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An Atlas of Different Distances Sets Polynomials of Graphs of Order at most Six

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Abstract: The different distances sets polynomial of a graph G of order p is defined as $D_d(G,x)=\sum_{i=1}^{p} d_d(G,i)x^i$, where $d_d(G,i)$ is the number of different distances sets polynomials of G of size i, [1]. We call the roots of different distances sets polynomial of a graph the different distances roots of that graph. In this article, we compute different distances sets polynomial of all graphs of order less than or equal six and their roots and present them in tables.

MSC: 05C76, 05C07, 05C05, 05C90.

Keywords: Different distances sets, different distances sets polynomials.

1. Introduction

The concept of different-distance sets of graph was introduced by H. M. Mulder [6], they are prove that a different-distance set induces either a special type of path or an independent set. and they present properties of different-distance sets, and consider the different-distance numbers of paths, cycles, Cartesian products of bipartite graphs, and Cartesian products of complete graphs.

Let $G=(V,E)$ be a graph. As usual $p=|V|$ and $q=|E|$ denote the number of vertices and edges of a graph G, respectively. In general we use $\langle X\rangle$ to denote the subgraph induced by the set of vertices X [3]. A graph G is connected if for every tow vertices $u,v\in V$, there exists a (u,v)-path in G. Otherwise G is called disconnected [4].

The open neighborhood and the closed neighborhood of v are denoted by $N(v)=\{u\in V : uv\in E\}$ and $N[v]=N(v)\cup\{v\}$, respectively. If $X\subseteq V$ then $N(X)=\cup_{v\in X}N(v)$ and $N[X]=N(X)\cup X$. The degree of a vertex v, in a graph G, is denoted $deg(v)$, and is defined to be the number of edges incident with v. In simple graphs, $deg(v)=|N(v)|$. The minimum degree of a graph G is denoted by δ, and the maximum degree is denoted by Δ. If $\delta=\Delta=r$ for any graph G, we say G is a regular graph of degree r. A vertex of degree equals to Zero in G is called an **isolated** vertex and a vertex of degree one is called a pendant vertex or an end vertex. An edge in a graph G is called a pendant edge if it is incident with a pendant

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vertex [3].

An independent Set ($astable$) in a graph G is a set of vertices no two of which are adjacent.An independent set in graph is maximum if the graph contains no larger stable set and maximal if the set cannot be extended to alarger stable set [2].

The distance $d(u,v)$ between any two vertices $u, v \in G$ is the minimum length of a u-v path [4]. Let $G = (V,E)$, be a connected, simple graph without loops, with vertex set V and edge set E. The order of G is the number $|V|$. Let u and v be two vertices in V. The degree $deg_G(u)$ of u is the number of neighbors of u, that is, vertices adjacent to u. The distance $d_G(u,v)$ between u and v is the length of a shortest u, v-path, or u, v-geodesic. The interval between u and v is the set $I_G(u,v) = \{w|d(u,w) + d(w,v) = d(u,v)\}$ [5].

A set S of vertices in a connected graph G is a different-distance set if $d(u,w) \neq d(v,w)$, for any vertex w in $V - S$, and for any two vertices u and v in S.

Note that this is equivalent to $u, v \in S, d(x,u) = d(x,v) \Rightarrow x \in S$ [6]. The minimum and maximum orders of different-distance sets S in G are called the lower and upper difference-distance numbers of G, and are denoted $dd(G)$ and $DD(G)$ respectively [1].

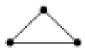
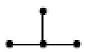
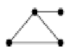
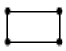
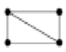
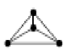
We assume that the entire vertex set $S = V$ is not a different-distance set, since there are no vertices in $V - S = \phi$ to demonstrate different distances to the vertices in S. Similarly, we exclude the empty set ϕ, because in this case nothing is to demonstrate. On the other hand, we assume that, for any nontrivial connected graph G, any singleton set $S = u$ is a different-distance set, since no vertex in $V - S$ is equidistant to two vertices in S [6]. Thus it follows, by definition, that for any connected graph G of order $n \geq 2$, we have $1 \leq dd(G) \leq DD(G) \leq n - 1$.

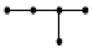
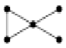
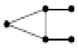
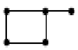
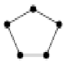
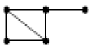
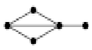
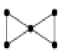
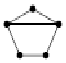
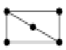
Let $D_d(G,i)$ be the family of different-distance sets polynomial of graph G with coordinately i and let $d_d(G,i) = |G_d(G,i)|$, Then the different-distance sets polynomial $D_d(G,x)$ of G is defined as $D_d(G,x) = \sum_{i=1}^{n} d_d(G,i)x^i$, where i is the size of different-distance sets of graph G [1].

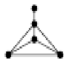
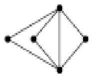
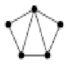
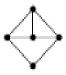
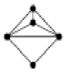
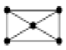
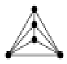
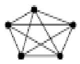
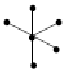
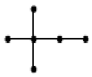
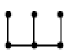
As all the type of graph polynomials, the analysis of the different-distance sets polynomial of graph can give use some information about graph [1], we have an atlas for different-distance sets polynomial and the different-distance sets roots of graph with order at most 6. In this article, we compute the different-distance sets polynomials and the root of different-distance sets of all connected graph of order less than or equal six and we present them in tables.

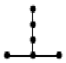
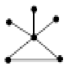
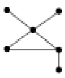
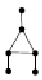
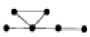
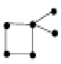
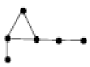
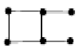
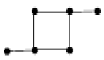
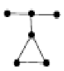
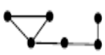
2. Main Results

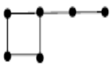
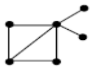
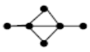
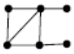
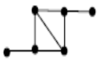
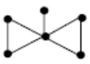
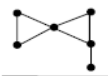
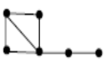
The table of different-distance sets polynomials and the root of different-distance sets of all connected graph of order less than or equal six are as follows.

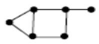
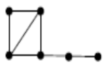
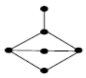
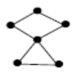
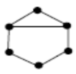
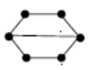
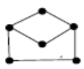
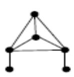
Graph	**Different Distances Sets Polynomial**	**Different Distances Sets Roots**
	x	0
	$2x$	0
	$2x^2+3x$	$0, \frac{-3}{2}$
	$3x$	0
	$4x^2+3x$	$0, -\frac{4}{3}$
	$2x^3+4x^2+4x$	$0, \frac{-1}{2}+i\frac{\sqrt{7}}{2}, \frac{-1}{2}-i\frac{\sqrt{7}}{2}$
	x^2+4x	$0, -4$
	$4x^2+4x$	$0, -1$
	$4x$	0
	$4x$	0
	$4x^2+5x$	$0, -\frac{5}{4}$

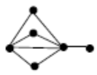
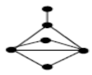
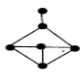
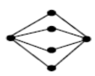
Graph	Different Distances Sets Polynomial	Different Distances Sets Roots
	x^3+6x^2+5x	$0,-1,-5$
	$2x^4+3x^3+6x^2+5x$	$0,-1,\ -\frac{1}{4}+i\frac{39}{4},-\frac{1}{4}-i\frac{39}{4}$
	$2x^2+5x$	$0,-\frac{5}{2}$
	$5x^2+5x$	$0,-\frac{5}{2}$
	x^3+4x^2+5x	$0,-2+i,-2-i$
	$6x^2+5x$	$0,-\frac{5}{6},$
	$5x$	0
	$2x^2+5x$	$0,-\frac{5}{2}$
	x^2+5x	$0,-5$
	$5x$	0
	$2x^2+5x$	$0,-5$
	$6x^2+5x$	$0,-\frac{5}{6}$

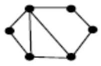
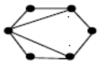
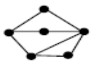
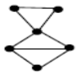
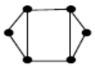
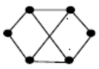
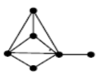
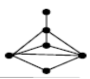
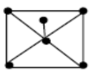
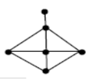
Graph	Different Distances Sets Polynomial	Different Distances Set Roots
	$2x^2 + 5x$	$0, -\frac{5}{2}$
	$5x$	0
	$5x$	0
	$2x^2 + 5x$	$0, -\frac{5}{2}$
	$5x$	0
	$5x$	0
	$5x$	0
	$5x$	0
	$5x^2 + 6x$	$0, -\frac{5}{6}$
	$x^3 + 8x^2 + 6x$	$0, -4 + \sqrt{10}, -4 - \sqrt{10}$
	$9x^2 + 6x$	$0, -\frac{2}{3}$
	$2x^3 + 9x^2 + 6x$	$0, \frac{-9+\sqrt{33}}{4}, \frac{-9-\sqrt{33}}{4}$

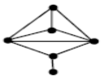
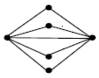
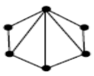
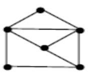
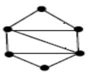
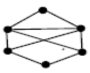
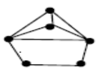
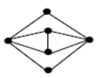
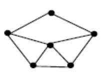
Graph	Different Distances Sets Polynomial	Different Distances Set Roots
	$2x^3 + 7x^2 + 6x$	$0, -\frac{3}{2}, -2$
	$2x^5 + 3x^4 + 4x^3 + 9x^2 + 6x$	$0, -1, -1.24045, 0.37022 + i1.51044, 0.37022 - i1.51044$
	$3x^2 + 6x$	$0, \sqrt{2i}, -\sqrt{2i}$
	$3x^2 + 6x$	$0, -2$
	$3x^2 + 6x$	$0, -2$
	$x^3 + 6x^2 + 6x$	$0, -3 + \sqrt{3}, -3 - \sqrt{3}$
	$8x^2 + 6x$	$0, -\frac{3}{4}$
	$x^3 + 5x^2 + 6x$	$0, -2, -3$
	$8x^2 + 6x$	$0, -\frac{3}{4}$
	$8x^2 + 6x$	$0, -\frac{3}{4}$
	$8x^2 + 6x$	$0, -\frac{3}{4}$
	$x^4 + 2x^3 + 6x^2 + 6x$	$0, -1.19128, -0.40436 + i2.20751, -0.40436 - i2.20751$

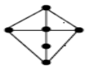
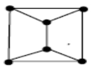
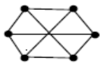
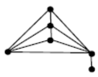
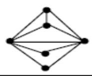
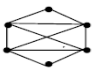
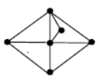
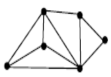
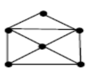
Graph	Different Distances Sets Polynomial	Different Distances Set Roots
	$x^3 + 9x^2 + 6x$	$0, \frac{-9+\sqrt{57}}{2}, \frac{-9-\sqrt{57}}{2}$
	$x^2 + 6x$	$0, -6$
	$9x^2 + 6x$	$0, -\frac{2}{3}$
	$8x^2 + 6x$	$0, -\frac{3}{4}$
	$2x^2 + 6x$	$0, -3$
	$2x^2 + 6x$	$0, -3$
	$4x^2 + 6x$	$0, -\frac{3}{2}$
	$4x^2 + 6x$	$0, -\frac{3}{2}$
	$x^2 + 6x$	$0, -6$
	$3x^2 + 6x$	$0, -2$
	$x^2 + 6x$	$0, -6$
	$x^3 + 5x^2 + 6x$	$0, -2, -3$

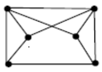
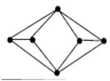
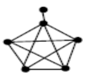
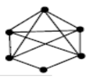
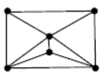
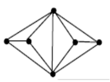
Graph	Different Distance Sets Polynomial	Different Distance Set Roots
	$7x^2 + 6x$	$0, -\frac{6}{7}$
	$4x^2 + 6x$	$0, -\frac{3}{2}$
	$x^3 + 5x^2 + 6x$	$0, -2, -3$
	$3x^2 + 6x$	$0, -2$
	$9x^2 + 6x$	$0, -\frac{-2}{3}$
	$6x^2 + 6x$	$0, -1$
	$x^2 + 6x$	$0, -6$
	$9x^2 + 6x$	$0, -\frac{2}{3}$
	$6x$	0
	$5x^2 + 6x$	$0, -\frac{6}{5}$
	$2x^2 + 6x$	$0, -3$
	$2x^2 + 6x$	$0, -3$

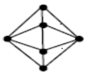
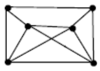
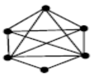
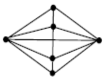
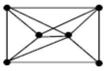
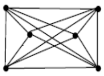
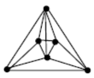
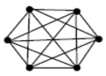
Graph	Different Distance Sets Polynomial	Different Distance Set Roots
	$x^2 + 6x$	$0, -6$
	$x^2 + 6x$	$0, -6$
	$x^2 + 6x$	$0, -6$
	$3x^2 + 6x$	$0, -2$
	$2x^2 + 6x$	$0, -3$
	$3x^2 + 6x$	$0, -2$
	$5x^2 + 6x$	$0, -\frac{6}{5}$
	$x^3 + 4x^2 + 6x$	$0, -2 + \sqrt{2}i, -2 + \sqrt{2}i$
	$5x^2 + 6x$	$0, -\frac{6}{5}$
	$6x$	0
	$2x^2 + 6x$	$0, -3$
	$8x^2 + 6x$	$0 - \frac{3}{4}$

Graph	Different Distance Sets Polynomial	Different Distance Set Roots
	$3x^2 + 6x$	$0, -2$
	$6x$	0
	$3x^2 + 6x$	$0, -2$
	$x^2 + 6x$	$0, -6$
	$2x^2 + 6x$	$0, -3$
	$3x^2 + 6x$	$0, -2$
	$6x$	0
	$9x^2 + 6x$	$0, -\frac{2}{3}$
	$x^2 + 6x$	$0, -\frac{1}{6}$
	$x^2 + 6x$	$0, -\frac{1}{6}$
	$x^2 + 6x$	$0, -\frac{1}{6}$
	$2x^2 + 6x$	$0, -3$

Graph	Different Distance Sets Polynomial	Different Distance Set Roots
	$3x^2 + 6x$	$0, -2$
	$6x$	0
	$6x$	0
	$6x$	0
	$6x$	0
	$6x$	$0i$
	$x^2 + 6x$	$0, -6$
	$x^2 + 6x$	$0, -6$
	$3x^2 + 6x$	$0, -2$
	$x^2 + 6x$	$0, -6$
	$4x^2 + 6x$	$0, -\frac{3}{2}$
	$6x$	0

Graph	Different Distance Sets Polynomial	Different Distance Set Roots
	$x^2 + 6x$	$0, -6$
	$4x^2 + 6x$	$0, -\frac{3}{2}$
	$3x^2 + 6x$	$0, -2$
	$9x^2 + 6x$	$0 - \frac{3}{2}$
	$x^2 + 6x$	$0. - 6$
	$2x^2 + 6x$	$0, -3$
	$6x$	0
	$6x$	0
	$x^2 + 6x$	$0, -6$
	$6x$	0
	$6x$	0
	$6x$	0

Graph	**Different Distance Sets Polynomial**	**Different Distance Set Roots**
	$x^2 + 6x$	$0, -6$
	$2x^2 + 6x$	$0, -3$
	$6x$	0
	$x^2 + 6x$	$0, -6$
	$3x^2 + 6x$	$0, -2$
	$6x$	0
	$x^2 + 6x$	$0, -6$
	$6x$	0
	$6x$	0
	$2x^2 + 6x$	$0, -3$
	$6x$	0
	$6x$	0

Graph	Different Distance Sets Polynomial	Different Distance Sets Roots
	$6x$	0
	$6x$	0
	$x^2 + 6x$	$0, -6$
	$6x$	0
	$6x$	0
	$6x$	0
	$6x$	0
	$6x$	0
	$6x$	0
	$6x$	0
	$6x$	0
	$6x$	0

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[3] G. Chartrand and L. Lesniak, *Graph and Digraph*, 2^{nd} Edition, Chap. Hall/CRC, (1996).

[4] F. Harary, *Graph Theory*, Narosa Publishing House, New Delhi, (2001).

[5] H. M. Mulder, *The interval function of a graph*, Mathematical Centre Tract 132, Mathematisch Centrum, Amsterdam, (1980).

[6] H. M. Mulder, J. T. Hedetniemi and S. T. Hedetniemi, *Different-Distance Sets in a Graph*, (2019).