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Fuzzy Ideals and Fuzzy Congruences of a Generalised Lattice

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Abstract: This paper deals with the correspondence between fuzzy ideals, fuzzy filters and fuzzy congruences in a generalised lattice. Proved that lattice of fuzzy ideals (fuzzy filters) of a generalised lattice is isomorphic to the lattice of fuzzy ideals of a corresponding lattice. Finally observed that the lattice of fuzzy ideals of a type of generalised lattice is isomorphic to the lattice of fuzzy congruences of that.

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1. Introduction

Murty and Swamy [5] introduced the concept of a generalised lattice and Kishore [2, 3] developed the theory of generalised lattices that can play an intermediate role between theories of lattices and posets. Kishore [4] introduced the concepts of fuzzy subgeneralised lattice(or fuzzy generalised lattice), fuzzy ideal and fuzzy filter in a ganeralised lattice. This paper deals with the correspondence between fuzzy ideals, fuzzy filters and fuzzy congruences in a generalised lattice. In this paper section 2 contains preliminaries which are taken from the references. In section 3 proved the lattice of fuzzy ideals (fuzzy filters) of a generalised lattice is isomorphic to the lattice of fuzzy ideals of the lattice of lowerbounds (upperbounds) of subsets of the generalised lattice. In section 4 introduced the concept of a fuzzy congruence of a generalised lattice and discussed the correspondence between fuzzy ideals and fuzzy congruences in a generalised lattice. Finally proved that the lattice of fuzzy ideals of a generalized Boolean generalised lattice is isomorphic to the lattice of fuzzy congruences of that.

2. Preliminaries

The concepts of generalised lattice and distributive poset are known from [2, 3, 5]. For any finite subset A of a poset P, define $L(A) = \{x \in P \mid x \leq a \text{ for all } a \in A\}$ and $U(A) = \{x \in P \mid x \geq a \text{ for all } a \in A\}$. Then the sets $\mathcal{L}(P) = \{L(A) \mid A \text{ is a finite subset of } P\}$ and $\mathcal{U}(P) = \{U(A) \mid A \text{ is a finite subset of } P\}$ are semi lattices under set inclusion. If P is a generalised lattice then the posets $(\mathcal{L}(P), \subseteq)$ and $(\mathcal{U}(P), \subseteq)$ are dual isomorphic lattices. A generalised lattice P is distributive if and only if the lattice $\mathcal{L}(P)$ (or $\mathcal{U}(P)$) is distributive. Here onwards through out this paper P denotes a generalised lattice unless specified.

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Definition 2.1 ([4]). *A fuzzy set μ in P is said to be a fuzzy subgeneralised lattice if for any finite subset A of P, (i) $\mu(s) \geq min_{a\in A}\{\mu(a)\}$ for all $s \in mu(A)$ and (ii) $\mu(t) \geq min_{a\in A}\{\mu(a)\}$ for all $t \in ML(A)$.*

Definition 2.2 ([4]). *Let μ be a fuzzy subgeneralised lattice of P. μ is called a fuzzy ideal if $x \leq y$ in P implies $\mu(x) \geq \mu(y)$. μ is called a fuzzy filter if $x \leq y$ in P implies $\mu(x) \leq \mu(y)$.*

Definition 2.3 ([4]). *Let μ be a fuzzy set in P. Then the intersection of all fuzzy ideals of P containing μ is called fuzzy ideal generated by μ and it is denoted by $(\mu]$. The intersection of all fuzzy filters of P containing μ is called fuzzy filter generated by μ and it is denoted by $[\mu)$.*

3. Fuzzy Ideals and Fuzzy Filters of a Generalised Lattice

In this section we observe the correspondence between fuzzy ideals (fuzzy filters) of P and fuzzy ideals of $\mathcal{L}(P)$ ($\mathcal{U}(P)$).

Theorem 3.1. *Let μ be a fuzzy ideal of P. Define a map $\overline{\mu} : \mathcal{L}(P) \to [0, 1]$ by $\overline{\mu}(L(A)) = min\{\mu(r) \mid r \in ML(A)\}$. Then $\overline{\mu}$ is a fuzzy ideal of $\mathcal{L}(P)$.*

Proof. Let $L(A), L(B) \in \mathcal{L}(P)$. (i) Consider $\overline{\mu}(L(A) \vee L(B)) = \overline{\mu}(L(mu(ML(A) \cup ML(B)))) = \overline{\mu}(L(mu(C))) = min\{\mu(s) \mid s \in ML(mu(C))\}$ where $C = ML(A) \cup ML(B)$. Since μ is a fuzzy subgeneralised lattice of P, for any $s \in ML(mu(C))$, we have $\mu(s) \geq min\{\mu(t) \mid t \in mu(C)\} \geq min\{\mu(c) \mid c \in C\}$. Therefore $\overline{\mu}(L(A) \vee L(B)) \geq min\{\mu(c) \mid c \in C\} = min\{\overline{\mu}(L(A)), \overline{\mu}(L(B))\}$. (ii) Suppose $L(A) \subseteq L(B)$. Then for each $a \in ML(A)$, there exists $b \in ML(B)$ such that $\mu(a) \geq \mu(b)$. Therefore $\overline{\mu}(L(A)) = min\{\mu(a) \mid a \in ML(A)\} \geq min\{\mu(b) \mid b \in ML(B)\} = \overline{\mu}(L(B))$. (iii) Since $L(A) \wedge L(B) \subseteq L(A), L(B)$, by (ii) we have $\overline{\mu}(L(A) \wedge L(B)) \geq \overline{\mu}(L(A)), \overline{\mu}(L(B))$. Therefore $\overline{\mu}(L(A) \wedge L(B)) \geq min\{\overline{\mu}(L(A)), \overline{\mu}(L(B))\}$. Hence by (i), (ii) and (iii) we can say that $\overline{\mu}$ is a fuzzy ideal of $\mathcal{L}(P)$. □

By the similar steps of the proof of Theorem 3.1 we can prove the following theorem.

Theorem 3.2. *Let μ be a fuzzy filter of P. Define a map $\overline{\mu} : \mathcal{U}(P) \to [0, 1]$ by $\overline{\mu}(U(A)) = min\{\mu(r) \mid r \in mu(A)\}$. Then $\overline{\mu}$ is a fuzzy ideal of $\mathcal{U}(P)$.*

Note: Let η be a fuzzy ideal of $\mathcal{L}(P)$. Define a map $\eta_P : P \to [0, 1]$ by $\eta_P(x) = \eta(L(x))$. Then η_P is a fuzzy set on P and satisfies the conditions: (i) $x \leq y \Rightarrow \eta_P(x) \geq \eta_P(y)$ (ii) $\eta_P(t) \geq min\{\eta_P(x), \eta_P(y)\}$ for all $t \in ML\{x, y\}$ and (iii) $\eta_P(s) \geq min\{\eta_P(x), \eta_P(y)\}$ for all $s \in ML(mu\{x, y\})$. Consider the fuzzy ideal generated by η_P in P, which is denoted by $(\eta_P]$.

Theorem 3.3. *There is one-one correspondence between fuzzy ideals of P and fuzzy ideals of $\mathcal{L}(P)$.*

Proof. Let μ be a fuzzy ideal of P. By above theorem we have $\overline{\mu} : \mathcal{L}(P) \to [0, 1]$ defined by $\overline{\mu}(L(A)) = min\{\mu(r) \mid r \in ML(A)\}$ is a fuzzy ideal of $\mathcal{L}(P)$. Now consider the fuzzy set $\overline{\mu}_P : P \to [0, 1]$ defined by $\overline{\mu}_P(x) = \overline{\mu}(L(x))$. It is clear that $\overline{\mu}_P = \mu$ and $(\overline{\mu}_P] = (\mu] = \mu$. Therefore the correspondence will be $\mu \to \overline{\mu} \to (\overline{\mu}_P] = \mu$. Conversely suppose η is a fuzzy ideal of $\mathcal{L}(P)$. Consider the fuzzy ideal $\zeta = (\eta_P]$ of P and the fuzzy ideal $\overline{\zeta}$ of $\mathcal{L}(P)$. To show that $\overline{\zeta} = \eta$: Let $L(A) \in \mathcal{L}(P)$, then for any $r \in ML(A)$ we have $\zeta(r) \geq \eta_P(r) = \eta(L(r)) \geq \eta(L(A))$ and this implies $\overline{\zeta}(L(A)) = min\{\zeta(r) \mid r \in ML(A)\} \geq \eta(L(A))$. Therefore $\overline{\zeta} \geq \eta$. On the other hand we have $\eta(L(A)) = min\{\eta(L(a)) \mid a \in ML(A)\} = min\{\eta_P(a) \mid a \in ML(A)\} \leq min\{\zeta(a) \mid a \in ML(A)\} = \overline{\zeta}(L(A))$ and that is $\eta \leq \overline{\zeta}$. Therefore the converse correspondence will be $\eta \to (\eta_P] \to \overline{(\eta_P]} = \eta$. □

By the similar steps of the proof of theorem 3.3 we can prove the following theorem.

Theorem 3.4. *There is one-one correspondence between fuzzy filters of P and fuzzy ideals of $\mathcal{U}(P)$.*

Notations 3.5. *The set of all fuzzy ideals of P is denoted by $F\mathcal{I}(P)$ and the set of all fuzzy filters of P is denoted by $F\mathcal{F}(P)$. The set of all fuzzy ideals of $\mathcal{L}(P)$ is denoted by $F\mathcal{I}(\mathcal{L}(P))$ and the set of all fuzzy ideals of $\mathcal{U}(P)$ is denoted by $F\mathcal{I}(\mathcal{U}(P))$.*

In [4] we have intersection of any family of fuzzy ideals of a generalised lattice is again a fuzzy ideal. Therefore the poset $(F\mathcal{I}(P), \subseteq)$ is a complete lattice and for any $\mu_1, \mu_2 \in F\mathcal{I}(P)$, we have $\mu_1 \wedge \mu_2 = \mu_1 \cap \mu_2$ and $\mu_1 \vee \mu_2 = \bigcap\{\mu \in F\mathcal{I}(P) \mid \mu_1 \cup \mu_2 \subseteq \mu\}$.

Theorem 3.6. *(i) $F\mathcal{I}(P) \cong F\mathcal{I}(\mathcal{L}(P))$ and (ii) $F\mathcal{F}(P) \cong F\mathcal{I}(\mathcal{U}(P))$.*

Proof. (i) Define a map $\psi : F\mathcal{I}(P) \to F\mathcal{I}(\mathcal{L}(P))$ by $\psi(\mu) = \overline{\mu}$ where $\overline{\mu} : \mathcal{L}(P) \to [0, 1]$ defined by $\overline{\mu}(L(A)) = min\{\mu(r) \mid r \in ML(A)\}$. By theorem 3.3 we have ψ is a bijection. Now let $\mu_1, \mu_2 \in F\mathcal{I}(P)$ and suppose $\mu_1 \leq \mu_2$, then since $\mu_1(r) \leq \mu_2(r)$ for all $r \in ML(A)$ we get $\mu_1(L(A)) \leq \mu_2(L(A))$ for all $L(A) \in \mathcal{L}(P)$ and that is $\psi(\mu_1) \leq \psi(\mu_2)$. Therefore ψ is order preserving. Conversely suppose $\psi(\mu_1) \leq \psi(\mu_2)$, then we have $\mu_1 = (\overline{\mu}_1)_P \leq (\overline{\mu}_2)_P = \mu_2$. Therefore ψ is an order isomorphism of $F\mathcal{I}(P)$ onto $F\mathcal{I}\mathcal{L}(P)$. Similarly we can prove (ii). □

4. Fuzzy Congruences and Fuzzy Ideals of a Generalised Lattice

In this section we observe the correpondence between fuzzy congruences and fuzzy ideals of a generalised lattice.

Definition 4.1. *Let P be a poset and θ be a fuzzy subset of $P \times P$. Then θ is said to be a fuzzy equivalence relation on P if for any $x, y, z \in P$ we have (i) $\theta(x, x) = Sup\{\theta(y, z) \mid y, z \in P\}$ (ii) $\theta(x, y) = \theta(y, x)$ and (iii) $min\{\theta(x, z), \theta(z, y)\} \leq \theta(x, y)$.*

Definition 4.2. *Let P be a poset and θ be a fuzzy equivalence relation on P. Then θ is said to be a fuzzy meet congruence on P if there exists a fuzzy meet congruence $\overline{\theta}$ of $\mathcal{L}(P)$ such that $\theta = \overline{\theta} \cap (P \times P)$. θ is said to be a fuzzy join congruence on P if there exists a fuzzy meet congruence $\overline{\theta}$ of $\mathcal{U}(P)$ such that $\theta = \overline{\theta} \cap (P \times P)$. θ is said to be a fuzzy congruence on P if θ is a fuzzy meet congruence as well as fuzzy join congruence on P.*

Observe that the intersection of any family of fuzzy congruences of a poset is again a fuzzy congruence. The set of all fuzzy congruences of a poset P is denoted by $F\mathcal{C}(P)$. Then the poset $(F\mathcal{C}(P), \subseteq)$ is a complete lattice and for any $\theta_1, \theta_2 \in F\mathcal{C}(P)$, we have $\theta_1 \wedge \theta_2 = \theta_1 \cap \theta_2$ and $\theta_1 \vee \theta_2 = \bigcap\{\theta \in F\mathcal{C}(P) \mid \theta_1 \cup \theta_2 \subseteq \theta\}$.

Definition 4.3. *Let P be a generalised lattice and θ be a fuzzy equivalence relation on P. Then θ is said to be a fuzzy strong congruence on P if for all $x_1, x_2, y_1, y_2 \in P$ we have (iv) $\theta(s, t) \geq min\{\theta(x_1, y_1), \theta(x_2, y_2)\}$ for all $s \in mu\{x_1, x_2\}$ and $t \in mu\{y_1, y_2\}$ and (v) $\theta(s, t) \geq min\{\theta(x_1, y_1), \theta(x_2, y_2)\}$ for all $s \in ML\{x_1, x_2\}$ and $t \in ML\{y_1, y_2\}$.*

Consider the following conditions: (iv)$'$: $\theta(s, t) \geq min\{\theta(a, b) \mid a \in A, b \in B\}$ for all $s \in mu(A), t \in mu(B)$ and (v)$'$: $\theta(s, t) \geq min\{\theta(a, b) \mid a \in A, b \in B\}$ for all $s \in ML(A), t \in ML(B)$; for any finite subsets A, B of P.
Observe that (iv) $\Leftrightarrow$ (iv)$'$ and (v) $\Leftrightarrow$ (v)$'$.

Definition 4.4. *Let P be a generalised lattice and μ be a fuzzy ideal of P. Consider the corresponding fuzzy ideal $\overline{\mu}$ of $\mathcal{L}(P)$. Define a fuzzy relation $C(\mu)$ of P by $C(\mu)(x, y) = Sup\{\overline{\mu}(L(A)) \mid L(A) \vee L(x) = L(A) \vee L(y)$ for some finite subset A of $P\}$.*

Notations 4.5. *A is finite subset of P, is denoted by $A \lessdot P$.*

Lemma 4.6. *Let P be a distributive generalised lattice and μ be a fuzzy ideal of P. Then $C(\mu)$ is a fuzzy congruence on P.*

Proof. Let $\theta = C(\mu)$. To show that θ is a fuzzy equivalence relation on P : Let $x, y, z \in P$. (i) $\theta(x,x) = Sup\{\overline{\mu}(L(A)) \mid L(A) \vee L(x) = L(A) \vee L(y)$ for some $A \lessdot P\} = Sup\{\overline{\mu}(L(A)) \mid A \lessdot P\}$ and $\theta(y,z) = Sup\{\overline{\mu}(L(B)) \mid L(B) \vee L(y) = L(B) \vee L(z)$ for some $B \lessdot P\} \leq Sup\{\overline{\mu}(L(B)) \mid B \lessdot P\} = \theta(x,x)$. Therefore $\theta(x,x) = Sup\{\theta(y,z) \mid y, z \in P\}$. (ii) Clearly $\theta(x,y) = \theta(y,x)$. (iii) Suppose $L(A) \vee L(x) = L(A) \vee L(z), L(B) \vee L(z) = L(B) \vee L(y)$ and let $L(C) = L(A) \vee L(B)$. Then clearly $L(C) \vee L(x) = L(C) \vee L(y)$. Since $\overline{\mu}$ is a fuzzy ideal of $\mathcal{L}(P)$ we have $min\{\theta(x,z), \theta(z,y)\} = min\{Sup\{\overline{\mu}(L(A)) \mid L(A) \vee L(x) = L(A) \vee L(z)$ for some $A \lessdot P\}, Sup\{\overline{\mu}(L(B)) \mid L(B) \vee L(z) = L(B) \vee L(y)$ for some $B \lessdot P\}\} = Sup\{\overline{\mu}(L(A) \vee L(B)) \mid L(A) \vee L(x) = L(A) \vee L(z), L(B) \vee L(z) = L(B) \vee L(y)$ for some $A, B \lessdot P\} \leq Sup\{\overline{\mu}(L(C)) \mid L(C) \vee L(x) = L(C) \vee L(y)$ for some $C \lessdot P\} = \theta(x,y)$. Therefore θ is a fuzzy equivalence relation on P. Since $\mathcal{L}(P)$ is a distributive lattice, by lemma3.1[6], the fuzzy relation $\overline{\theta}$ on $\mathcal{L}(P)$ defined by $\overline{\theta}(L(X), L(Y)) = Sup\{\overline{\mu}(L(A)) \mid L(A) \vee L(X) = L(A) \vee L(Y)$ for some $A \lessdot P\}$ is a fuzzy congruence on $\mathcal{L}(P)$. Then $\overline{\theta} \cap (P \times P) = C(\mu)$ and $C(\mu)$ is a fuzzy congruence on P. □

Lemma 4.7. *Let P be a distributive generalised lattice and μ be a fuzzy ideal of P. Then $C(\mu)$ is a fuzzy strong congruence on P.*

Proof. By the Lemma 4.6, $\theta = C(\mu)$ is a fuzzy equivalence relation and fuzzy congruence on P. Then there exists a fuzzy congruence $\overline{\theta}$ on $\mathcal{L}(P)$ such that $\theta = \overline{\theta} \cap (P \times P)$. To show that θ is a fuzzy strong congruence on P : (iv) Let $x_1, x_2, y_1, y_2 \in P$. Consider $min\{\theta(x_1,y_1), \theta(x_2,y_2)\} = min\{\overline{\theta}(L(x_1), L(y_1)), \overline{\theta}(L(x_2), L(y_2))\} \leq \overline{\theta}(L(x_1) \vee L(x_2), L(y_1) \vee L(y_2)) = \overline{\theta}(L(mu\{x_1,x_2\}), L(mu\{y_1,y_2\}))$. For any $s \in mu\{x_1,x_2\}, t \in mu\{y_1,y_2\}$ we have $\overline{\theta}(L(s), L(t)) \geq min\{\overline{\theta}(L(mu\{x_1,x_2\}), L(mu\{y_1,y_2\}), \overline{\theta}(L(s), L(t))\}$. Therefore $\theta(s,t) \geq min\{\theta(x_1,y_1), \theta(x_2,y_2)\}$. Similarly we can prove (v). □

Definition 4.8. *Let P be a generalised lattice and θ be a fuzzy congruence on P. Then there exists a fuzzy congruence $\overline{\theta}$ on $\mathcal{L}(P)$ such that $\theta = \overline{\theta} \cap (\times P)$. Define the fuzzy subset $I(\theta) : P \to [0,1]$ of P by $I(\theta)(x) = inf\{\overline{\theta}(L(x) \wedge L(Y), L(x)) \mid Y \lessdot P\}$.*

Lemma 4.9. *Let P be a distributive generalised lattice and θ is a fuzzy congruence on P. Then $I(\theta)$ is a fuzzy ideal of P.*

Proof. Let $\mu = I(\theta)$. To show that μ is a fuzzy ideal of P : (i) Let $x, y \in P$, $s \in mu\{x,y\}$ and Z be a finite subset of P. Since $\mathcal{L}(P)$ is a distributive lattice we have $\overline{\theta}(L(s) \wedge L(Z), L(s)) = \overline{\theta}(((L(x) \vee L(y)) \wedge L(Z)) \vee (L(s) \wedge L(Z)), L(s) \vee L(s)) \geq min\{\overline{\theta}((L(x) \wedge L(Z)) \vee (L(y) \wedge L(Z)), L(s)), \overline{\theta}((L(s) \wedge L(Z)), L(s))\} \geq \overline{\theta}((L(x) \wedge L(Z)) \vee (L(y) \wedge L(Z)), L(x) \vee L(s)) \geq min\{\overline{\theta}(L(x) \wedge L(Z), L(x)), \overline{\theta}(L(y) \wedge L(Z), L(s))\} \geq \overline{\theta}(L(x) \wedge L(Z), L(x))$. Similarly $\overline{\theta}(L(s) \wedge L(Z), L(s)) \geq \overline{\theta}(L(y) \wedge L(Z), L(y))$. Therefore $\mu(s) \geq min\{\mu(x), \mu(y)\}$. (ii) Let $x, y \in P$ and $x \leq y$. Then $\mu(x) = inf\{\overline{\theta}(L(x) \wedge L(Z), L(x)) \mid Z \lessdot P\} \geq inf\{min\{\overline{\theta}(L(x), L(x)), \overline{\theta}(L(y) \wedge L(Z), L(y))\} \mid Z \lessdot P\} = \mu(y)$. (iii) Let $x, y \in P$ and $t \in ML\{x,y\}$. Then by (ii) we have $\mu(t) \geq min\{\mu(x), \mu(y)\}$. □

Theorem 4.10. *Let P be a distributive generalised lattice with 0. If μ is a fuzzy ideal of P then $\mu = I(C(\mu))$.*

Proof. Let $\theta = C(\mu)$. Then by Lemmas 4.5 and 4.6, θ is a fuzzy congruence and later by lemma 4.8 $I(\theta)$ is a fuzzy ideal of P. To show that $\mu = I(\theta)$: For any $x \in P$ and any finite subset Y of P we have $\overline{\theta}(L(x) \wedge L(Y), L(x)) = Sup\{\overline{\mu}(L(A)) \mid L(A) \vee (L(x) \wedge L(Y)) = L(A) \vee L(x)$ for some $A \lessdot P\} \geq \overline{\mu}(L(x)) = \mu(x)$. Then $I(\theta)(x) \geq \mu(x)$ for all $x \in P$, that is $I(\theta) \geq \mu$. Conversely for any $x \in P$ we have $I(\theta)(x) \leq \overline{\theta}(L(x) \wedge L(0), L(x)) = Sup\{\overline{\mu}(L(A)) \mid L(A) = L(A) \vee L(x)$ for some $A \lessdot P\} = \overline{\mu}(L(x)) = \mu(x)$. Therefore $I(\theta) \leq \mu$ and hence $I(\theta) = \mu$. □

Lemma 4.11. *Let P be a generalised lattice, μ_1, μ_2 be a fuzzy ideals and θ_1, θ_2 be fuzzy congruences of P. Then (i) $\mu_1 \subseteq \mu_2 \Rightarrow C(\mu_1) \subseteq C(\mu_2)$ (ii) $\theta_1 \subseteq \theta_2 \Rightarrow I(\theta_1) \subseteq I(\theta_2)$.*

5. Fuzzy Congruences and Fuzzy Ideals of a Generalized Boolean Generalised Lattice (gBgl)

Recall that a complemented distributive lattice is called a Boolean algebra and a relatively complemented distributive lattice bounded below is called a generalized Boolean algebra.

Definition 5.1. *A complemented distributive generalised lattice is called a Boolean generalised lattice.*

Definition 5.2. *A generalised lattice P is called a generalized Boolean generalised lattice (gBgl) if $\mathcal{L}(P)$ is a generalized Boolean algebra.*

Theorem 5.3. *Let P be a gBgl and θ be a fuzzy congruence on P. Then $C(I(\theta)) = \theta$.*

Proof. There exists a fuzzy congruence $\overline{\theta}$ on $\mathcal{L}(P)$ such that $\theta = \overline{\theta} \cap (P \times P)$. Since $\mathcal{L}(P)$ is a generalized Boolean algebra, by theorem 3.2[6] we have $C(I(\overline{\theta})) = \overline{\theta}$. Let $\eta = I(\overline{\theta})$ and $\mu = I(\theta)$. Then $\mu = (\eta_P]$ and $\overline{\mu} = \eta$. Therefore $\theta = \overline{\theta} \cap (P \times P) = C(I(\overline{\theta})) \cap (P \times P) = C(\eta) \cap (P \times P) = C(\overline{\mu}) \cap (P \times P) = C(\mu) = C(I(\theta))$. □

Theorem 5.4. *Let P be a gBgl. Then $(F\mathcal{I}(P), \wedge, \vee) \cong (F\mathcal{C}(P), \wedge, \vee)$.*

Proof. Define $f : F\mathcal{C}(P) \to F\mathcal{I}(P)$ by $f(\theta) = I(\theta)$. To show that f is onto: Let $\mu \in F\mathcal{I}(P)$. Then by Lemma 4.6, $C(\mu) \in F\mathcal{C}(P)$ such that $f(C(\mu)) = I(C(\mu)) = \mu$. To show that f is one-one: Let $\theta_1, \theta_2 \in F\mathcal{C}(P)$ and suppose $f(\theta_1) = f(\theta_2)$. Then by theorem5.3 $\theta_1 = C(I(\theta_1)) = C(I(\theta_2)) = \theta_2$. To show that f is $\wedge-$homomorphism: Let $\theta_1, \theta_2 \in F\mathcal{C}(P)$ and $\theta = \theta_1 \cap \theta_2$. Let $I(\overline{\theta_1}) = \eta_1, I(\overline{\theta_2}) = \eta_2$ and $I(\overline{\theta}) = \eta$. Since $\mathcal{L}(P)$ is a generalized Boolean algebra, we have $\eta = \eta_1 \cap \eta_2$. Then $\eta_P = \eta_{1_P} \cap \eta_{2_P}$ and $I(\theta) = (\eta_P] = (\eta_{1_P}] \cap (\eta_{2_P}] = I(\theta_1) \cap I(\theta_2)$. Therefore $f(\theta_1 \wedge \theta_2) = f(\theta_1) \wedge f(\theta_2)$. To show that f is $\vee-$homomorphism: Let $\theta_1, \theta_2 \in F\mathcal{C}(P)$. Then by lemma4.10 $I(\theta_1 \vee \theta_2)$ is an upper bound of $\{I(\theta_1), I(\theta_2)\}$. Let μ be any upper bound of $\{I(\theta_1), I(\theta_2)\}$. Then $\theta_1 = C(I(\theta_1)) \subseteq C(\mu),\ \theta_2 = C(I(\theta_2)) \subseteq C(\mu)$ and $I(\theta_1 \vee \theta_2) \subseteq I(C(\mu)) = \mu$. Therefore $f(\theta_1 \vee \theta_2) = I(\theta_1 \vee \theta_2) = I(\theta_1) \vee I(\theta_2) = f(\theta_1) \vee f(\theta_2)$. □

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References

[1] N. Ajmal and K. V. Thomas, *Fuzzy lattices*, Information sciences, 79(1994), 271-291.

[2] P. R. K. Kishore, M. K. Murty, V. S. Ramani and M. D. P. Patnaik, *On generalised lattices*, Southeast Asian Bulletin of Mathematics, 33(2009), 1091-1104.

[3] P. R. K. Kishore, *Distributive generalised lattices*, International Journal of Computational Cognition, 7(3)(2009), 23-26.

[4] P. R. K. Kishore, *Fuzzy generalised lattices*, International Journal of Mathematical Sciences & Applications, 9(3)(2015), 1-9.

[5] M. K. Murty and U. M. Swamy, *Distributive partially ordered sets*, The Aligarh Bull. of Math., 8(1978), 1-16.

[6] Bo. Yuan and W. Wu, *Fuzzy ideals on a distributive lattice*, Fuzzy Sets and Systems, 35(1990), 231-240.