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Virtual Tour of a Travelling Salesman

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Abstract: The main focus of this paper is to study the fuzzy travelling salesman problems in the true environment of fuzzy decision variables through the virtual modifier function in association with a modifier coefficient from a network of cities in a complete graph or complete digraph or connected graph.

MSC: 90B06; 90C08.

Keywords: Fuzzy decision variable; Modifier coefficient; Virtual modifier function.

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1. Introduction

The fuzzy travelling salesman problems consists of a salesman and a set of cities in a network (complete graph or complete digraph or connected graph). The salesman has to visit each one of the cities starting from a certain one (e.g. hometown) and returning to the same city. The main objective of the problem is that; the salesman wishes to minimise the total length or cost of the trip. It is a particular case of assignment problem in optimisation. A large number of methods and algorithms has been developed to solve fuzzy travelling salesman problems; these approaches do always generates the optimal solution. Instead, they will often consistently find good solutions to the problems in the true environmental conditions. These good solutions are typically considered to be good enough simply because they are the best that can be found in a reasonable amount of time. Generally the road and traffic conditions are critical in most of the developing countries. In a case study [3]; Nwobi-Okoye study the classical travelling salesman problems using the fuzzy decision variables by dividing the road and traffic conditions into five linguistic variables. On the basis of this key idea the fuzzy travelling salesman problems has been studied using fuzzy decision variables. The remainder of this paper is penned as follows: A brief note on the fuzzy numbers, defuzzification process and magnitude ranking method has been recited in Section 2. The fuzzy travelling salesman problems in the true environment of fuzzy decision variable has been setup with some definitions and a Theorem in Section 3. The obtained solution is compared and analysed in Section 4. The paper end with a conclusion in Section 5.

2. Preliminaries

In 1965 Zadeh [7]; defined triangular and trapezoidal fuzzy number in association with membership function. These definitions have wide range of applications in fuzzy theory. A triangular fuzzy number can also be defined as a pair of bounded

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monotonic increasing and decreasing along with left and right continuous functions respectively. In 1981 Yagar [6]; proposed Robust ranking index for the defuzzification process in fuzzy theory. In 2017 Selvi et al. [5]; defined triangular fuzzy number on the basis of its local index number and fuzziness index functions. Some definitions has been revisited to make this article self contained.

Definition 2.1 ([7]). *A fuzzy number $\tilde{A} = (a,\ b,\ c)$ with membership function $\mu_{\tilde{A}}(x)$ of the first form*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}\ , & a \leq x < b\ ; \\ 1\ , & x = b\ ; \\ \frac{c-x}{c-b}\ , & b < x \leq c\ ; \\ 0\ , & otherwise. \end{cases} ; \qquad \mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}\ , & a \leq x < b\ ; \\ 1\ , & b \leq x \leq c\ ; \\ \frac{d-x}{d-c}\ , & c < x \leq d\ ; \\ 0\ , & otherwise. \end{cases} ;$$

Form-1. Triangular Fuzzy number *Form-2. Trapezoidal fuzzy number*

is called a triangular fuzzy number and a fuzzy number $\tilde{A} = (a,\ b,\ c,\ d)$ with membership function $\mu_{\tilde{A}}(x)$ of the second form is called a trapezoidal fuzzy number.

Defuzzification is the process of finding a singleton value (crisp value) that represent the average value of the triangular fuzzy number. A triangular fuzzy number $\tilde{a} \in F(\mathbb{R})$ can also be represented as a pair $\tilde{a} = (\underline{a},\ \overline{a})$ of the functions $(\underline{a}(r),\ \overline{a}(r)),\ 0 \leq r \leq 1$ such that $\underline{a}(r),\ \overline{a}(r)$ are bounded monotonic increasing right and bounded decreasing left continuous functions with $\underline{a}(r) < \overline{a}(r),\ for\ all\ r.$

Definition 2.2 ([5]). *For an arbitrary fuzzy number $\tilde{a} = (\underline{a},\ \overline{a})$, the number $a_0 = \frac{\underline{a}(r)+\overline{a}(r)}{2}$; $r = 1$ is said to be location index number of $\tilde{a}$. Two continuous functions $a_* = a_0 - \underline{a}$ and $a^* = \overline{a} - a_0$ are called left fuzziness index functions and right fuzziness index functions respectively. Hence every triangular fuzzy number $\tilde{a}$ can be represented by $\tilde{a} = (\underline{a},\ a_0,\ \overline{a}) = (a_0,\ a_*,\ a^*)$.*

Definition 2.3 ([5]). *For a triangular fuzzy number $(\underline{a},\ a_0,\ \overline{a}) = (a_0,\ a_*,\ a^*)$ magnitude of triangular fuzzy number is given by equation*

$$Mag(\underline{a},\ a_0,\ \overline{a}) = \frac{1}{2}\int_0^1 (\underline{a} + a_0 + \overline{a})f(r)\ dr = \frac{1}{2}\int_0^1 (a^* + 3a_0 - a_*)f(r)\ dr. \tag{1}$$

Where $f(r)$ is a non-negative increasing function on $[0,1]$ with $f(0) = 0,\ f(1) = 1$ and $\int_0^1 f(r)\ dr = \frac{1}{2}$.

Definition 2.4 ([6]). *For triangular convex fuzzy number $\tilde{a}$ the Robust ranking index is defined by*

$$R(\tilde{a}) = \frac{1}{2}\int_0^1 (a_\alpha^L,\ a_\alpha^U)d\alpha \tag{2}$$

where $(a_\alpha^L,\ a_\alpha^U) = (b-a)\alpha + a + c - (c-b)\alpha = 2b\alpha + (a+c)(1-\alpha)$ is the α-label cut of the fuzzy number $\tilde{a}$.

Definition 2.5 ([1]). *For a trapezoidal fuzzy number $\tilde{A} = (a,\ b,\ c,\ d)$ the average and detailed coefficients namely the scaling and wavelet coefficients can be calculated using $\alpha = \frac{a+b+c+d}{4}$, $\beta = \frac{(a+b)-(c+d)}{4}$, $\gamma = \frac{a-b}{2}$ and $\delta = \frac{c-d}{4}$ and call this new 4-tuple as $R(\tilde{A}) = (\alpha,\ \beta,\ \gamma,\ \delta)$.*

3. Fuzzy Travelling Salesman Problem with Fuzzy Decision Variables

Generally network of a salesman (complete graph or complete digraph or connected graph) is useful to develop an assignment matrix (mathematical model). Assuming a salesman has to visit n cities ($C_i,\ i = 1,\ 2,\ \ldots\ n$). Salesman wishes to start from a particular city, visit each city once and then return to his starting point. The classical travelling salesman problems are particular case of linear programming deals with crisp parameters. But in real life problems the information are of uncertain, imprecise and vague in nature. Now fuzzy numbers(triangular, trapezoidal) are being used to deal with the impreciseness and uncertainty aspects of information. The decision variables in fuzzy travelling salesman problems are replace by fuzzy decision variables to minimise the risk of loss of information during the defuzzification process. The network in the figure-1 representation is useful to develop the assignment cost matrix.

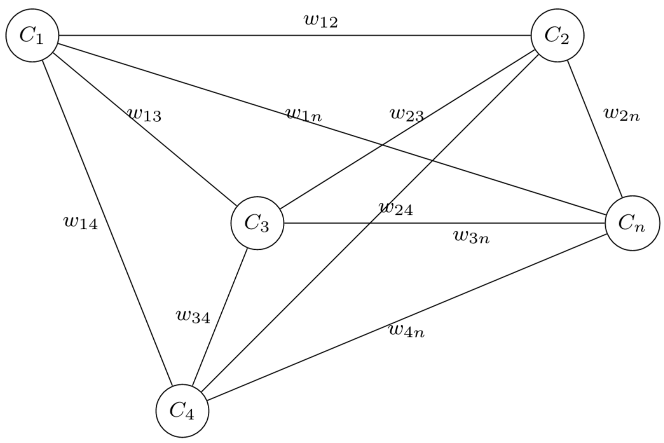

Figure 1. **Network of a salesman (complete graph)**

Let $w_{ij},\ i \neq j$ (edges of the network) be the distance or time or cost of going from city i to city j. Let the fuzzy decision variable x_{ijt} be 1, if the salesman has to travels from city i to city j, otherwise let it be zero. Mathematically,

$$x_{ijt} = \begin{cases} 1, & \text{city } i \text{ to city } j \text{ is used in step } t; \\ 0, & \text{otherwise.} \end{cases}$$

and subject to additional constraints that x_{ijt} is so chosen that, no city is visited twice before all the cities are visited. In particular, the salesman has no permission to visit city i to city i. Mathematically,

$$w_{ij} = \begin{cases} (\tilde{c}_{ij};\ \theta), & \text{city} \quad i \quad \text{to} \quad \text{city} \quad j; \\ \infty, & \text{city} \quad i \quad \text{to} \quad \text{city} \quad i. \end{cases}$$

where θ is the virtual coefficient (see Definition 3.1), $\tilde{c}_{ij} = (a_{ij},\ b_{ij},\ c_{ij})$ or $(a_{ij},\ b_{ij},\ c_{ij},\ d_{ij})$; are fuzzy triangular or trapezoidal number respectively. Now the travelling salesman problem can be stated in the form of $n \times n$ symmetric cost matrix of real numbers for complete graph as stated below:

$$\begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{bmatrix} \infty & w_{12} & \ldots & w_{1n} \\ w_{21} & \infty & \ldots & w_{2n} \\ \vdots & \vdots & & \vdots \\ w_{n1} & w_{n2} & \ldots & \infty \end{bmatrix} \end{array}$$

Now linear programming model of fuzzy travelling salesman problem with fuzzy decision variables can be stated as

$$
\begin{aligned}
\text{Min. } Z &= \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=1}^{n} w_{ij}\cdot x_{ijt} \\
\text{subject to constraints:} & \\
\sum_{i=1}^{n} x_{ijt} &= 1;\ \ j,\ t=1,\ 2,\ \dots\ n,\ i\neq j, \\
\sum_{j=1}^{n} x_{ijt} &= 1;\ \ i,\ t=1,\ 2,\ \dots\ n,\ i\neq j, \\
x_{ijt} &= 0 \ \text{ or } \ 1, \\
w_{ij} &= (\tilde{c}_{ij};\ \theta),\ i\neq j, \\
\tilde{c}_{ij} &= (a_{ij},\ b_{ij},\ c_{ij}) \text{ or } (a_{ij},\ b_{ij},\ c_{ij},\ d_{ij}).
\end{aligned}
$$

Where

$$
\begin{aligned}
n &= \text{the number of stops to be visited (the number of nodes in the network),} \\
i,\ j &= \text{indices of stops that can take integer values from 1 to } n, \\
t &= \text{the time period or step in the route between the stops,} \\
\tilde{c}_{ij} &= \text{the distance or cost from stop i to stop j,}
\end{aligned}
$$

$$
x_{ijt} = \begin{cases} 1, & \text{if the edge of the network from } i \text{ to } j \text{ is used in step } t \text{ of the route;} \\ 0, & \text{Otherwise.} \end{cases}
$$

The time of travel could be obtained by the equation:

$$
T = \frac{D}{S} \tag{3}
$$

Where T = time of travel, D = distance, S = speed. But when the conditions of edges(distance or cost or time) deviates from the good condition, the edges of the network will be tripled in worst cases scenarios. These false edges are known as virtual edges (V_E) in the network N. The actual edges of the network can be represented by the membership function in figure-2 that are categorises into five such as very small(VS), small(S), medium(M), large(L) and very large(VL) associated with weights $a_0,\ a_1,\ a_2,\ a_3,\ a_4 \in \mathbb{N}$ respectively; where $a_{k+1} = (k+2)a_k,\ k = 0,\ 1,\ 2,\ 3,\ a_0$ = initial value and $a_0 < \text{Mag}(\tilde{c}_{ij}) < a_4$ for all $i,\ j = 1,\ 2,\ \dots\ n$.

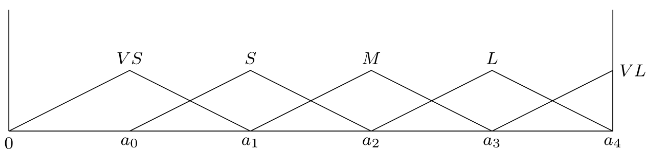

Figure 2. **Membership function for actual edges**

Definition 3.1. *Let N be the network of a salesman with set of vertices($V(N)$) and set of edges($E(N)$) together with the information regarding the road and traffic condition. Now divide the edges in to five categories namely poor(P), fair(F), good(G), excellent(E) and outstanding(O) in figure-3 on the basis of there true information.*

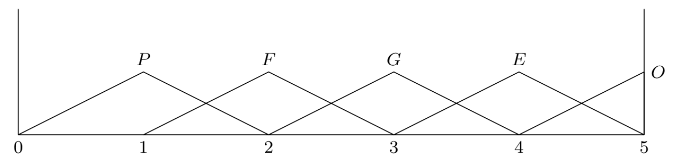

Figure 3. **Membership function for road and traffic conditions**

Using the relation of speed, distance and time we can generate a modifier coefficient (θ) that associate with each of the five categories of edges(road) conditions with weights (w_p, w_f, w_g, w_e and w_o) respectively as in the table-1 under the assumption that the speed will be 60_{kmph} in good conditions.

Virtual coefficient	Edge conditions				
	w_p	w_f	w_g	w_e	w_o
θ	3	1.5	1	0.75	0.6

Table 1. Modifier.

Definition 3.2. *Let N be the network of a salesman with set of vertices($V(N)$) and set of edges($E(N) = \{w_{ij} : i,\ j = 1,\ 2,\ \ldots\ n\}$). Define a virtual modifier function $f : E(N) \to \mathbb{N}$ such that*

$$f(w_{ij}) = \left[\frac{2}{5} + Mag(w_{ij})\right]; \quad Mag(w_{ij}) = \ \theta \cdot Mag(\tilde{c}_{ij}); \quad w_{ij} \in E(N)$$

*where $\theta \in \{w_p,\ w_f,\ w_g,\ w_e,\ w_o\}$, $[\cdot]$ is greatest integer function, $\frac{2}{5}$ is the **Palsu's** number and*

$$Mag(\tilde{c}_{ij}) = \begin{cases} Mag(a_{ij},\ b_{ij},\ c_{ij}), & \text{for triangular fuzzy number;} \\ Mag(a_{ij},\ b_{ij},\ c_{ij},\ d_{ij}), & \text{for trapezoidal fuzzy number.} \end{cases}$$

$Mag(a_{ij},\ b_{ij},\ c_{ij})$ and $Mag(a_{ij},\ b_{ij},\ c_{ij},\ d_{ij})$ are magnitude ranking of triangular and trapezoidal fuzzy number.

The modifier coefficient θ in Definition 3.1 for any network can be redefined according to the requirement and it will act as a catalyst in the defuzzzification process. The process of defuzzification has been defined in association with the modifier coefficient in the Definition 3.2.

Theorem 3.3. *Let N be the network of a salesman with set of vertices ($V(N)$) and set of edges ($E(N) = \{w_{ij} : i,\ j = 1,\ 2,\ \ldots\ n\}$) together with the information regarding the road and traffic condition. Let a virtual modifier function $f : E(N) \to \mathbb{N}$ define on the edges of the network. Then the function modifies all the edges of the network and the decision preference will be smaller virtual edge(Ve_0) to larger(Ve_9).*

Proof. Let N be network of a salesman with set of vertices ($V(N)$) and set of edges ($E(N)$) together with the information regarding the road and traffic condition of a network with

$$w_{ij} = \begin{cases} (\tilde{c}_{ij};\ \theta), & \text{city} \quad i \quad \text{to} \quad \text{city} \quad j; \\ \infty, & \text{city} \quad i \quad \text{to} \quad \text{city} \quad i. \end{cases}$$

where θ is the virtual coefficient, $\tilde{c}_{ij} = (a_{ij},\ b_{ij},\ c_{ij})$ or $(a_{ij},\ b_{ij},\ c_{ij},\ d_{ij})$; are fuzzy triangular and trapezoidal number respectively. Now the travelling salesman problem can be stated in the form of $n \times n$ symmetric cost matrix of real numbers for complete graph as stated below:

$$
\begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_n \end{array}
\begin{array}{c}
\begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\
\begin{bmatrix}
\infty & w_{12} & \dots & w_{1n} \\
w_{21} & \infty & \dots & w_{2n} \\
\vdots & \vdots & & \vdots \\
w_{n1} & w_{n2} & \dots & \infty
\end{bmatrix}
\end{array}
$$

The actual edges of the network can be categorises into five such as very small(VS), small(S), medium(M), large(L) and very large(VL). Now divide the edges in to five categories namely poor(P), fair(F), good(G), excellent(E) and outstanding(O) on the basis of there true information with a modifier coefficient(θ) that associate with each of the five categories of edges(roads) conditions with weights(w_p, w_f, w_g, w_e and w_o); under the assumption that the speed will be 60_{kmph} in good conditions. Let a virtual modifier function $f : E(N) \to \mathbb{N}$ such that

$$f(w_{ij}) = \left[\frac{2}{5} + \text{Mag}(w_{ij})\right]; \quad \text{Mag}(w_{ij}) = \ \theta \cdot \text{Mag}(\tilde{c}_{ij}); \quad w_{ij} \in E(N)$$

Where θ is the virtual coefficient chosen from the modifier Table 1. The function modifies all the edges of the network on the basis of there conditions; particularly in worst cases (poor condition) the actual edge will be enhanced (virtual edge) by multiple of three and in perfect cases (outstanding condition) the actual will be contract (virtual edge) by a factor of 0.6 except the edges are in good conditions. Hence a set of virtual edges i.e., $\{Ve_i : \ i = 1, \ 2, \ \dots \ 9\}$ will be generated with integer weights such that $a_0 \le f(w_{ij}) \le 3 \cdot a_4$. Now the membership function for virtual edges of the network can be estimated and represent in Figure-4. Since the edges (poor condition) are enhanced by multiple of three, so these are not suitable for any decision making situations because it will affect the optimal solution in original environment. Therefore the virtual modifier function not only modify the edges but also preserve the true information with it.

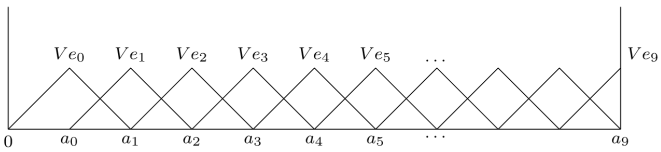

Figure 4. **Membership function for virtual edges**

Hence the decision preference will be greater interest on smaller virtual edge (Ve_0) to larger (Ve_9). This proves the Theorem. □

Example 3.4. *Consider a network of five cities* ($C_i, \quad i = 1, \ 2, \ 3, \ 4, \ 5$) *with a set of edges* ($E(N) = \{w_{ij} : i, \ j = \ 1, \ 2, \ 3, \ 4, \ 5\}$) *for a travelling salesman. Let the distance travelled (cost) are represented by fuzzy quantifiers with true information about the road and traffic conditions characterise by five linguistic variables such as poor(P), fair(F), good(G), excellent(E) and outstanding(O). The edges of the network are* $w_{12} = (2, \ 3, \ 4; \ P) = \ w_{42}$, $w_{13} = (5, \ 6, \ 7; \ E) = \ w_{32} = \ w_{45}$, $w_{14} = (2, \ 3, \ 7; \ F)$, $w_{15} = (1, \ 3, \ 4; \ P) = \ w_{51}$, $w_{21} = (3, \ 4, \ 6; \ F)$, $w_{23} = (3, \ 5, \ 6; \ G)$, $w_{24} = (2, \ 3, \ 8; \ F)$, $w_{25} = (2, \ 4, \ 5; \ F) = \ w_{53}$, $w_{31} = (6, \ 7, \ 8; \ O) = \ w_{54}$, $w_{34} = (4, \ 6, \ 7; \ G)$, $w_{35} = (4, \ 5, \ 7; \ G)$, $w_{41} = (2, \ 3, \ 12; \ G)$, $w_{43} = (6, \ 6, \ 7; \ O)$, $w_{52} = (3, \ 3, \ 4; \ F)$ *and* $w_{ii} = \infty; \ i = 1, \ 2, \ 3, \ 4, \ 5$.

Solution. The following cost matrix with road and traffic condition can be obtained from the network of cities for a travelling salesman problem.

$$\begin{array}{c|ccccc} & C_1 & C_2 & C_3 & C_4 & C_5 \\ \end{array}$$
$$\begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array}\begin{bmatrix} \infty & (2,\ 3,\ 4;\ w_p) & (5,\ 6,\ 7;\ w_e) & (2,\ 3,\ 7;\ w_f) & (1,\ 3,\ 4;\ w_p) \\ (3,\ 4,\ 6;\ w_f) & \infty & (3,\ 5,\ 6;\ w_g) & (2,\ 3,\ 8;\ w_f) & (2,\ 4,\ 5;\ w_f) \\ (6,\ 7,\ 8;\ w_o) & (5,\ 6,\ 7;\ w_e) & \infty & (4,\ 6,\ 7;\ w_g) & (4,\ 5,\ 7;\ w_g) \\ (2,\ 3,\ 12;\ w_g) & (2,\ 3,\ 4;\ w_p) & (6,\ 6,\ 7;\ w_o) & \infty & (5,\ 6,\ 7;\ w_e) \\ (1,\ 3,\ 4;\ w_p) & (3,\ 3,\ 4;\ w_f) & (2,\ 4,\ 5;\ w_f) & (6,\ 7,\ 8;\ w_o) & \infty \end{bmatrix}$$

Now using the Robust's magnitude ranking method we have the following actual edges (distance travel) of the network are

$$\text{Mag}(\tilde{c}_{12}) = \text{Mag}(\tilde{c}_{42}) = 3;\ \text{Mag}(\tilde{c}_{13}) = \text{Mag}(\tilde{c}_{32}) = \text{Mag}(\tilde{c}_{45}) = 6;\ \text{Mag}(\tilde{c}_{14}) = 3.75;$$
$$\text{Mag}(\tilde{c}_{15}) = 2.75;\ \text{Mag}(\tilde{c}_{21}) = 4.25;\ \text{Mag}(\tilde{c}_{23}) = 4.75;\ \text{Mag}(\tilde{c}_{24}) = 4;$$
$$\text{Mag}(\tilde{c}_{25}) = \text{Mag}(\tilde{c}_{53}) = 3.75;\ \text{Mag}(\tilde{c}_{31}) = \text{Mag}(\tilde{c}_{54}) = 7;$$
$$\text{Mag}(\tilde{c}_{34}) = 5.25;\ \text{Mag}(\tilde{c}_{35}) = 5.75;\ \text{Mag}(\tilde{c}_{41}) = 5;$$
$$\text{Mag}(\tilde{c}_{43}) = 6.5;\ \text{Mag}(\tilde{c}_{52}) = 4.25.$$

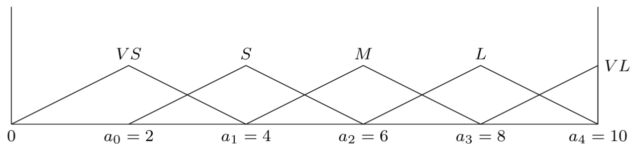

Figure 5. **Membership function for actual edges the Example 3.4**

Now using the virtual modifier function along with virtual coefficient and Robust's magnitude ranking method we have the following virtual edges(distance travel) are

$$f(w_{12}) = f(w_{42}) = 9;\ f(w_{13}) = f(w_{32}) = f(w_{45}) = 4;\ f(w_{14}) = 6;\ f(w_{15}) = 8;$$
$$f(w_{21}) = 6;\ f(w_{23}) = 5;\ f(w_{24}) = 6;\ f(w_{25}) = f(w_{53}) = 6;\ f(w_{34}) = 5;$$
$$f(w_{31}) = f(w_{54}) = 4;\ f(w_{35}) = 6;\ f(w_{41}) = 5;\ f(w_{43}) = 4;\ f(w_{52}) = 6.$$

Thus clearly in decision making situation the smaller values are preferable instead of larger because of their better road and traffic conditions.

Computational steps

The computational steps to get optimal solution from fuzzy travelling salesman problems with fuzzy decision variables is divided into two stages. For the sack of simplicity, first we convert the fuzzy travelling salesman with fuzzy decision variables(F-TSP) to classical travelling salesman problems(C-TSP) with virtual parameters and then optimal solution obtained by using any standard method.

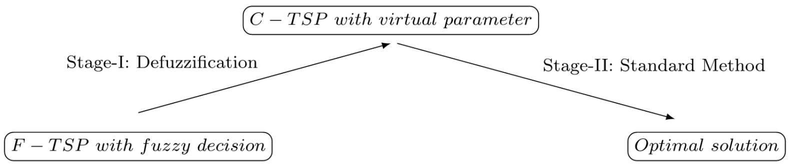

Figure 6. **Algorithmic model**

Phase-I Defuzzification: Now convert the fuzzy travelling salesman problems with fuzzy decision variables in to classic travellings salesman problems with virtual parameters(crisp value) using the virtual modifier function in Definition 3.2. The figure-4 will describe the membership function for virtual edge(road) conditions.

phase-II Direct approach: Any standard method can be useful to solve the virtual assignment cost matrix to get the optimal solution.

Example 3.5. *From the Example 3.4; we have the following actual and virtual assignment cost matrix respectively*

$$
\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array}
\begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} \infty & 3 & 6 & 3.75 & \boxed{2.75} \\ \boxed{4.25} & \infty & 4.75 & 4 & 3.75 \\ 7 & 6 & \infty & \boxed{5.25} & 5.75 \\ 5 & \boxed{3} & 6.5 & \infty & 6 \\ 2.75 & 4.25 & \boxed{3.75} & 7 & \infty \end{bmatrix} \end{array}
\qquad
\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array}
\begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} \infty & 9 & 4 & \boxed{6} & 8 \\ \boxed{6} & \infty & 5 & 6 & 6 \\ 4 & 4 & \infty & 5 & \boxed{6} \\ 5 & 9 & \boxed{4} & \infty & 4 \\ 8 & \boxed{6} & 6 & 4 & \infty \end{bmatrix} \end{array}
$$

Using the direct method proposed by Mohanta [2]; we get the optimal solution of the actual problem is $C_1 \to C_5 \to C_3 \to C_4 \to C_2 \to C_1$ *with optimal distance travelled (cost) is* 19, *its corresponding fuzzy optimal distance is* $(12,\ 20,\ 26)$ *and for virtual problem is* $C_1 \to C_4 \to C_3 \to C_5 \to C_2 \longrightarrow C_1$ *with optimal distance travelled (cost) is* 28 *with corresponding fuzzy optimal cost is* $(18,\ 21,\ 31)$.

4. Result analysis

The optimal tour obtained from the actual cost matrix and virtual cost matrix (see the following figure-6) are comparable with one another to get the best alternative.

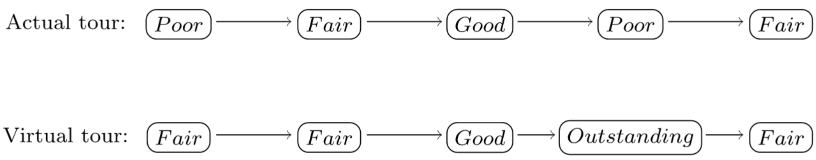

Figure 7. **Optimal sequence**

Since the actual tour contains some edges of poor road and traffic conditions that will affect the optimal tour in the true environment and the optimal cost will rise from 19 to 34. But the optimal cost for the virtual tour will be all most same because it contains better road and traffic conditions as compare to the actual tour.

5. Conclusion

This article mainly generate an alternative optimal tour of the travelling salesman in a network of cities having prior information about the road and traffic conditions of the edges. Finally it can be claimed that optimal tour is one of the best alternative to the salesman of its kind and can be completed in a reasonable short amount of time. The application of virtual modifier function can be extended to assignment problems, transportation problems and all other linear programming problems under the environment of fuzzy decision variables.

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