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Cordiality of Transformation Graphs of Path

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Abstract: A graph is said to be cordial if it has a 0-1 vertex labeling that satisfies certain properties. In this paper we show that transformation graphs of path are cordial.

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Keywords: Total Graph, Transformation Graph, Cordial Graphs.

1. Introduction

All graphs G considered here are finite, undirected and simple. We refer to [1] for unexplained terminology and notations. In 2001, Wu and Meng [3] introduced some new graphical transformations which generalizes the concept of the total graph. As is the case with the total graph, these generalizations referred to as *transformation graphs* G^{xyz} have $V(G) \cup E(G)$ as the vertex set. The adjacency of two of its vertices is determined by adjacency and incidence nature of the corresponding elements in G. Let α, β be two elements of $V(G) \cup E(G)$. Then associativity of α and β is taken as $+$ if they are adjacent or incident in G, otherwise $-$. Let xyz be a 3-permutation of the set $\{+, -\}$. The pair α and β is said to correspond to x or y or z of xyz if α and β are both in $V(G)$ or both are in $E(G)$, or one is in $V(G)$ and the other is in $E(G)$ respectively. Thus the *transformation graph* G^{xyz} of G is the graph whose vertex set is $V(G) \cup E(G)$. Two of its vertices α and β are adjacent if and only if their associativity in G is consistent with the corresponding element of xyz.

In particular the transformation graph G^{++-} of G is the graph with vertex set $V(G) \cup E(G)$ in which the vertices u and v are joined by an edge if one of the following holds

(1). both $u, v \in V(G)$ and u and v are adjacent in G

(2). both $u, v \in E(G)$ and u and v are adjacent in G

(3). one is in $V(G)$ and the other is in $E(G)$ and they are not incident with each other in G.

The transformation graphs are investigated in [4], [5] and [6].

A *graph labeling* is an assignment of integers to the vertices or edges, or both, subject to certain conditions. A mapping $f : V(G) \rightarrow \{0, 1\}$ is called a binary labeling of the graph G. For each $v \in V(G)$, $f(v)$ is called the vertex label of the

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vertex v under f and for an edge uv the induced edge labeling $g : E(G) \to \{0, 1\}$ is given by $g(uv) = |f(u) - f(v)|$. Then f is called a *cordial labeling* of G if the number of vertices labeled 0 and the number of vertices labeled 1 differs by at most 1, and, the number of edges labeled 0 and the number of edges labeled 1 differs by at most 1. A graph G is cordial if it admits a cordial labeling. This concept is introduced by Cahit [2].

For convenience, the transformation graph G^{xyz} is partitioned into $G^{xyz} = S_x(G) \cup S_y(G) \cup S_z(G)$ where $S_x(G)$, $S_y(G)$ and $S_z(G)$ are the edge-induced subgraphs of G^{xyz}. The edge set of each of which is respectively determined by x, y and z of the permutation xyz. $S_x(G) \cong G$ when x is $+$ and $S_x(G) \cong \overline{G}$ when x is $-$. $S_y(G) \cong L(G)$ when y is $+$ and $S_y(G) \cong \overline{L(G)}$ when y is $-$. When z is $+$, $\alpha, \beta \in V(G^{xyz})$ are adjacent in $S_z(G)$ if they are incident with each other in G. When z is $-$, α, β are adjacent in $S_z(G)$ if they are not incident in G.

The following notations are used in relation to labeling of G^{xyz}:

Let V_0 and V_1 denote the set of vertices of G^{xyz} labeled 0 and 1. E_0 and E_1 denote set of edges of G^{xyz} labeled 0 and 1 respectively. $E_0(S_x)$ and $E_1(S_x)$ denote set of edges labeled 0 and 1 in $S_x(G)$. Similar meanings are associates with $E_0(S_y)$, $E_1(S_y)$, $E_0(S_z)$ and $E_1(S_z)$.

For P_n^{xyz} $xyz \in \{++-, +--, -++, -+-, --+, ---, +++\}$ we define a vertex labeling $f : V(P_n^{xyz}) \to \{0, 1\}$ and specify the induced edge labeling $g : E(P_n^{xyz}) \to \{0, 1\}$ then show that f is a cordial labeling.

2. Cordiality of Transformation Graphs of Path

Let $P_n = v_1 - v_2 - v_3 - \ldots - v_n$ be the path on n vertices and $e_j = v_j v_{j+1}$ $(1 \le j \le n-1)$ be the edges of P_n.

Theorem 2.1.

(a). For any positive integer $n \ge 3$ the transformation graph P_n^{xyz} where $xyz \in \{++-, +--, -++, -+-, --+, ---, +++\}$ are cordial.

(b). For $3 \le n \le 10$ or $n \ge 11$ and $n \equiv 0, 1, 2, 3, 4, 5, 6 (mod\ 8)$, P_n^{+-+} is cordial.

Proof.

(a). In each case defined a binary labeling $f : V(P_n^{xyz}) \to \{0, 1\}$ as follows:

Case 1: When $xyz = ++-$.

For $3 \le n \le 8$, vertices are labeled as in Table 1, which admits cordial labeling of P_n^{++-}.

n	$f(v_1)f(v_2)...f(v_n)$	$f(e_1)f(e_2)...f(e_{n-1})$	$\lvert V_0\rvert \sim \lvert V_1\rvert$	$\lvert E_0\rvert \sim \lvert E_1\rvert$
3	100	01	$3 \sim 2 = 1$	$3 \sim 2 = 1$
4	1110	000	$4 \sim 3 = 1$	$6 \sim 5 = 1$
5	11110	0000	$5 \sim 4 = 1$	$9 \sim 10 = 1$
6	111110	00001	$5 \sim 6 = 1$	$15 \sim 14 = 1$
7	0111110	000001	$7 \sim 6 = 1$	$21 \sim 20 = 1$

Table 1. **Cordial labeling of P_n^{++-} for the case $3 \le n \le 8$.**

For $n \ge 8$, express n as $n \equiv 0, 1, 2, 3 (mod\ 4)$ Let $n = 4r, n = 4r+1, n = 4r+2$ *or* $n = 4r+3$). Then the vertices of P_n^{++-} are labeled as in Table 2.

<table>
<tr><th>n</th><th>$f(v_i)$</th><th>$f(e_i)$</th><th>$|V_0| \sim |V_1|$</th></tr>
<tr><td>$n = 4r$</td><td rowspan="3">$\begin{cases} 0 & i=1 \\ 0 & r+3 \leq i \leq r+3+\left\lfloor \frac{n-8}{2} \right\rfloor \\ 0 & i=n \\ 1 & otherwise \end{cases}$</td><td rowspan="4">$\begin{cases} 0 & r \leq i \leq r+4+\left\lceil \frac{n-8}{2} \right\rceil \\ 1 & otherwise \end{cases}$</td><td rowspan="4">1</td></tr>
<tr><td>$n = 4r + 1$</td></tr>
<tr><td>$n = 4r + 2$</td></tr>
<tr><td>$n = 4r + 3$</td><td>$\begin{cases} 0 & i=1 \\ 0 & r+4 \leq i \leq r+4+\left\lfloor \frac{n-8}{2} \right\rfloor \\ 0 & i=n \\ 1 & otherwise \end{cases}$</td></tr>
</table>

Table 2. **Cordial labeling of P_n^{++-} for the case $n \geq 8$.**

where $|V_0| = \lceil (n-8)/2 \rceil + 7 + \lfloor (n-8)/2 \rfloor + 1 = n$ and $|V_1| = 2n - 9 - \lceil (n-8)/2 \rceil - \lfloor (n-8)/2 \rfloor = n - 1$

For $n \geq 8$, induced edge mapping $g : E(P_n^{++-}) \to \{0, 1\}$ is given below:

S_x receives the labeling :

$$g(v_i v_{i+1}) = \begin{cases} 1 & i = 1, n-1, r+2, r+3+\lfloor \frac{n-8}{2} \rfloor \\ 0 & otherwise \end{cases}$$

S_y receives the labeling :

$$g(e_j e_{j+1}) = \begin{cases} 1 & j = r-1 \\ 1 & j = r+4+\lceil \frac{n-8}{2} \rceil \\ 0 & otherwise \end{cases}$$

S_z receives the labeling :

when $n \equiv 0, 1, 2 \ (mod \ \ 4)$

for each $1 \leq j \leq r-1$; $r+5+\lceil \frac{n-8}{2} \rceil \leq j \leq n-1$ and $i \neq j, \ j+1$

$$g(e_j v_i) = \begin{cases} 1 & i = 1, n \\ 1 & r+3 \leq i \leq r+3+\lfloor \frac{n-8}{2} \rfloor \\ 0 & otherwise \end{cases}$$

for each $r \leq j \leq r+4+\lceil \frac{n-8}{2} \rceil$ and $i \neq j, \ j+1$

$$g(e_j v_i) = \begin{cases} 0 & i = 1, n \\ 0 & r+3 \leq i \leq r+3+\lfloor \frac{n-8}{2} \rfloor \\ 1 & otherwise \end{cases}$$

when $n \equiv 3 (mod \ \ 4)$

for each $1 \leq j \leq r-1$; $r+5+\lceil \frac{n-8}{2} \rceil \leq j \leq n-1$ and $i \neq j, \ j+1$

$$g(e_j v_i) = \begin{cases} 1 & i = 1, n \\ 1 & r+4 \leq i \leq r+4+\lfloor \frac{n-8}{2} \rfloor \\ 0 & otherwise \end{cases}$$

for each $r \leq j \leq r+4+\lceil \frac{n-8}{2} \rceil$ and $i \neq j, j+1$

$$g(e_j v_i) = \begin{cases} 0 & i = 1, n \\ 0 & r+4 \leq i \leq r+4+\lfloor \frac{n-8}{2} \rfloor \\ 1 & otherwise \end{cases}$$

$|E_0|$ and $|E_1|$ for the case $n \geq 8$ are calculated as in Table 3.

n	$\|E_0(S_x)\|$	$\|E_0(S_y)\|$	$\|E_0(S_z)\|$	$\|E_1(S_x)\|$	$\|E_1(S_y)\|$	$\|E_1(S_z)\|$	$\|\|E_0\|-\|E_1\|\|$
$n=4r$	n-5	n-4	$2nr-n+9-6r$	4	2	$2nr+n-7-6r$	1
$n=4r+1$			$2nr-n-4r+7$			$2nr+2n-8r-8$	1
$n=4r+2$			$2nr-6r+6$			$2nr+2n-6r-10$	1
$n=4r+3$			$2nr-4r+5$			$2nr+3n-8r-6$	1

Table 3. **$|E_0|$ and $|E_1|$ of P_n^{++-} for the case $n \geq 8$.**

Therefore P_n^{++-} is cordial.

Case 2: When $xyz = +++$.

$f(v_i) = 1 \quad for\ 1 \leq i \leq n$

$f(e_i) = 0 \quad for\ 1 \leq i \leq n-1$

Therefore $|V_0| = n-1$ and $|V_1| = n$ and hence $||V_0|-|V_1|| = 1$. The induced edge labeling $g : E(P_n^{+++}) \rightarrow \{0,1\}$ for the edges of

- $S_x \cong P_n$: $g(v_i v_{i+1}) = 0$, $1 \leq i \leq n-1$
- $S_y \cong L(P_n)$: $g(e_j e_{j+1}) = 0$, $1 \leq j \leq n-2$ and
- S_z : $g(e_j v_i) = 1$ for $1 \leq j \leq n-1$ and $i = j, j+1$.

Clearly,

$$|E_0| = |E_0(S_x)| + |E_0(S_y)| + |E_0(S_z)| = (n-1) + (n-2) + 0 = 2n-3$$

$$|E_1| = |E_1(S_x)| + |E_1(S_y)| + |E_1(S_z)| = 0 + 0 + 2(n-1) = 2n-2$$

and $|E_0| - |E_1| = -1$. Thus P_n^{+++} is cordial.

Case 3: When $xyz = -+-$.

For $3 \leq n < 8$, the vertices of P_n^{-+-} are labeled as in Table 4 which admits cordial labeling.

n	$f(v_1)f(v_2)...f(v_n)$	$f(e_1)f(e_2)...f(e_{n-1})$	$\|V_0\| \sim \|V_1\|$	$\|E_0\| \sim \|E_1\|$
3	010	10	$3 \sim 2 = 1$	$2 \sim 2 = 0$
4	0101	101	$3 \sim 4 = 1$	$5 \sim 6 = 1$
5	01010	1010	$5 \sim 4 = 1$	$10 \sim 11 = 1$
6	010101	10010	$6 \sim 5 = 1$	$16 \sim 17 = 1$
7	0101010	100101	$7 \sim 6 = 1$	$25 \sim 25 = 0$

Table 4. **Cordial labeling of P_n^{-+-} for the case $n < 8$.**

For $n \geq 8$ the vertices of P_n^{-+-} are labeled as in Table 5.

n	$f(e_i)$	$f(v_i)$	$\lvert V_0\rvert \sim \lvert V_1\rvert$
$n=8r$ $n=8r+1$ $n=8r+2$ $n=8r+3$ $n=8r+4$ $n=8r+5$	$\begin{cases} 0 & r+2\le i\le 2r+2 \\ 0 & 2r+4\le i\equiv 0(\text{mod } 2)\le n-1 \\ 1 & otherwise \end{cases}$	$\begin{cases} 1 & i\equiv 0,2,4,6(\text{mod } 8) \\ 0 & otherwise \end{cases}$	1
$n=8r+6$ $n=8r+7$	$\begin{cases} 0 & r+2\le i\le 2r+3 \\ 0 & 2r+5\le i\equiv 1(\text{mod } 2)\le n-1 \\ 1 & otherwise \end{cases}$		1

Table 5. **Cordial labeling of P_n^{-+-} for the case $n \geq 8$.**

Therefore $|V_0| = \frac{n}{2} + r + 1 + (n - 2r - 4)/2 = n - 1$ and $|V_1| = \frac{n}{2} + r + 1 + (n - 2r - 2)/2 = n$.

For $n \geq 8$, the induced edge mapping $g : E(P_n^{-+-}) \to \{0, 1\}$ for the edges of

- $S_x \cong \overline{P_n}$: for each $1 \leq i \leq n$
$$g(v_iv_j) = \begin{cases} 0 & j = i+2, i+4, \ldots, (n-1) \ or \ n \\ 1 & j = i+3, i+5, \ldots, n \ or (n-1) \end{cases}$$

- $S_y \cong L(P_n)$:
$$g(e_je_{j+1}) = \begin{cases} 0 & 1 \leq j \leq r \\ 0 & r+2 \leq j \leq (2r+2) \ if \ n \equiv 6, 7(mod\ 8) \\ 0 & r+2 \leq j \leq (2r+1) \ if \ n \equiv 0, 1, 2, 3, 4, 5(mod\ 8) \\ 1 & otherwise \end{cases}$$

- S_z : **when** $n \equiv 6, 7(mod\ 8)$ Let $n = 8r + 6$ or $n = 8r + 7$

  for each $1 \leq j \leq r + 1$ or $2r + 4 \leq j \equiv 0(mod\ 2) \leq n - 1$ and $i \neq j, j + 1$
  $$g(e_jv_i) = \begin{cases} 0 & i \equiv 1(mod\ 2) \\ 1 & i \equiv 0(mod\ 2) \end{cases}$$
  for each $(r + 2 \leq j \leq 2r + 3)$ or $(2r + 5 \leq j \equiv 1(mod\ 2) \leq n - 1)$ and $i \neq j, j + 1$
  $$g(e_jv_i) = \begin{cases} 0 & i \equiv 0(mod\ 2) \\ 1 & i \equiv 1(mod\ 2) \end{cases}$$
  when $n \equiv 0, 1, 2, 3, 4, 5(mod\ 8)$

  Let $n = 8r + k \quad 0 \leq k \leq 5$

  for each $(1 \leq j \leq r + 1)$ or $(2r + 3 \leq j \equiv 1(mod\ 2) \leq n - 1)$ and $i \neq j, j + 1$
  $$g(e_jv_i) = \begin{cases} 0 & i \equiv 1(mod\ 2) \\ 1 & i \equiv 0(mod\ 2) \end{cases}$$
  for each $(r + 2 \leq j \leq 2r + 2)$ or $(2r + 4 \leq j \equiv 0(mod\ 2) \leq n - 1)$ and $i \neq j, j + 1$
  $$g(e_jv_i) = \begin{cases} 0 & i \equiv 0(mod\ 2) \\ 1 & i \equiv 1(mod\ 2) \end{cases}$$

$|E_0|$ and $|E_1|$ in the above different cases are listed in the Tables 6, 7 and their difference in Table 8.

<table>
<tr><th>n</th><th>$|E_0(S_x)|$</th><th>$|E_0(S_z)|$</th><th>$|E_1(S_x)|$</th><th>$|E_1(S_z)|$</th></tr>
<tr><td>Even</td><td>$2\sum_{i=1}^{\frac{n-2}{2}} i$</td><td rowspan="2">$\frac{(n-1)(n-2)}{2}$</td><td>$\frac{n-2}{2}+2\sum_{i=1}^{\frac{n-4}{2}} i$</td><td rowspan="2">$\frac{(n-1)(n-2)}{2}$</td></tr>
<tr><td>Odd</td><td>$\frac{n-1}{2}+2\sum_{i=1}^{\frac{n-2}{2}} i$</td><td>$2\sum_{i=1}^{\frac{n-3}{2}} i$</td></tr>
</table>

Table 6. $|E_0|$ **and** $|E_1|$ **of** S_x **and** S_z **in** P_n^{-+-} **for the case** $n \geq 8$.

n	$\|E_0(S_y)\|$	$\|E_1(S_y)\|$
$n \equiv 0,1,2,3,4,5 \pmod 8$	$2r$	n-$2r$-2
$n \equiv 6,7 \pmod 8$	$2r+1$	n-$2r$-3

Table 7. $|E_0|$ **and** $|E_1|$ **of** S_y **in** P_n^{-+-} **for the case** $n \geq 8$.

<table>
<tr><th colspan="2">n</th><th>$|E_0| \sim |E_1|$</th><th>$|E_0| \sim |E_1|$</th></tr>
<tr><td rowspan="4">Even</td><td>$n=8r$</td><td rowspan="3">$\frac{8r-n+2}{2}$</td><td>1</td></tr>
<tr><td>$n=8r+2$</td><td>0</td></tr>
<tr><td>$n=8r+4$</td><td>1</td></tr>
<tr><td>$n=8r+6$</td><td>$\frac{8r-n+6}{2}$</td><td>0</td></tr>
<tr><td rowspan="4">Odd</td><td>$n=8r+1$</td><td rowspan="3">$\frac{8r-n+3}{2}$</td><td>1</td></tr>
<tr><td>$n=8r+3$</td><td>0</td></tr>
<tr><td>$n=8r+5$</td><td>1</td></tr>
<tr><td>$n=8r+7$</td><td>$\frac{8r-n+7}{2}$</td><td>0</td></tr>
</table>

Table 8. $|E_0| \sim |E_1|$ **in** P_n^{-+-} **for the case** $n \geq 8$.

Therefore P_n^{-+-} is cordial.

Case 4: When $xyz = -++$ Here the binary labeling $f : V(P_n^{-++}) \to \{0,1\}$ is same as given to P_n^{-+-}.

For $n \geq 8$, induced edge mapping $g : E(P_n^{-++}) \to \{0,1\}$ is given below:

S_x and S_y of P_n^{-++} receives the labeling as mentioned in the case of P_n^{-+-}.

S_z receives the labeling as below :

when $n \equiv 0,1,2,3,4,5 \ (mod\ 8)$

for $2r+3 \leq j \leq n-1$: $\quad g(e_j v_{j+1}) = 0$ and $g(e_j v_j) = 1$

for $1 \leq j \leq r+1$: $g(e_j v_{j+1}) = \begin{cases} 0 & j\ odd \\ 1 & j\ even \end{cases}$ and

$$g(e_jv_j) = \begin{cases} 1 & j\ odd \\ 0 & j\ even \end{cases}$$

for $r+2 \leq j \leq 2r+3$ ($when\ n \equiv 6,7 (mod\ n)$) or $2r+2$ ($otherwise$)

$$g(e_jv_{j+1}) = \begin{cases} 0 & j\ even \\ 1 & j\ odd \end{cases} \quad \text{and} \quad g(e_jv_j) = \begin{cases} 1 & j\ even \\ 0 & j\ odd \end{cases}$$

when $n \equiv 6(mod\ 8)$ let $n = 8r+6$

for $2r+4 \leq j \leq n-1$: $g(e_jv_{j+1}) = 1$ and $g(e_jv_j) = 0$

when $n \equiv 7(mod\ 8)$ let $n = 8r+7$

for $2r+4 \leq j \leq n-2$: $g(e_jv_{j+1}) = 0$ and $g(e_jv_j) = 1$

In P_n^{-++}, $|E_0(S_x)|, |E_1(S_x)|, |E_0(S_y)|$ and $|E_1(S_y)|$ are same as P_n^{-+-} mentioned in Case 3 but $|E_0(S_z)| = |E_1(S_z)| = n$, so that $|E_0|$ and $|E_1|$ for different cases satisfies $|E_0| \sim |E_1| \leq 1$.

Therefore P_n^{-++} is cordial.

Case 5: When $xyz = --+$

For $3 \leq n \leq 7$ the vertices of P_n^{--+} are labeled as in Table 9 which admits cordial labeling.

n	$f(v_1)f(v_2)...f(v_n)$	$f(e_1)f(e_2)...f(e_{n-1})$	$\|V_0\| \sim \|V_1\|$	$\|E_0\| \sim \|E_1\|$
3	101	10	$2 \sim 3 = 1$	$3 \sim 2 = 1$
4	1010	110	$3 \sim 4 = 1$	$5 \sim 5 = 0$
5	10101	1100	$4 \sim 5 = 1$	$8 \sim 9 = 1$
6	101010	11001	$5 \sim 6 = 1$	$13 \sim 13 = 0$
7	0101010	110011	$6 \sim 7 = 1$	$17 \sim 17 = 0$

Table 9. **Cordial labeling of P_n^{--+} for the case $n \leq 7$.**

For $n \geq 8$ the vertices of P_n^{--+} are labeled as follows:

$$for\ all x \in V(P_n^{--+}),\ f(x) == \begin{cases} 0 & if\ x = v_i,\ \ i \equiv 0,2\ (mod\ 4) \\ 0 & if\ x = e_i\ \ i \equiv 0,3\ (mod\ 4) \\ 1 & otherwise \end{cases}$$

For $n \geq 8$, induced edge mapping $g : E(P_n^{--+}) \to \{0,1\}$ for the edges of

- S_x : for each $1 \leq i \leq n$

  $$g(v_iv_j) = \begin{cases} 1 & j = i+2, i+4, \ldots, (n-1)\ or n \\ 0 & j = i+3, i+5, \ldots, n\ or (n-1) \end{cases}$$

- S_y : for each $j \equiv 0,3\ (mod\ 4)$

  $$g(e_je_k) = \begin{cases} 0 & j+3 \leq k \equiv 0,3\ (mod\ 4) \leq n-1 \\ 1 & otherwise \end{cases}$$

  for each $j \equiv 1,2\ (mod\ 4)$

  $$g(e_je_k) = \begin{cases} 0 & j+3 \leq k \equiv 1,2\ (mod\ 4) \leq n-1 \\ 1 & otherwise \end{cases}$$

- S_z : $g(e_jv_{j+1}) = \begin{cases} 0 & j \equiv 2,3\ (mod\ 4) \\ 1 & otherwise \end{cases}$

$$g(e_jv_j) = \begin{cases} 0 & j \equiv 0,1\ (mod\ 4) \\ 1 & otherwise \end{cases}$$

$|E_0|$ and $|E_1|$ of P_n^{--+} for the case $n \geq 7$ are given in Table 10

<table>
<tr><th colspan="2">n</th><th>$|E_0(S_x)|$</th><th>$|E_0(S_y)|$</th><th>$|E_0(S_z)|$</th><th>$|E_1(S_x)|$</th><th>$|E_1(S_y)|$</th><th>$|E_1(S_z)|$</th><th>$|E_0| \sim |E_1|$</th></tr>
<tr><td colspan="2">Even</td><td>$2\sum^{\frac{n-2}{2}} i$</td><td>$2\sum^{\frac{n-4}{2}} i$</td><td rowspan="3">n-1</td><td>$\frac{n-2}{2} + 2\sum^{\frac{n-4}{2}} i$</td><td>$\frac{n-2}{2} + 2\sum^{\frac{n-4}{2}} i$</td><td rowspan="3">$n$-1</td><td>0</td></tr>
<tr><td rowspan="2">Odd</td><td>$n = 3+4r$</td><td rowspan="2">$\frac{n-1}{2} + 2\sum_{i=1}^{\frac{n-3}{2}} i$</td><td>$4\sum_{i=1}^{r} (2i-1)$</td><td rowspan="2">$2\sum_{i=1}^{\frac{n-3}{2}} i$</td><td>$2\sum_{i=1}^{2r-1} i + n - 3$</td><td>1</td></tr>
<tr><td>$n = 1+4r$</td><td>$4\sum_{i=1}^{r} 2i$</td><td>$2\sum_{i=1}^{2r} i + n - 2$</td><td>1</td></tr>
</table>

Table 10. **$|E_0|$ and $|E_1|$ of P_n^{--+} for the case $n \geq 7$**

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Therefore P_n^{--+} is cordial.

Case 6: When $xyz = ---$

Define a binary labeling $f : V(P_n^{---}) \to \{0, 1\}$ same as given to P_n^{--+} in Case 5 which admits cordiality.

For $n \geq 8$,induced edge mapping $g : E(P_n^{---}) \to \{0, 1\}$ is:

S_x and S_y of receives the labeling same as in case of P_n^{--+}.

S_z receives the labeling :

$$\text{for each } j \equiv 1,2 \ (mod\ 4) \ g(e_j v_i) = \begin{cases} 0 & i \equiv 0,2 \ (mod\ 4) \\ 1 & otherwise \end{cases}$$

$$\text{for each } j \equiv 0,3 \ (mod\ 4) \ g(e_j v_i) = \begin{cases} 0 & i \equiv 1,3 \ (mod\ 4) \\ 1 & otherwise \end{cases}$$

In P_n^{---}, $|E_0(S_x)|, |E_1(S_x)|, |E_0(S_y)|$ and $|E_1(S_y)|$ are same as P_n^{-+-} mentioned in Case 3 but $|E_0(S_z)| = n\left(\lfloor \frac{n}{2} \rfloor\right) - 1$ $|E_1(S_z)| = n\left(\lceil \frac{n}{2} - 1\right)$, so that $|E_0|$ and $|E_1|$ for different cases satisfies $||E_0| - |E_1|| \in \{0, 1\}$

Case 7: When $xyz = +--$

For $3 \leq n < 11$, the vertices of P_n^{+--} are labeled as in Table 11 which admits cordial labeling.

n	$f(v_1)f(v_2)...f(v_n)$	$f(e_1)f(e_2)...f(e_{n-1})$	$\lvert V_0\rvert \sim \lvert V_1\rvert$	$\lvert E_0\rvert \sim \lvert E_1\rvert$
3	000	11	$3 \sim 2 = 1$	$2 \sim 2 = 0$
5	01100	0101	$5 \sim 4 = 1$	$10 \sim 9 = 0$
6	011001	01010	$6 \sim 7 = 1$	$16 \sim 15 = 1$
7	0111001	010101	$6 \sim 7 = 1$	$23 \sim 24 = 1$
8	0111001	010101	$8 \sim 7 = 1$	$32 \sim 32 = 0$
9	011100101	0101010	$8 \sim 9 = 1$	$42 \sim 43 = 1$
10	0111001010	010101010	$10 \sim 9 = 1$	$54 \sim 55 = 1$

Table 11. **Cordial labeling of P_n^{+--} for the case $n < 11$.**

For $n \geq 11$

(i). When n is odd and $n \equiv 3, 5, 7, 1(mod\ 8)$, each n is expressed in the form $n - 11 \equiv 0, 2, 4, 6(mod\ 8)$ respectively. (i.e., $n = 11 + 8r$, $n = 11 + 8r + 2$, $n = 11 + 8r + 4$, $n = 11 + 8r + 6$)

$f(v_1) = 0$

$$f(v_i) = \begin{cases} 0 & i = 1 \\ 0 & r+5 \leq i \leq 2r+7 \\ 0 & i = 2r+9, 2r+11, 2r+13, ..., n \\ 1 & otherise \end{cases}$$

when $n \equiv 3(mod\ 8)$, $f(e_1) = 1$ and when $n \equiv 5, 7, 1(mod\ 8)$, $f(e_1) = 0$

$$f(e_i) = \begin{cases} 0 & i = 3, 5, 7, ..., n-1 \\ 1 & otherise \end{cases}$$

(ii). When n is even and $n \equiv 4, 6, 0, 2, (mod\ 8)$, each n is expressed in the form $n - 11 \equiv 1, 3, 5, 7(mod\ 8)$ (i.e., $n = 11 + 8r + 1, \quad n = 11 + 8r + 3, \quad n = 11 + 8r + 5, \quad n = 11 + 8r + 7$)

$f(v_1) = 0$

when $n \equiv 4(mod\ 8)$, $f(v_{r+5}) = 1$ and

when $n \equiv 6, 0, 2(mod\ 8)$, $f(v_{r+5}) = 0$

$$f(v_i) = \begin{cases} 0 & r+6 \leq i \leq 2r+7 \\ 0 & i = 2r+9, 2r+11, ..., n \\ 1 & otherise \end{cases}$$

$$f(e_i) = \begin{cases} 0 & i = 3, 5, 7, ..., n-1 \\ 1 & otherise \end{cases}$$

For both odd and even n, $||V_0| - V_1|| = 1$.

For $n \geq 11$, induced edge mapping $g : E(P_n^{+--}) \to \{0, 1\}$ for the edges of

- S_x: $$g(v_i v_{i+1}) = \begin{cases} 0 & 2 \leq i \leq r+3 \ if\ n\ is\ \ odd \\ 0 & 2 \leq i \leq r+4 \ ifn\ is\ \ even \\ 0 & r+5 \leq i \leq 2r+6 \\ 1 & otherwise \end{cases}$$
- S_y: When $n - 11 \equiv 4, 5, 6, 7, 0, 1\ (mod\ 8)$

  for each $1 \leq j \leq n-1, \quad k \neq j, j+1$

  $$g(e_j e_k) = \begin{cases} 0 & k = j+2, j+4, j+6, \ldots, (n-1)\ or\ (n-2) \\ 1 & otherwise \end{cases}$$

  When $n - 11 \equiv 0(mod\ 8)$

  $$g(e_1 e_k) = \begin{cases} 0 & k = 4, 6, 8, \ldots, (n-1) \\ 1 & k = 3, 5, 7, 9, \ldots, (n-2) \end{cases}$$

  for each $2 \leq j \leq n-1, \quad k \neq j, j+1$

  $$g(e_j e_k) = \begin{cases} 0 & k = j+2, j+4, j+6, \ldots, (n-1)\ or\ (n-2) \\ 1 & otherwise \end{cases}$$
- S_z: When $n - 11 \equiv 1, 2, 3, 4, 5, 6(mod\ 8)$

  for each $j \equiv 1\ (mod\ 2), \quad k \neq j, j+1$

  $$g(e_j v_k) = \begin{cases} 0 & r+5 \leq k \leq 2r+7 \\ 0 & 2r+9 \leq k \equiv 1\ (mod\ 2) \leq n \\ 1 & otherwise \end{cases}$$

  for each $j \equiv 0\ (mod\ 2), \quad k \neq j, j+1$

$$g(e_jv_k) = \begin{cases} 1 & r+5 \le k \le 2r+7 \\ 1 & 2r+9 \le k \equiv 1 \ (mod\ 2) \le n \\ 0 & otherwise \end{cases}$$

When $n-11 \equiv 0(mod\ 8)$

for each $j = 3, 5, 7, ..., (n-2)$ $k \neq j, j+1$

$$g(e_jv_k) = \begin{cases} 0 & r+5 \le k \le 2r+7 \\ 0 & 2r+9 \le k \equiv 1 \ (mod\ 2) \le n \\ 1 & otherwise \end{cases}$$

for each $j = 1, 2, 4, 6, 8, ..., (n-1)$ $k \neq j, j+1$

$$g(e_jv_k) = \begin{cases} 1 & r+5 \le k \le 2r+7 \\ 0 & 2r+9 \le k \equiv 1 \ (mod\ 2) \le n \\ 0 & otherwise \end{cases}$$

When $n-11 \equiv 1(mod\ 8)$

for each $j = 1, 3, 5, 7, ..., (n-1)$ $k \neq j, j+1$

$$g(e_jv_k) = \begin{cases} 0 & r+6 \le k \le 2r+7 \\ 0 & 2r+9 \le k \equiv 1 \ (mod\ 2) \le n \\ 1 & otherwise \end{cases}$$

for each $j = 2, 4, 6, 8, ..., (n-2)$ $k \neq j, j+1$

$$g(e_jv_k) = \begin{cases} 1 & r+6 \le k \le 2r+7 \\ 1 & 2r+9 \le k \equiv 1 \ (mod\ 2) \le n \\ 0 & otherwise \end{cases}$$

For odd n, $|E_0|$ and $|E_1|$ can be calculated as in the above cases and $|E_0| - |E_1| = 0$.

Similar to the above discussion , $||E_0| - |E_1|| \le 1$ when n is even.

Therefore P_n^{+--} is cordial.

(b). When $xyz = +-+$

Here we show that P_n^{+-+} is cordial for $n \equiv 0, 1, 2, 3, 4, 5, 6(mod\ 8)$.

Here we express $n \equiv 0, 1, 2, 3, 4, 5, 6(mod\ 8)$ as $n-11 \equiv 0, 1, 2, 3, 5, 6, 7(mod\ 8)$.

For $4 \le n < 11$ we label the vertices as in Table12 which admits cordial labeling of P_n^{+-+}

n	$f(v_1)f(v_2)...f(v_n)$	$f(e_1)f(e_2)...f(e_{n-1})$	$\|V_0\| \sim \|V_1\|$	$\|E_0\| \sim \|E_1\|$
4	0110	110	$4 \sim 3 = 1$	$5 \sim 6 = 1$
5	01100	0101	$5 \sim 4 = 1$	$8 \sim 7 = 1$
6	011001	01010	$6 \sim 5 = 1$	$11 \sim 10 = 1$
7	0110010	010101	$7 \sim 6 = 1$	$14 \sim 14 = 0$
8	01100101	0101010	$8 \sim 7 = 1$	$18 \sim 18 = 0$
9	011001010	01010101	$9 \sim 8 = 1$	$22 \sim 23 = 1$
10	0110010101	010101010	$10 \sim 9 = 1$	$27 \sim 28 = 1$

Table 12. **Cordial labeling of P_n^{+-+} for the case $n < 7$.**

For $n \ge 11$

(i) When $n \equiv 3\ (mod\ 8)$ we express n as $n - 11 \equiv 0 (mod\ 8)$ Let $n = 11 + 8r$.

$$f(v_i) = \begin{cases} 0 & i = 1 \\ 0 & r+5 \leq i \leq 2r+7 \\ 0 & i = 2r+9, 2r+11, 2r+13, ..., n \\ 1 & otherise \end{cases}$$

$$f(e_i) = \begin{cases} 0 & i = 1, 3, 5, 7, ..., n-4 \\ 0 & i = n-2 \\ 1 & otherise \end{cases}$$

which admits cordial labeling.

(ii) When $n \equiv 0, 1, 2, 5, 6\ (mod\ 8)$ we express n as $n - 11 \equiv 5, 6, 7, 0, 1 (mod\ 8)$. Let $n = 13 + 8r, 14 + 8r, 15 + 8r, 16 + 8r, 17 + 8r$. We define binary labeling for these n values same as defined in case of P_n^{+--} which admits cordial labeling.

□

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