \\ 0 & \text{if } \gamma_{2a}(v_i, v_j) \neq 0,\ (v_i, v_j) \in E,\ a \in A \end{cases}$$

That is $\overline{\mu}_{2a}(v_i, v_j) = min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\}$ and $\overline{\gamma}_{2a}(v_i, v_j) = max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\}$ for every $(v_i, v_j) \in \overline{E}$ and for every $a \in A$. Thus $\overline{G}^*_{A,\overline{V},\overline{E}}$ is a strong IFSG. □

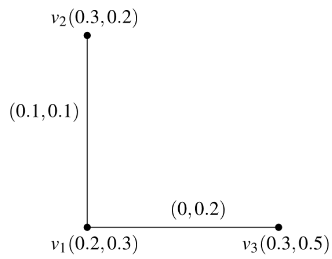
Definition 4.10. *Let $G^*_{A,V,E}$ be an IFSG. The* **ifs $-\ \phi\ -$ Complement** *of $G^*_{A,V,E}$ is defined as $\overline{G}^{*\phi}_{A,\overline{V}^\phi,\overline{E}^\phi} = (\overline{V}^\phi, \overline{E}^\phi, (A, \overline{\mu}^\phi_1), (A, \overline{\gamma}^\phi_1), (A, \overline{\mu}^\phi_2), (A, \overline{\gamma}^\phi_2))$ where*

(i) $\overline{\mu}^\phi_{1a}(v_i) = \mu_{1a}(v_i)$; $\overline{\gamma}^\phi_{1a}(v_i) = \gamma_{1a}(v_i)$ for every $v_i \in V$ and for every $a \in A$.

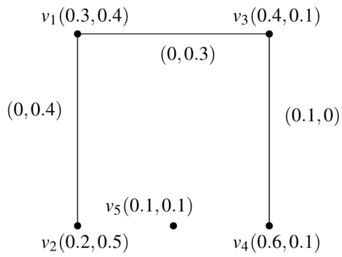
(ii) $$\overline{\mu}^\phi_{2a}(v_i, v_j) = \begin{cases} min\{\mu_{1a}(v_i), \mu_{1a}(v_j)\} - \mu_{2a}(v_i, v_j) & \text{if } \mu_{2a}(v_i, v_j) \neq 0,\ a \in A \\ 0 & \text{if } \mu_{2a}(v_i, v_j) = 0,\ a \in A \end{cases}$$

$$\overline{\gamma}^\phi_{2a}(v_i, v_j) = \begin{cases} max\{\gamma_{1a}(v_i), \gamma_{1a}(v_j)\} - \gamma_{2a}(v_i, v_j) & \text{if } \gamma_{2a}(v_i, v_j) \neq 0\ ,\ a \in A \\ 0 & \text{if } \gamma_{2a}(v_i, v_j) = 0,\ a \in A \end{cases}$$

Example 4.11. *The ifs-ϕ-complement of the IFSG $G^*_{A,V,E}$ of Example 4.3 is represented as $\overline{G}^{*\phi}_{A,\overline{V}^\phi,\overline{E}^\phi} = (\overline{V}^\phi, \overline{E}^\phi, (A, \overline{\mu}^\phi_1), (A, \overline{\gamma}^\phi_1), (A, \overline{\mu}^\phi_2), (A, \overline{\gamma}^\phi_2))$, which is described in Table 5 and Figure 5.*



5(a) ifs-ϕ- complement of IFSG corresponding to the parameter a_1.



5(b) ifs-ϕ-complement of IFSG corresponding to the parameter a_2

Figure 5:

(a)

$\overline{\mu}^{\phi}_{1a}$	v_1	v_2	v_3	v_4	v_5
a_1	0.2	0.3	0.3	0	0
a_2	0.3	0.2	0.4	0.6	0.1

(b)

$\overline{\gamma}^{\phi}_{1a}$	v_1	v_2	v_3	v_4	v_5
a_1	0.3	0.2	0.5	0	0
a_2	0.4	0.5	0.1	0.1	0.1

(c)

$\overline{\mu}^{\phi}_{2a}$	(v_1,v_2)	(v_1,v_3)	(v_3,v_4)
a_1	0.1	0	0
a_2	0	0	0.1

(d)

$\overline{\gamma}^{\phi}_{2a}$	(v_1,v_2)	(v_1,v_3)	(v_3,v_4)
a_1	0.1	0.2	0
a_2	0.4	0.3	0

Table 5:

Note 4.12. *For any IFSG $G^*_{A,V,E}$, $\overline{\overline{G}^{*\phi}}_{A,\overline{\overline{V}^{\phi}}^{\phi},\overline{\overline{E}^{\phi}}^{\phi}} \ncong G^*_{A,V,E}$. Consider the IFSG $G^*_{A,V,E}$ and $\overline{G}^{*\phi}_{A,\overline{V}^{\phi},\overline{E}^{\phi}}$ of Example 4.11, consider the edge (v_3,v_5) in $\overline{G}^{*\phi}_{A,\overline{V}^{\phi},\overline{E}^{\phi}}$, $\overline{\overline{\mu}^{\phi}_{2a}}^{\phi}(v_3,v_5)$ not equal to $\mu_{2a}(v_3,v_5)$ in $G^*_{A,V,E}$ for the parameter a_2.*

Proposition 4.13. *The ifs-ϕ-complement $G^{*\phi}_{A,\overline{V}^{\phi},\overline{E}^{\phi}}$ of an IFSG $G^*_{A,V,E}$ satisfies the following properties.*

(i) $O(\overline{G}^{*\phi}_{A,\overline{V}^{\phi},\overline{E}^{\phi}}) = O(G^*_{A,V,E})$.

(ii)
$$\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j}(\overline{\mu}^{\phi}_{2a}(v_i,v_j)\right)+\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j}\mu_{2a}(v_i,v_j)\right)=\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j}(min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\}\right)$$
and,
$$\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j}(\overline{\gamma}^{\phi}_{2a}(v_i,v_j)\right)+\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j}\gamma_{2a}(v_i,v_j)\right)=\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j}(max\{\gamma_{1a}(v_i),\gamma_{1a}(v_j)\}\right)$$
for every $(v_i,v_j)\in E$ and for every $a\in A$.

(iii) Vertex set of $\overline{G}^{*\phi}_{A,\overline{V}^{\phi},\overline{E}^{\phi}}$ is same as the vertex set of $G^*_{A,V,E}$.

(iv) The number of elements in the edge set of $\overline{G}^{*\phi}_{A,\overline{V}^{\phi},\overline{E}^{\phi}}$ is less than or equal to the number elements in the edge set in $G^*_{A,V,E}$.

(v) If the crisp subgraph corresponding to every parameter $a\in A$ is complete, then $\overline{G}^{*\phi}_{A,\overline{V}^{\phi},\overline{E}^{\phi}}$ and $\overline{G}^*_{A,\overline{V},\overline{E}}$ are same.

Proof.

(i) From the definition of $\overline{G}^{*\phi}_{A,\overline{V}^{\phi},\overline{E}^{\phi}}$, $O(\overline{G}^{*\phi}_{A,\overline{V}^{\phi},\overline{E}^{\phi}}) = O(G^*_{A,V,E})$.

(ii) $$\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}\overline{\mu}^{\phi}_{2a}(v_i,v_j)\right)=\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\}-\mu_{2a}(v_i,v_j))\right)$$

$$=\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})\right)-\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(\mu_{2a}(v_i,v_j))\right)$$

$$\Rightarrow\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}\overline{\mu}^{\phi}_{2a}(v_i,v_j)\right)+\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(\mu_{2a}(v_i,v_j))\right)=\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})\right)$$

$$\Rightarrow\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(\overline{\mu}^{\phi}_{2a}(v_i,v_j)+\mu_{2a}(v_i,v_j)\right)=\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\}\right)$$

$$Similarly,\ \sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(\overline{\gamma}^{\phi}_{2a}(v_i,v_j)+\gamma_{2a}(v_i,v_j)\right)=\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(max\{\gamma_{1a}(v_i),\gamma_{1a}(v_j)\}\right).$$

(iii) From the definition of $\overline{G}^{*\phi}_{A,\overline{V}^{\phi},\overline{E}^{\phi}}$ itself.

(iv) Example 4.11 illustrates the property. □

Definition 4.14. *An IFSG is said to be ifs-self complementary if* $G^*_{A,V,E}\cong\overline{G}^*_{A,\overline{V},\overline{E}}$.

Theorem 4.15. *Let* $G^*_{A,V,E}$ *be an ifs-self complementary IFSG, then*

$$\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}\mu_{2a}(v_i,v_j)\right)=\frac{1}{2}\left(\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})\right)\right)\ and$$
$$\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}\gamma_{2a}(v_i,v_j)\right)=\frac{1}{2}\left(\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(max\{\gamma_{1a}(v_i),\gamma_{1a}(v_j)\})\right)\right)$$

for every $(v_i,v_j)\in E$ *and for every* $a\in A$.

Proof. Let $G^*_{A,V,E}$ be a self complementary IFSG, then $G^*_{A,V,E}\cong\overline{G}^*_{A,\overline{V},\overline{E}}$. Then there exists an isomorphism $h:V\to\overline{V}$ such that $\overline{\mu}_{1a}(v_i)=\mu_{1a}(h(v_i))$, $\overline{\gamma}_{1a}(v_i)=\gamma_{1a}(h(v_i))$ and $\overline{\mu}_{2a}(h(v_i),h(v_j))=\mu_{2a}(v_i,v_j)$ and $\overline{\gamma}_{2a}(h(v_i),h(v_j))=\gamma_{2a}(v_i,v_j)$ for every $v_i,v_j\in V$ *and for every* $a\in A$. By the definition of ifs-complement of an IFSG, $\overline{\mu}_{2a}(h(v_i),h(v_j))=min(\mu_{1a}(h(v_i),\mu_{1a}(h(v_j))-\mu_{2a}(h(v_i),h(v_j))$ for every $v_i,v_j\in V$ and for every $a\in A$.

$$\Rightarrow\qquad \mu_{2a}(v_i,v_j)=\min(\mu_{1a}(v_i),\mu_{1a}(v_j))-\mu_{2a}(v_i,v_j)$$

$$\Rightarrow\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}\mu_{2a}(v_i,v_j)\right)=\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})\right)-\left(\sum_{a\in A}\sum_{v_i,v_j\in V,v_i\neq v_j}\mu_{2a}(v_i,v_j)\right)$$

$$\Rightarrow 2\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}\mu_{2a}(v_i,v_j)\right)=\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})\right)$$

$$\Rightarrow\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}\mu_{2a}(v_i,v_j)\right)=\frac{1}{2}\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})\right).$$

Also, $\overline{\gamma}_{2a}(h(v_i),h(v_j))=max(\gamma_{1a}(h(v_i),\gamma_{1a}(h(v_j))-\gamma_{2a}(h(v_i),h(v_j))$ for every $v_i,v_j\in V$ and for every $a\in A$.

$$\Rightarrow\qquad \gamma_{2a}(v_i,v_j)=max(\gamma_{1a}(v_i),\gamma_{1a}(v_j))-\gamma_{2a}(v_i,v_j)$$

$$\Rightarrow\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}\gamma_{2a}(v_i,v_j)\right)=\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(max\{\gamma_{1a}(v_i),\gamma_{1a}(v_j)\})\right)-\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}\gamma_{2a}(v_i,v_j)\right)$$

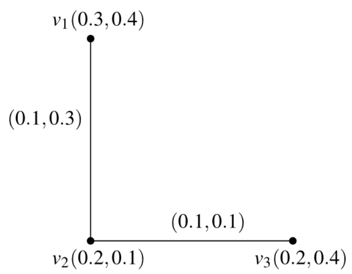
$$\Rightarrow 2\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}\gamma_{2a}(v_i,v_j)\right)=\sum_{a\in A}\left(\sum_{v_i,v_j\in V,v_i\neq v_j}(max\{\gamma_{1a}(v_i),\gamma_{1a}(v_j)\})\right)$$

$$\Rightarrow \sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j}\gamma_{2a}(v_i,v_j)\right)=\frac{1}{2}\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j}(max\{\gamma_{1a}(v_i),\gamma_{1a}(v_j)\})\right).$$

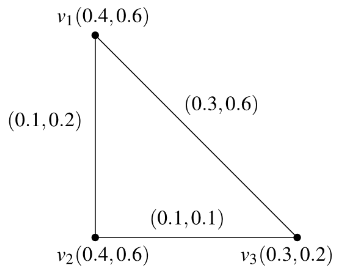
□

Remark 4.16. *The condition in the theorem 4.15 is not sufficient for an IFSG to be self complementary. The Example 4.17 illustrates this.*

Example 4.17. *Consider the IFSG $G^*_{A,V,E}=(V,E,(A,\mu_1),\ (A,\gamma_1),\ (A,\mu_2),\ (A,\gamma_2))$ described in Table 6 and Figure 6, where $V=\{v_1,v_2,v_3\}$ be the vertex set, $E=\{(v_1,v_2),(v_1,v_3),(v_2,v_3)\}$ be the edge set and let $A=\{a_1,a_2\}$ be the parameter set. The ifs-complement of $G^*_{A,V,E}$ is $\overline{G}^*_{A,\overline{V},\overline{E}}=(\overline{V},\overline{E},(A,\overline{\mu}_1),(A,\overline{\gamma}_1),\ (A,\overline{\mu}_2),\ (A,\overline{\gamma}_2))$, which is described in Table 7 and Figure 7.*



6(a) IFSG corresponding to the parameter a_1.



6(b) IFSG corresponding to the parameter a_2.

Figure 6:

(a)

μ_{1a}	v_1	v_2	v_3
a_1	0.3	0.2	0.2
a_2	0.4	0.4	0.3

(b)

γ_{1a}	v_1	v_2	v_3
a_1	0.4	0.1	0.4
a_2	0.6	0.6	0.2

(c)

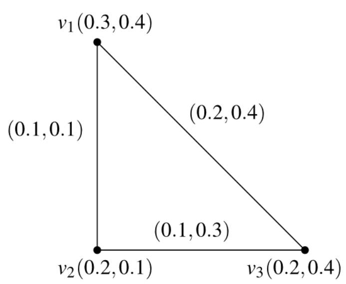
μ_{2a}	(v_1,v_2)	(v_1,v_3)	(v_2,v_3)
a_1	0.1	0	0.1
a_2	0.1	0.3	0.1

(d)

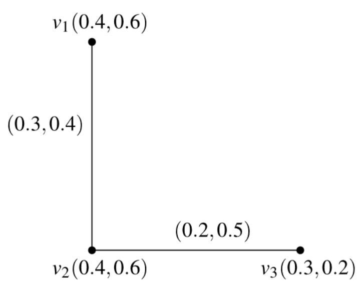
γ_{2a}	(v_1,v_2)	(v_1,v_3)	(v_2,v_3)
a_1	0.3	0	0.1
a_2	0.2	0.6	0.1

Table 6:

Figure 7:



7(a) Ifs-complement of IFSG corresponding to the parameter a_1.



7(b) Ifs-complement of IFSG corresponding to the parameter a_2.

(a)

μ_{1a}	v_1	v_2	v_3
a_1	0.3	0.2	0.2
a_2	0.4	0.4	0.3

(b)

γ_{1a}	v_1	v_2	v_3
a_1	0.4	0.1	0.4
a_2	0.6	0.6	0.2

(c)

μ_{2a}	(v_1, v_2)	(v_1, v_3)	(v_2, v_3)
a_1	0.1	0.2	0.1
a_2	0.3	0	0.2

(d)

γ_{2a}	(v_1, v_2)	(v_1, v_3)	(v_2, v_3)
a_1	0.1	0.4	0.3
a_2	0.4	0	0.5

Table 7:

In this example,

$$\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} \mu_{2a}(v_i,v_j)\right) = 0.1+0.1+0.1+0.1+0.3 = 0.7$$

$$\frac{1}{2}\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} (min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})\right) = \frac{1}{2}(0.2+0.2+0.4+0.3+0.3) = 0.7.$$

Also,

$$\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} \gamma_{2a}(v_i,v_j)\right) = 0.3+0.1+0.2+0.6+0.1 = 1.3$$

$$\frac{1}{2}\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} (max\{\gamma_{1a}(v_i),\gamma_{1a}(v_j)\})\right) = \frac{1}{2}(0.4+0.4+0.6+0.6+0.6) = 1.3$$

Thus

$$\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} \mu_{2a}(v_i,v_j)\right) = \frac{1}{2}\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} (min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})\right) \text{ and}$$
$$\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} \gamma_{2a}(v_i,v_j)\right) = \frac{1}{2}\sum_{a\in A}\left(\sum_{v_i,v_j\in V, v_i\neq v_j} (max\{\gamma_{1a}(v_i),\gamma_{1a}(v_j)\})\right).$$

But $G^*_{A,V,E}$ is not an ifs-self complementary intuitionistic fuzzy soft graph.

Theorem 4.18. *If $G^*_{A,V,E}$ be an IFSG such that $\mu_{2a}(v_i,v_j) = \frac{1}{2}(min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})$ and $\gamma_{2a}(v_i,v_j) = \frac{1}{2}(max\{\gamma_{1a}(v_i),\gamma_{1a}(v_j)\})$ for every $(v_i,v_j)\in E$ and for every $a\in A$, then $G^*_{A,V,E}$ be an ifs-self complementary IFSG.*

Proof. Let $G^*_{A,V,E}$ be an IFSG such that $\mu_{2a}(v_i,v_j) = \frac{1}{2}(min\{\mu_{1a}(v_i),\mu_{1a}(v_j)\})$ and $\gamma_{2a}(v_i,v_j) = \frac{1}{2}(max\{\gamma_{1a}(v_i),\gamma_{1a}(v_j)\})$ for every $(v_i,v_j)\in E$ and for every $a\in A$. Then $G^*_{A,V,E}\cong \overline{G}^*_{A,\overline{V},\overline{E}}$ by the identity map on V. □

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