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The Use of Adomian Decomposition Method for Solving Nonlinear Wave-Like Equation With Variable Coefficients

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Abstract: The Adomian Decomposition Method (ADM) has been widely applied in solving partial differential equations which represent various phenomena in engineering and physics. In this paper, nonlinear wave-like equations with variable coefficients are solved using Sawi Adomian De-composition Method (SADM). Sawi decomposition method is a combined form of Sawi transform method and the Adomian decomposition method. The nonlinear term can easily be handled with the help of Adomian polynomials which is considered to be a significant advantage of this technique over the other methods. We shall show that SADM is able to solve this type of equations effectively and accurately.

Keywords: Sawi transform, Sawi A domian decomposition, Wave-like equation.

1. Introduction

Differential equations are two types: linear differential equations and non-linear differential equations. Non-linear differential equations are the most complex in the solution compared with linear differential equations due to the presence of non-linear part in them. So we find that a lot of researchers are working to develop new methods to solve this kind of equations.

The most frequent used methods for investigating linear and non-linear differential equations are: Adomian decomposition method (ADM) [1, 2] variational iteration method (VIM) [3], generalized differential transform method (GDTM) [4], homotopy analysis method (HAM) [5], homotopy perturbation method (HPM) [6], homotopy decomposition method [7]. Also, there are some other classical solution techniques such as Laplace transform method, Mohand transform [8], and Kamal transform method [9].

To cite a few, Nuruddeen [10] utilized the Elzaki Decomposition Method as a reliable technique for Solving Linear and Nonlinear Schrodinger Equations. In Chi-Min Liu and Ray-Yeng Yang [11] Application of Adomian Decomposition Method was used to Bounded and Unbounded Stokes' Problems. In Agom, [12] Application of Adomian Decomposition Method was used to Solving Second Order Nonlinear Ordinary Differential Equations. In [13] Fractional natural decomposition method was used to for solving a certain class of nonlinear time-fractional wave-like equations with variable coeffcients .Also, Afgan Aslanov [14] used the A Homotopy-Analysis as a method to search for the solutions of Nonlinear Wave-Like Equations with Variable Coefficients.

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In this paper, we intend to couple Sawi transform established recently by Mohand Mahgoub [15] with the a well-known systematic; the Adomian decomposition method. Sawi transform is known for its effectiveness in solving linear ordinary differential equations, linear partial differential equation and integral equations. While on the other hand, the Adomian decomposition method is a well-known method for solving linear and nonlinear, homogeneous and nonhomogeneous differential and partial differential equations, integro-differential and fractional differential equations that gives exact solutions in form of a convergent series. Sawi Transform of partial derivative is derived, and its applicability was demonstrated using four different partial equations in [13]. Based on the work in [2] we apply KADM to solve nonlinear wave-like equations with variable coefficients.

2. Definition and Derivations of Sawi Transform of Derivatives

Sawi transform of the function $f(t)$ is defined as

$$S\left[f\left(t\right)\right]=\frac{1}{v^{2}}\int_{0}^{\infty}f\left(t\right)e^{\frac{-t}{v}}dt=\ R\left(v\right),\quad t>0\qquad k_{1}\leq v\leq k_{2} \tag{1}$$

To obtain Sawi transform of partial derivatives we use integration by parts as follows:

$$\begin{aligned}S\left[\frac{\partial f(x,t)}{\partial t}\right]&=\frac{1}{v^{2}}\int_{0}^{\infty}\frac{\partial f(x,t)}{\partial t}e^{\frac{-t}{v}}dt\\&=\frac{1}{v^{2}}\lim_{p\to\infty}\int_{0}^{p}e^{\frac{-t}{v}}\frac{\partial f(x,t)}{\partial t}dt\\&=\frac{1}{v^{2}}\lim_{p\to\infty}\left(\left[e^{\frac{-t}{v}}f\left(x,t\right)\right]_{0}^{p}+\frac{1}{v}\int_{0}^{p}e^{\frac{-t}{v}}f\left(x,t\right)dt\right)\\&=\frac{1}{v^{2}}\left[-f\left(x,0\right)+vR(x,v)\right]\end{aligned}$$

Thus,

$$S\left[\frac{\partial f(x,t)}{\partial t}\right]=v^{-1}R\left(x,v\right)-v^{-2}f\left(x,0\right) \tag{2}$$

To find $S\left[\frac{\partial^{2}f(x,t)}{\partial t^{2}}\right]$, let $\frac{\partial f(x,t)}{\partial t}=g(x,t)$, then by using Equation (2) we have:

$$S\left[\frac{\partial^{2}f(x,t)}{\partial t^{2}}\right]=S\left[\frac{\partial g(x,t)}{\partial t}\right]=v^{-1}S\left[\frac{\partial g(x,t)}{\partial t}\right]-v^{-2}g(x,0)$$

$$S\left[\frac{\partial^{2}f(x,t)}{\partial t^{2}}\right]=v^{-2}R\left(x,v\right)-v^{-2}\frac{\partial f\left(x,0\right)}{\partial t}-v^{-3}f(x,0) \tag{3}$$

We can easily extend this result to the nth partial derivative by using mathematical induction. Now, we assume the $f(x,t)$ is piecewise continuous and is of exponential order. Then,

$$S\left[\frac{\partial f(x,t)}{\partial x}\right]=\frac{1}{v^{2}}\int_{0}^{\infty}e^{\frac{-t}{v}}\frac{\partial f(x,t)}{\partial x}dt$$

Using the Leibniz' rule

$$S\left[\frac{\partial f(x,t)}{\partial x}\right]=\frac{1}{v^{2}}\int_{0}^{\infty}e^{\frac{-t}{v}}\frac{\partial f(x,t)}{\partial x}dt=\frac{1}{v^{2}}\frac{\partial}{\partial x}\int_{0}^{\infty}e^{\frac{-t}{v}}f\left(x,t\right)dt$$

Thus,

$$S\left[\frac{\partial f(x,t)}{\partial x}\right]=\frac{d}{dx}\left(R\left(x,v\right)\right) \tag{4}$$

Also we can find:

$$S\left[\frac{\partial^{2}f(x,t)}{\partial x^{2}}\right]=\frac{d^{2}}{dx^{2}}\left(R\left(x,v\right)\right) \tag{5}$$

And

$$S\left[\frac{\partial^{n}f(x,t)}{\partial x^{n}}\right]=\frac{d^{n}}{dx^{n}}\left(R\left(x,v\right)\right) \tag{6}$$

3. Basic Idea of Sawi Adomian Decomposition Method (SADM)

The general form of nonlinear non-homogeneous partial differential equation can be considered as the follow:

$$Du(x,t) + Ru(x,t) + Nu(x,t) = f(x,t) \tag{7}$$

With the following initial conditions

$$u(x,0) = h(x), \quad ut(x,0) = g(x)$$

Where D is the second order linear differential operator $D = \frac{\partial^2}{\partial t^2}$, is the linear differential operator of less order than D, N represents the general non-linear differential operator and $f(x,t)$ is the source term. Taking Sawi transform (denoted throughout this paper by $S(.)$) on both sides of Equation (7), to get:

$$S[Du(x,t) + Ru(x,t) + Nu(x,t)] = S[f(x,t)] \tag{8}$$

Using the differentiation property of Sawi transform and above initial conditions, we have:

$$S[u(x,t)] = v^2 S[f(x,t)] + \frac{1}{v}h(x) + g(x) - v^2 S[Ru(x,t) + Nu(x,t)] \tag{9}$$

Operating with Sawi inverse transform as in [2], on both sides of Eq.(9) gives:

$$u(x,t) = G(x,t) - S^{-1}[v^2 S[Ru(x,t) + Nu(x,t)]] \tag{10}$$

Where $G(x,t)$ represents the term arising from the source term and the prescribed initial condition. Now, we apply the Adomian decomposition method as in [14]

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{11}$$

And the nonlinear term can be decomposed as:

$$Nu(x,t) = \sum_{n=0}^{\infty} A_n(u) \tag{12}$$

Where $A_n(u)$ are Adomian polynomials of $u_0, u_1, u_2 \ldots, u_n$ and it given by the following formula:

$$A_n = \frac{1}{n!}\frac{\partial^n}{\partial \lambda^n}\left[N\left(\sum_{n=0}^{\infty} \lambda^i u_i\right)\right]_{\lambda=0}, \; n = 0, 1, 2, \ldots \tag{13}$$

Substituting Equation (12) and (11) in Equation (10) we get:

$$\sum_{n=0}^{\infty} u_n(x,t) = G(x,t) - S^{-1}\left[v^2 S\left[R\sum_{n=0}^{\infty} u_n(x,t) + \sum_{n=0}^{\infty} A_n(u)\right]\right] \tag{14}$$

On comparing both sides of the Equation (14), we get

$$u_0(x,t) = G(x,t),$$

$$u_1(x,t) = -S^{-1}\left[v^2 S[Ru_0(x,t) + A_0(u)]\right],$$

$$u_2(x,t) = -S^{-1}\left[v^2 S[Ru_1(x,t) + A_1(u)]\right],$$

$$u_3(x,t) = -S^{-1}\left[v^2 S[Ru_2(x,t) + A_2(u)]\right],$$
$$\vdots$$

In general the recursive relation is given by

$$u_0(x,t) = G(x,t),$$
$$u_{n+1}(x,t) = -S^{-1}\left[v^2 S\left[Ru_n(x,t) + A_n(u)\right]\right], \quad n \geq 0 \tag{15}$$

Finally, applying Sawi transform of the right hand side Equation (15) and then taking inverse Sawi transform, we get $u_0, u_1, u_2, \ldots$ which are the series form of the desired solutions.

4. Numerical Examples

In this section we discuss some examples to illustrate Sawi transform Decomposition Method.

Example 4.1. *Let's consider the second-dimensional nonlinear wave-like equation with variable coefficients [16]*

$$u_{tt} = \frac{\partial^2}{\partial x\partial y}(u_{xx}u_{yy}) - \frac{\partial^2}{\partial x\partial y}(xyu_x u_y) - u \tag{16}$$

With the initial condition;

$$u(x,y,0) = e^{xy}, u_t(x,y,0) = e^{xy} \tag{17}$$

Applying Sawi transform of both sides of Equation (16), we obtain,

$$S[u_{tt}(x,y,t)] = S\left[\frac{\partial^2}{\partial x\partial y}(u_{xx}u_{yy}) - \frac{\partial^2}{\partial x\partial y}(xyu_x u_y) - u\right] \tag{18}$$

Using the differential property of Sawi transform. Equation (18) can be written as:

$$S[u(x,y,t)] = \frac{1}{v}e^{xy} + e^{xy} + v^2 S\left[\frac{\partial^2}{\partial x\partial y}(u_{xx}u_{yy}) - \frac{\partial^2}{\partial x\partial y}(xyu_x u_y) - u\right] \tag{19}$$

The inverse of Sawi transform implies that:

$$u(x,y,t) = e^{xy} + te^{xy} + S^{-1}\left[v^2 S\left[\frac{\partial^2}{\partial x\partial y}(u_{xx}u_{yy}) - \frac{\partial^2}{\partial x\partial y}(xyu_x u_y) - u\right]\right] \tag{20}$$

Following the technique, if we assume an infinite series solution of the form Equations (11) and (12), we obtain

$$\sum_{n=0}^{\infty} u_n(x,y,t) = e^{xy} + te^{xy} + S^{-1}\left[v^2 S\left[\frac{\partial^2}{\partial x\partial y}\sum_{n=0}^{\infty} A_n(u) - \frac{\partial^2}{\partial x\partial y}\left(xy\sum_{n=0}^{\infty} B_n(u)\right) - \sum_{n=0}^{\infty} u_n(x,y,t)\right]\right] \tag{21}$$

Where $A_n(u)$ and $B_n(u)$ are Adomian polynomials that represent nonlinear term. So Adomian polynomials are given as follows:

$$A_n(u) = u_{xx}u_{yy} \quad and \quad B_n(u) = u_x u_y$$

Using (13). The few components of the Adomian polynomials are given as follows:

$$A_0 = (u_0)_{xx}(u_0)_{yy}$$

$$A_1 = (u_1)_{xx}(u_0)_{yy} + (u_0)_{xx}(u_1)_{yy}$$
$$A_2 = (u_2)_{xx}(u_0)_{yy} + (u_1)_{xx}(u_1)_{yy} + (u_0)_{xx}(u_2)_{yy}$$
$$\vdots$$
$$B_0 = (u_0)_x(u_0)_y$$
$$B_1 = (u_1)_x(u_0)_y + (u_0)_x(u_1)_y$$
$$B_2 = (u_2)_x(u_0)_y + (u_1)_x(u_1)_y + (u_0)_x(u_2)_y$$
$$\vdots$$

From the relationship in (15), we obtain

$$\begin{aligned} u_0(x,y,t) &= G(x,t) = e^{xy} + te^{xy} \\ u_1(x,y,t) &= S^{-1}\left[v^2 S\left[\frac{\partial^2}{\partial x\partial y}A_0(u) - \frac{\partial^2}{\partial x\partial y}(xyB_0(u)) - u_0(x,y,t)\right]\right] \\ &= \frac{-t^2e^{xy}}{2} - \frac{t^3e^{xy}}{6} \\ u_2(x,y,t) &= S^{-1}\left[v^2 S\left[\frac{\partial^2}{\partial x\partial y}A_1(u) - \frac{\partial^2}{\partial x\partial y}(xyB_1(u)) - u_1(x,y,t)\right]\right] \\ &= \frac{t^4e^{xy}}{24} + \frac{t^5e^{xy}}{120} \\ &\vdots \end{aligned}$$

Which in closed form gives exact solution

$$u(x,y,t) = \sum_{n=0}^{\infty} u_n(x,y,t) = e^{xy}\left(1 + t - \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} + \dots\right) \tag{22}$$

Thus,

$$u(x,y,t) = e^{xy}(\sin t + \cos t) \tag{23}$$

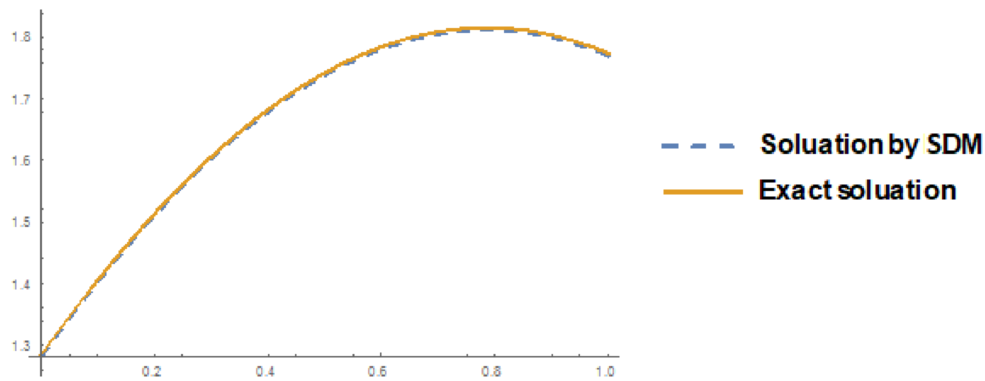

Figure 1. **Example 4.1 Solution by SDM and exact solution**

As shown by figure 1, comparison of the obtained results with those of the exact solution, reveals that SADM leads to accurate solutions for Example 4.1.

Example 4.2. *Let's consider the nonlinear partial differential equation*

$$u_{tt} = u^2 \frac{\partial^2}{\partial x^2}(u_x u_{xx} u_{xxx}) - u_x^2 \frac{\partial^2}{\partial x^2}\left(u_{xx}^3\right) - 18u^5 + u, \quad 0 < x < 1, \ t > 0 \tag{24}$$

With the initial condition;

$$u(x,0) = e^x, u_t(x,0) = e^x \tag{25}$$

Applying Sawi transform of both sides of Equation (25), we obtain,

$$S[u_{tt}(x,t)] = S\left[u^2 \frac{\partial^2}{\partial x^2}(u_x u_{xx} u_{xxx}) - u_x^2 \frac{\partial^2}{\partial x^2}\left(u_{xx}^3\right) - 18u^5 + u\right] \tag{26}$$

Using the differential property of Sawi transform. Equation (26) can be written as:

$$S[u(x,t)] = \frac{1}{v}e^x + e^x + v^2 S\left[u^2 \frac{\partial^2}{\partial x^2}(u_x u_{xx} u_{xxx}) - u_x^2 \frac{\partial^2}{\partial x^2}\left(u_{xx}^3\right) - 18u^5 + u\right] \tag{27}$$

The inverse of Sawi transform implies that:

$$u(x,t) = e^x + te^x + S^{-1}\left[v^2 S\left[u^2 \frac{\partial^2}{\partial x^2}(u_x u_{xx} u_{xxx}) - u_x^2 \frac{\partial^2}{\partial x^2}\left(u_{xx}^3\right) - 18u^5 + u\right]\right] \tag{28}$$

Following the technique, if we assume an infinite series solution of the form Equations (11) and (12), we obtain

$$\sum_{n=0}^{\infty} u_n(x, \ t) = e^x + te^x + S^{-1}\left[v^2 S\left[\sum_{n=0}^{\infty} A_n(u) - \sum_{n=0}^{\infty} B_n(u) - 18\sum_{n=0}^{\infty} C_n(u) + \sum_{n=0}^{\infty} u_n(x,t)\right]\right] \tag{29}$$

Where $A_n(u)$, $B_n(u)$ and $C_n(u)$ are Adomian polynomials that represent nonlinear term. So Adomian polynomials are given as follows:

$$A_n(u) = u^2 \frac{\partial^2}{\partial x^2}(u_x u_{xx} u_{xxx}), \qquad B_n(u) = u_x^2 \frac{\partial^2}{\partial x^2}\left(u_{xx}^3\right) \quad \textit{and} \quad C_n(u) = u^5$$

Using (13). The few components of the Adomian polynomials are given as follows:

$$\begin{aligned}
A_0 &= {u_0}^2 \frac{\partial^2}{\partial x^2}({u_0}_x u_{0xx} u_{0xxx}) \\
A_1 &= u_1 u_0 \frac{\partial^2}{\partial x^2}({u_0}_x u_{0xx} u_{0xxx}) + {u_0}^2 \frac{\partial^2}{\partial x^2}({u_1}_x u_{0xx} u_{0xxx}) + ({u_0}_x u_{1xx} u_{0xxx}) + ({u_0}_x u_{0xx} u_{1xxx}) \\
&\ \vdots \\
B_0 &= (u_0)_x^2 \frac{\partial^2}{\partial x^2}\left((u_0)_{xx}^3\right) \\
B_1 &= 2(u_0)_x (u_1)_x \frac{\partial^2}{\partial x^2}\left((u_0)_{xx}^3\right) + (u_0)_x^2 \frac{\partial^2}{\partial x^2}\left(3(u_0)_{xx}^2 (u_1)_{xx}\right) \\
&\ \vdots \\
C_0 &= (u_0)^5 \\
C_1 &= 5(u_0)^5 u_1 \\
&\ \vdots
\end{aligned}$$

From the relationship in (15), we obtain

$$u_0(x,t) = e^x + te^x$$

$$\begin{aligned} u_1(x,t) &= S^{-1}\left[v^2 S\left[A_0(u) - B_0(u) - 18C_0(u) + u_0(x,t)\right]\right] \\ &= \frac{t^2e^x}{2} + \frac{t^3e^x}{6} \\ u_2(x,t) &= S^{-1}\left[v^2 S\left[A_1(u) - B_1(u) - 18C_1(u) + u_1(x,t)\right]\right] \\ &= \frac{t^4e^x}{24} + \frac{t^5e^x}{120} \\ &\vdots \end{aligned}$$

Which in closed form gives exact solution

$$u(x,\ t) = \sum_{n=0}^{\infty} u_n(x,t) = e^x\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} + \ldots\right) \tag{30}$$

Thus,

$$u(x,t) = e^{x+t} \tag{31}$$

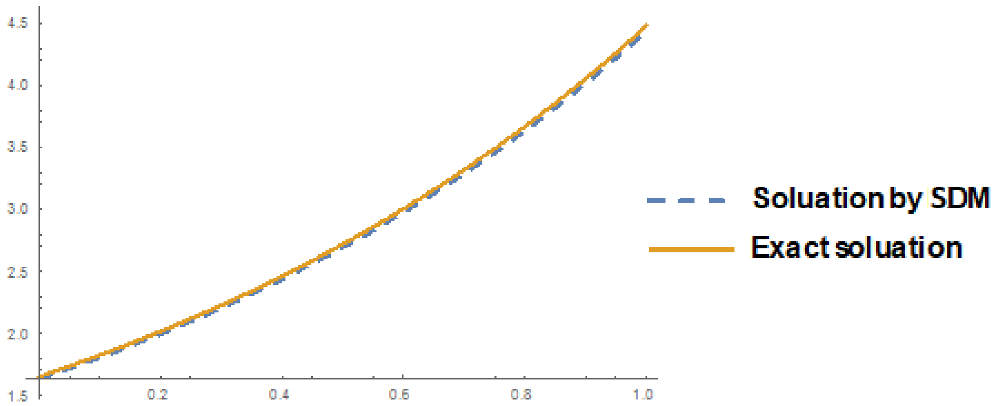

Figure 2. **Example 4.2 Solution by SDM and exact Solution**

As shown by figure 2, comparison of the obtained results with those of the exact solution, reveals that SADM leads to accurate solutions for Example 4.2.

Example 4.3. *Let's consider the nonlinear partial differential equation*

$$u_{tt} = x^2\frac{\partial}{\partial x}(u_x u_{xx}) - x^2\left(u_{xx}^2\right) - u, \quad 0 \le x \le 1, \quad t > 0 \tag{32}$$

With the initial condition;

$$u(x,0) = 0, \quad u_t(x,0) = x^2 \tag{33}$$

Applying Sawi transform of both sides of Equation (32), we obtain,

$$S\left[u_{tt}(x,t)\right] = S\left[x^2\frac{\partial}{\partial x}(u_x u_{xx}) - x^2\left(u_{xx}^2\right) - u\right] \tag{34}$$

Using the differential property of Sawi transform . Eq. (34) can be written as:

$$S\left[u(x,t)\right] = x^2 + v^2 S\left[x^2\frac{\partial}{\partial x}(u_x u_{xx}) - x^2\left(u_{xx}^2\right) - u\right] \tag{35}$$

The inverse of Sawi transform implies that:

$$u(x,t) = tx^2 + S^{-1}\left[v^2 S\left[x^2 \frac{\partial}{\partial x}(u_x u_{xx}) - x^2\left(u_{xx}^2\right) - u\right]\right] \tag{36}$$

Following the technique, if we assume an infinite series solution of the form Equations (11) and (12), we obtain

$$\sum_{n=0}^{\infty} u_n\,(x,\ t) = tx^2 + S^{-1}\left[v^2 S\left[x^2 \frac{\partial}{\partial x}\sum_{n=0}^{\infty} A_n\,(u) - x^2 \sum_{n=0}^{\infty} B_n\,(u) - \sum_{n=0}^{\infty} u_n(x,t)\right]\right] \tag{37}$$

Where $A_n\,(u)$ and $B_n\,(u)$ are Adomian polynomials that represent nonlinear term. So Adomian polynomials are given as follows:

$$A_n\,(u) = (u_x u_{xx}), \quad B_n\,(u) = \left(u_{xx}^2\right)$$

Using (13). The few components of the Adomian polynomials are given as follows:

$$\begin{aligned}
&A_0 = (u_{0_x})(u_{0xx})\\
&A_1 = (u_{0_x})\,(u_{1xx}) + (u_{1_x})(u_{0xx})\\
&A_2 = (u_{0_x})\,(u_{2xx}) + (u_{1_x})\,(u_{1xx}) + (u_{2_x})(u_{0xx})\\
&\quad\vdots\\
&B_0 = (u_0)_{xx}^2\\
&B_1 = 2(u_0)_{xx}(u_1)_{xx}\\
&B_2 = (u_1)_{xx}^2 + 2(u_0)_{xx}(u_2)_{xx}\\
&\quad\vdots
\end{aligned}$$

From the relationship in (15), we obtain

$$\begin{aligned}
&u_0\,(x,t) = tx^2\\
&u_1\,(x,t) = S^{-1}\left[v^2 S\left[\ x^2 \frac{\partial}{\partial x} A_0\,(u) - x^2 B_0\,(u) - u_0(x,t)\right]\right] = -\frac{t^3 x^2}{6}\\
&u_2\,(x,t) = S^{-1}\left[v^2 S\left[\ x^2 \frac{\partial}{\partial x} A_1\,(u) - x^2 B_1\,(u) - u_1(x,t)\right]\right] = \frac{t^5 x^2}{120}\\
&u_3\,(x,t) = S^{-1}\left[v^2 S\left[\ x^2 \frac{\partial}{\partial x} A_2\,(u) - x^2 B_2\,(u) - u_2(x,t)\right]\right] = -\frac{t^7 x^2}{5040}\\
&\quad\vdots
\end{aligned}$$

Which in closed form gives exact solution

$$u\,(x,t) = \sum_{n=0}^{\infty} u_n\,(x,t) = x^2\left(1 - \frac{t^3}{3!} - \frac{t^5}{5!} - \frac{t^7}{7!} - \ldots\right) \tag{38}$$

Thus,

$$u\,(x,t) = x^2 \sin t \tag{39}$$

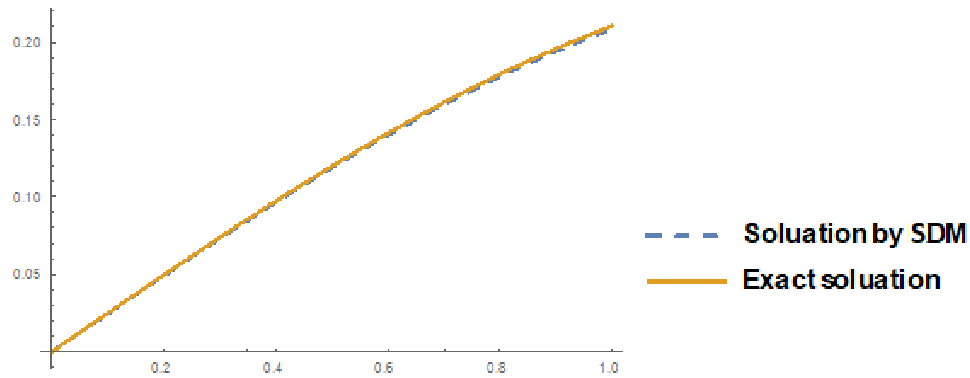

Figure 3. **Example 4.3 Solution by SDM and exact solution**

As shown by figure 3, comparison of the obtained results with those of the exact solution, reveals that SADM leads to accurate solutions for Example 4.3.

5. Conclusion

In the present paper, the coupling, Sawi decomposition method would give exact solutions for the nonlinear Wave-Like equation with variable coefficients as the effectiveness of both Sawi transform and Adomian decomposition method.

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