Int. J. Math. And Appl., 8(3)(2020), 217–223
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

Regularity of Intuitionistic Fuzzy Soft Hypergraphs

RD Beulah[1,*] and T. Saradhadevi[1]

1 Department of Mathematics, Sri GVG Visalakshi College For Women, Udumalpet, Tamilnadu, India.

Abstract: An Intuitionistic Fuzzy Soft Set(IFSS) is an extension of fuzzy soft set to deal with vague information corresponding to their different parameters. The IFSS is the most efficient tool to deal with uncertain information than fuzzy soft set. Hypergraphs are used to represent almost any complex situation involving objects and a relationship among them. The concept of IFSS is applied to hypergraphs and presented the notion of Intuitionistic Fuzzy Soft Hypergraphs(IFSHGs). Further, we defined regular, totally regular and perfectly regular IFSHGs with their properties and also illustrated with its examples.

MSC: 05C72, 05C65

Keywords: Intuitionistic Fuzzy Soft Hypergraphs, Regular, Totally Regular, Perfectly Regular.

1. Introduction

The concept of soft set theory for dealing with uncertainty from the viewpoint of parametrization was initiated by Molodtsov [7] in 1999. Maji [4] et al. introduced concept of soft set theory and extended to fuzzy soft set. The noble concept of Intuitionistic fuzzy set was introduced by Atanassov [1] in 1999. After that Maji [5] et al proposed intuitionistic fuzzy soft set as an extension fuzzy soft set. The idea of graph theory was introduced by Euler. The idea of graphs is generalized to a hypergraphs, that is, a set V of vertices together with a collection of subsets of V. In 1976, Berge [2] introduced the concept of fuzzy hypergraphs. In 2009, Nagoorgani [8] introduced regular fuzzy graphs. Parvathi [9, 10] et al. introduced the concept of intuitionistic fuzzy graphs, intuitionistic fuzzy hypergraphs. Later in 2018, regular and totally regular intuitionistic fuzzy hypergraphs were proposed by Pradeepa [11]. In 2014, Thumbakara [13] and George discussed the concept of soft graphs in the specific way. In 2015, The concept of fuzzy soft graphs was introduced by Mohinta and samanta [6]. Then intuitionistic fuzzy soft graphs were developed by several authors [3, 12]. Recently, intuitionistic fuzzy soft hypergraphs(IFSHGs) was introduced in 2018 by Thilagavathi [14]. In this paper, the concepts like Regular, Totally Regular, Perfectly Regular, Uniform IFSHGs are illustrated with examples. Also proved that an IFSHG is regular and totally regular IFSHGs if $(\mathfrak{T}_\mu, \mathfrak{T}_\nu)$ is a constant function.

Notation list

- U be the universe set and Φ be the set of all parameters.
- $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ is an IFSHGs.

* *E-mail: beulahrd2019@yahoo.com*

- $\langle \mathfrak{N}_\mu, \mathfrak{N}_\nu \rangle$ or simply $\langle \mu_i, \nu_i \rangle$ denotes the degrees of membership and nonmembership of the vertex $v_i \in V$, such that $0 \leq \mathfrak{N}_\mu + \mathfrak{N}_\nu \leq 1$.
- $\langle \mathfrak{S}_\mu, \mathfrak{S}_\nu \rangle$ or simply $\langle \mu_{ij}, \nu_{ij} \rangle$ denotes the degrees of membership and nonmembership of the hyperedge $v_i, v_j \in V \times V$, such that $0 \leq \mathfrak{S}_\mu + \mathfrak{S}_\nu \leq 1$.
- $P(V \times V)$ is an intuitionistic fuzzy power set.
- $P(V)$ and $P(E)$ be the set of all intuitionistic fuzzy soft set over vertices V and hyperedges E respectively.
- The support of an intutionistic fuzzy soft set V in $\mathfrak{S}$ is denoted by $supp\mathfrak{E}_j(\phi_i) = \{v_i/\mathfrak{S}_\mu(\phi_i) > 0$ and $\mathfrak{S}_\nu(\phi_i) > 0, \phi_i \in \Phi\}$.
- $\big(\mathfrak{T}_\mu : \mathfrak{N} \to [0,1], \mathfrak{T}_\nu : \mathfrak{N} \to [0,1]\big)$ is a constant function

Throughout this paper, these notations are used.

1.1. Preliminaries

The basic definitions relating to intuitionistic fuzzy set, intuitionistic fuzzy soft set, intuitionistic fuzzy hypergraphs and intuitionistic fuzzy soft hypergraphs are dealt in this section.

Definition 1.1 ([1]). *Let a set E be fixed. An intuitionistic fuzzy set (IFS) V in E is an object of the form $V = \{\langle v_i, \mu_i(v_i), \nu_i(v_i)\rangle / v_i \in E\}$, where the function $\mu_i : E \to [0,1]$ and $\nu_i : E \to [0,1]$ determine the degree of membership and the degree of non-membership of the element $v_i \in E$, respectively and for every $\nu_i \in E$, $0 \leq \mu_i(v_i) + \nu_i(v_i) \leq 1$.*

Definition 1.2 ([9]). *Let E be the fixed set and $V = \{\langle v_i, \mu_i(v_i), \nu_i(v_i)\rangle \,|v_i \in V\}$ be an IFS. Six types of Cartesian products of n subsets (crisp sets) $V_1, V_2, \cdots, V_n$ of V over E are defined as follows,*

$$V_{i_1} \times_1 V_{i_2} \times_1 V_{i_3} \cdots \times_1 V_{i_n} = \left\{\left\langle (v_1, v_2, \cdots, v_n), \prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \nu_i \right\rangle |v_1 \in V_1, \cdots, v_n \in V_n\right\},$$

$$V_{i_1} \times_2 V_{i_2} \times_2 V_{i_3} \cdots \times_2 V_{i_n} = \left\{\left\langle (v_1, v_2, \cdots, v_n), \sum_{i=1}^{n} \mu_i - \sum_{i\neq j} \mu_i\mu_j + \sum_{i\neq j\neq k} \mu_i\mu_j\mu_k - \cdots + (-1)^{n-2} \times \sum_{i\neq j\neq k\cdots\neq n} \mu_i\mu_j\mu_k\cdots\mu_n + (-1)^{n-1}\prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \nu_i \right\rangle |v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\right\},$$

$$V_{i_1} \times_3 V_{i_2} \times_3 V_{i_3} \cdots \times_3 V_{i_n} = \left\{\left\langle (v_1, v_2, \cdots, v_n), \prod_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \nu_i - \sum_{i\neq j} \nu_i\nu_j + \sum_{i\neq j\neq k} \nu_i\nu_j\nu_k - \cdots + (-1)^{n-2} \sum_{i\neq j\neq k\cdots\neq n} \nu_i\nu_j\nu_k\cdots\nu_n + (-1)^{n-1}\prod_{i=1}^{n} \nu_i \right\rangle |v_1 \in V_1, \cdots, v_n \in V_n\right\},$$

$$V_{i_1} \times_4 V_{i_2} \times_4 V_{i_3} \cdots \times_4 V_{i_n} = \left\{\left\langle (v_1, v_2, \cdots, v_n), min(\mu_1, \mu_2, \cdots, \mu_n), max(\nu_1, \nu_2, \cdots, \nu_n) \right\rangle |v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\right\},$$

$$V_{i_1} \times_5 V_{i_2} \times_5 V_{i_3} \cdots \times_5 V_{i_n} = \left\{\left\langle (v_1, v_2, \cdots, v_n), max(\mu_1, \mu_2, \cdots, \mu_n), min(\nu_1, \nu_2, \cdots, \nu_n) \right\rangle |v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\right\},$$

$$V_{i_1} \times_6 V_{i_2} \times_6 V_{i_3} \cdots \times_6 V_{i_n} = \left\{\left\langle (v_1, v_2, \cdots, v_n), \frac{\sum_{i=1}^{n} \mu_i}{n}, \frac{\sum_{i=1}^{n} \nu_i}{n} \right\rangle |v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\right\}$$

It must be noted that $v_i \times_s v_j$ is an IFS, where $s = 1,2,3,4,5,6$.

Definition 1.3 ([5]). *If $M \subseteq \Phi$ and $\mathcal{IF}^U$ denotes the set of all intuitionistic fuzzy sets of U. A pair (F, M) is called an intuitionistic fuzzy soft set over U, where intutionistic fuzzy approximation function is given by $F = (F_\mu, F_\nu) : M \to \mathcal{IF}^U$.*

Definition 1.4 ([3, 10]). *An intuitionistic fuzzy soft graph (IFSG) on a nonempty set V is an ordered 3-tuple $G = (F, K, \Phi)$ such that*

- *(F, Φ) is an intuitionistic fuzzy soft set over V.*
- *(K, Φ) is an intuitionistic fuzzy relation on V. That is $K : \Phi \to P(V \times V)$.*

- $(F(\phi), K(\phi))$ *is an intuitionistic fuzzy soft subgraph, for all* $\phi \in \Phi$.

That is,

1. $K_\mu(\phi)(uv) \leq min\{F_\mu(\phi)(u), K_\mu(\phi)(v)\}$
2. $K_\nu(\phi)(uv) \leq max\{F_\nu(\phi)(u), K_\nu(\phi)(v)\}$,

such that $0 \leq K_\mu(\phi)(uv) + K_\nu(\phi)(uv) \leq 1$, *for every* $\phi \in \Phi$ *and* $u,\ v \in V$.

Note: The fifth cartesian product has been used throughout this paper,
$V_{i_1} \times_5 V_{i_2} \times_5 V_{i_3} \cdots \times_5 V_{i_n} = \{\langle (v_1, v_2, \cdots, v_n), max(\mu_1, \mu_2, \cdots, \mu_n), min(\nu_1, \nu_2, \cdots, \nu_n)\rangle | v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\}$.

Definition 1.5. *An intuitionistic fuzzy soft hypergraphs(IFSHGs)* $\tilde{H} = (H^*, \mathfrak{N}, \mathfrak{S}, \Phi)$ *is an ordered 4-tuple, such that*

- $H^* = \langle V, E\rangle$ *is a intutionistic fuzzy hypergraph.*
- $(\mathfrak{N}, \Phi)$ *is an intuitionistic fuzzy soft set over* V.
- $(\mathfrak{S}, \Phi)$ *is an intuitionistic fuzzy relation on* V. *That is* $\mathfrak{S} : R \to P(V \times V)$.
- $(\mathfrak{N}(\phi), \mathfrak{S}(\phi))$ *is an intuitionistic fuzzy soft subhypergraph, for all* $\phi \in \Phi$.

That is,

1. $\mathfrak{S}_\mu(\phi)(x_1, ..., x_n) \leq max\{\mathfrak{N}_\mu(\phi)(x_1), \mathfrak{N}_\mu(\phi)(x_2), ..., \mathfrak{N}_\mu(\phi)(x_n)\}$
2. $\mathfrak{S}_\nu(\phi)(x_1, ..., x_n) \leq min\{\mathfrak{N}_\nu(\phi)(x_1), \mathfrak{N}_\nu(\phi)(x_2), ..., \mathfrak{N}_\nu(\phi)(x_n)\}$,

such that $0 < \mathfrak{S}_\mu(\phi)(x_1, ..., x_n) + \mathfrak{S}_\nu(\phi)(x_1, ..., x_n) \leq 1$, *for all* $\phi \in \Phi$ *and* $x_1, ..., x_n \in V$. *Where,* $\mathfrak{S}_\mu(\phi)(x_1, ..., x_n)$ *denotes the degree of membership and* $\mathfrak{S}_\nu(\phi)(x_1, ..., x_n)$ *denotes the degree of non-membership of vertex to intuitionistic fuzzy soft hyperedge* $\mathfrak{T}_j$.
Intuitionistic fuzzy soft hypergraph is denoted by $\tilde{H} = (\mathfrak{N}(\phi), \mathfrak{S}(\phi))$ *or* $\tilde{H} = \left\{\tilde{H}(\phi_1), \tilde{H}(\phi_2), ..., \tilde{H}(\phi_n)\right\}$. *In other words, an intuitionistic fuzzy soft hypergraphs is a parameterized family of intuitionistic fuzzy hypergraphs.*

Example 1.6. *Consider an IFSHG* $\tilde{H} = (H^*, \mathfrak{N}, \mathfrak{S}, \Phi)$, *such that* $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ *and* $E = \{S_1, S_2, S_3, S_4, S_5\}$. *Let* $\Phi = \{\phi_1, \phi_2\}$ *be a parameter set. Let* $(\mathfrak{N}, \Phi)$ *be an intuitionistic fuzzy soft set over* V *with its approximate function* $\mathfrak{N} : \Phi \to P(V)$. *Let* $(\mathfrak{S}, \Phi)$ *be an intuitionistic fuzzy soft set over* E *with its approximate function* $\mathfrak{S} : \Phi \to P(E)$. *An IFSHG* $\tilde{H} = \left\{\tilde{H}(\phi_1), \tilde{H}(\phi_2)\right\}$ *is shown in Figure 1.*

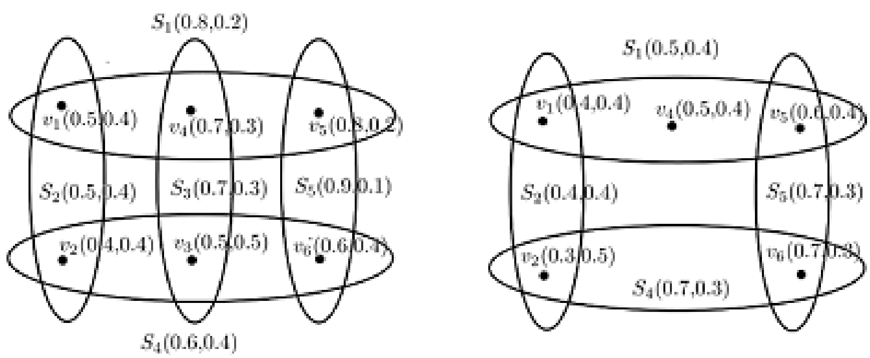

Figure 1. $\tilde{H} = \left\{\tilde{H}(\phi_1), \tilde{H}(\phi_2)\right\}$

2. Regular Intuitionistic Fuzzy Soft Hypergraphs

Definition 2.1. *The order of an IFSHG is* $\mathcal{O}(\tilde{H}) = \left[\sum_{\phi_i \in \Phi}\left(\sum_{v \in V} \mathfrak{N}_\mu(v)\right), \sum_{\phi_i \in \Phi}\left(\sum_{v \in V} \mathfrak{N}_\nu(v)\right)\right]$.

Definition 2.2. *The size of an IFSHG is*

$$\mathcal{S}(\tilde{H}) = \left[\sum_{\phi_i \in \Phi}\left(\sum_{v_1 \ldots v_n \in \mathfrak{T}} \mathfrak{T}_\mu(\phi_i)(v_1 \ldots v_n)\right), \sum_{\phi_i \in \Phi}\left(\sum_{v_1 \ldots v_n \in E} \mathfrak{T}_\nu(\phi_i)(v_1 \ldots v_n)\right)\right].$$

Definition 2.3. *The open neighborhood of a vertex $v_i(\phi_i)$ in the intuitionistic fuzzy soft hypergraph is denoted by $O_N(\phi_i)(v_1 \ldots v_n)$ and it is defined by the set of adjacent vertices of $v_i(\phi_i)$ excluding that vertex corresponding to the parameter.*

Definition 2.4. *The closed neighbourhood of a vertex $v_i(\phi_i)$ in the intuitionistic fuzzy soft hypergraph is denoted by $C_N\left[(\phi_i)(v_1 \ldots v_n)\right]$ and it is defined by the set of adjacent vertices of $v_i(\phi_i)$ including that vertex corresponding to the parameter.*

Example 2.5. *For the above Example 1.6, the open neighborhood of a vertex $v_2(\phi_1) = v_1, v_3, v_6$ and $v_2(\phi_2) = v_1, v_6$. The closed neighborhood of a vertex $v_2(\phi_1) = v_1, v_2, v_3, v_6$ and $v_2(\phi_2) = v_1, v_2, v_6$.*

Definition 2.6. *If $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ be an Intuitionistic Fuzzy Soft Hypergraph, then the degree of open neighborhood for a vertex $v_i(\phi_i)$ is denoted by $degO_N\left(v_i(\phi_i)\right)$ and it is defined by $degO_N\left(v_i(\phi_i)\right) = \left(deg_\mu v_i(\phi_i), deg_\nu\left(v_i(\phi_i)\right)\right)$, where $deg_\mu\left(v_i(\phi_i)\right) = \sum_{v_i \in \mathfrak{N}} \mathfrak{N}_\mu(v_i(\phi_i))$ and $deg_\nu\left(v_i(\phi_i)\right) = \sum_{v_i \in \mathfrak{N}} \mathfrak{N}_\nu(v_i(\phi_i))$.*

Definition 2.7. *If $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ be an Intuitionistic Fuzzy Soft Hypergraph, then the degree of closed neighborhood for a vertex $v_i(\phi_i)$ is denoted by $degC_N[v_i(\phi_i)]$ and it is defined by $degC_N[v_i(\phi_i)] = (deg_\mu[v_i(\phi_i)], deg_\nu[v_i(\phi_i)])$, where $deg_\mu[v_i(\phi_i)] = deg_\mu(v_i(\phi_i)) + \mathfrak{N}_\mu(v_i(\phi_i))$ and $deg_\nu[v_i(\phi_i)] = deg_\nu(v_i(\phi_i)) + \mathfrak{N}_\nu(v_i(\phi_i))$.*

Example 2.8. *For the above Example 1.6, the degree of open neighborhood for a vertex $v_2(\phi_1) = (1.6, 1.3)$ and $v_2(\phi_2) = (1.1, 0.7)$. The degree of closed neighborhood for a vertex $v_2(\phi_1) = (2.0, 1.7)$ and $v_2(\phi_2) = (1.4, 1.2)$.*

Definition 2.9. *Let $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ be an IFSHG. If all the vertices in $\mathfrak{N}$ have the same degree of open neighborhood degree $(k_i, k_i^{'})$ for the corresponding parameters, then $\tilde{H}$ is said to be $(k_i, k_i^{'})$ - regular Intuitionistic Fuzzy Soft Hypergraph.*

Remark 2.10. *Any intuitionistic fuzzy soft hypergraphs with two vertices and one hyperedge is regular.*

Definition 2.11. *Let $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ be an IFSHG. If all the vertices in $\mathfrak{N}$ have the same degree of closed neighborhood degree $(p_i, p_i^{'})$ for the corresponding parameters, then $\tilde{H}$ is said to be $(p_i, p_i^{'})$ - totally regular Intuitionistic Fuzzy Soft Hypergraph.*

Definition 2.12. *Let $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ be an IFSHG. IF $\tilde{H}$ is $(k_i, k_i^{'})$ - regular and $(p_i, p_i^{'})$ - totally regular Intuitionistic Fuzzy Soft Hypergraph, then it is said to be perfectly regular IFSHG.*

Example 2.13. *Consider an IFSHG $\tilde{H} = (H^*, \mathfrak{N}, \mathfrak{S}, \Phi)$, such that $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_2, v_2v_3, v_3v_4, v_1v_4\}$. Let $\Phi = \{\phi_1\}$ be a parameter set. Let $(\mathfrak{N}, \Phi)$ be an intuitionistic fuzzy soft set over V with its approximate function $\mathfrak{N} : \Phi \to P(V)$.*

$$\mathfrak{N}(\phi_1) = \{v_1\langle 0.7, 0.3\rangle, v_2\langle 0.7, 0.3\rangle, v_3\langle 0.7, 0.3\rangle, v_4\langle 0.7, 0.3\rangle\}$$

Let $(\mathfrak{S}, \Phi)$ be an intuitionistic fuzzy soft set over E with its approximate function $\mathfrak{S} : \Phi \to P(E)$.

$$\mathfrak{S}(\phi_1) = \{v_1v_2\langle 0.8, 0.2\rangle, v_2v_3\langle 0.8, 0.2\rangle, v_3v_4\langle 0.8, 0.2, v_1v_4\langle 0.8, 0.2\rangle\}.$$

The open neighborhood degree of the vertices for the parameter ϕ_1 are same. That is, $deg(v_1) = deg(v_2) = deg(v_3) = deg(v_4) = (1.4, 0.6)$. Hence IFSHG is said to be regular of degree $(1.4, 0.6)$ (or) $(1.4, 0.6)$ - regular Intuitionistic Fuzzy Soft Hypergraph. The closed neighborhood degree of the vertices for the parameter ϕ_1 are same. That is, $deg(v_1) = deg(v_2) = deg(v_3) = deg(v_4) = (3.0, 1.0)$. Hence IFSHG is said to be totally regular of degree $(2.1, 0.9)$ (or) $(2.1, 0.9)$- totally regular Intuitionistic Fuzzy Soft Hypergraph.

Remark 2.14. *Any intuitionistic fuzzy soft hypergraphs with different membership and non-membership values need not to be regular or totally regular under parametrization.*

Theorem 2.15. *Let $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ be an IFSHG. Then $\big(\mathfrak{T}_\mu : \mathfrak{N} \to [0,1], \mathfrak{T}_\nu : \mathfrak{N} \to [0,1]\big)$ is a constant function iff the following conditions are equivalent.*

(i). $\tilde{H}$ is a regular IFSHG.

(ii). $\tilde{H}$ is totally regular IFSHG.

Proof. Suppose that $(\mathfrak{T}_\mu, \mathfrak{T}_\nu)$ be a constant function. Let $\mathfrak{T}_\mu(v_1) = \mathcal{C}_1$ and $\mathfrak{T}_\nu(v_1) = \mathcal{C}_2$ for the parameter $\phi_1 \in \Phi$ and $v_1 \in \mathfrak{T}$.
$(i) \implies (ii)$ Assume that $\tilde{H}$ is a $(k_i, k_i^{'})$-regular IFSHG. Let $deg_\mu(v_1(\phi_1)) = k_1$ and $deg_\nu(v_1(\phi_1) = k_1^{'}$. Then we have,

$$deg_\mu[v_1(\phi_1)] = deg_\mu(v_1(\phi_1)) + \mathfrak{T}_\mu(v_1(\phi_1)) \;\; and \;\; deg_\nu[v_1(\phi_1)] = deg_\nu(v_1(\phi_1)) + \mathfrak{T}_\mu(v_1)(\phi_1))$$

Thus $deg_\mu[v_1(\phi_1)] = k_1 + \mathcal{C}_1$ and $deg_\nu[v_1(\phi_1)] = k_1^{'} + \mathcal{C}_2$. Hence $\tilde{H}$ is totally regular IFSHG.
$(ii) \implies (i)$ Assume that $\tilde{H}$ is a $(p_i, p_i^{'})$-totally regular IFSHG. Let $deg_\mu[v_1(\phi_1)] = p_1$ and $deg_\nu[v_1(\phi_1)] = p_1^{'}$. Then we have

$$deg_\mu[v_1(\phi_1)] = deg_\mu(v_1(\phi_1)) + \mathfrak{T}_\mu(v_1(\phi_1)) \;\; and \;\; deg_\nu[v_1(\phi_1)] = deg_\nu(v_1(\phi_1)) + \mathfrak{T}_\mu(v_1)(\phi_1))$$

$\implies deg_\mu(v_1(\phi_1)) + \mathfrak{T}_\mu(v_1(\phi_1)) = p_1, deg_\nu(v_1(\phi_1)) + \mathfrak{T}_\mu(v_1)(\phi_1)) = p_1^{'}$
$\implies deg_\mu(v_1(\phi_1)) + \mathcal{C}_1 = p_1, deg_\nu(v_1(\phi_1)) + \mathcal{C}_2 = p_1^{'}$
$\implies deg_\mu(v_1(\phi_1)) = p_1 - \mathcal{C}_1$, $deg_\nu(v_1(\phi_1)) = p_1^{'} - \mathcal{C}_2$, for $\phi_1 \in \Phi$ and $v_1 \in \mathfrak{T}$.
Thus $\tilde{H}$ is a regular IFSHG. Hence (i) and (ii) are equivalent.
Conversely, Assume that (i) and (ii) are equivalent. That is $\tilde{H}$ is a regular IFSHG iff $\tilde{H}$ is a totally regular IFSHG. Suppose that $(\mathfrak{T}_\mu, \mathfrak{T}_\nu)$ is not a constant function and $\mathfrak{T}_i(v_1)$ and $\mathfrak{T}_i(v_2)$ is not equal for some $v_1, v_2 \in \mathfrak{N}$ corresponding to the parameter ϕ_1. If $\tilde{H}$ is a $(k_i, k_i^{'})$ - regular IFSHG, then $deg(v_1)(\phi_1)) = (k_1, k_1^{'})$ for all $v_1 \in \mathfrak{T}_i$. Consider,

$$deg[v_1(\phi_1)] = deg(v_1(\phi_1)) + \mathfrak{T}(v_1(\phi_1)) = (k_1, k_1^{'}) + \mathfrak{T}(v_1(\phi_1)) \text{ and } deg[v_2(\phi_1)] = deg(v_2(\phi_1)) + \mathfrak{T}(v_2(\phi_1)) = (k_2, k_2^{'}) + \mathfrak{T}(v_2(\phi_1)).$$

Then $\mathfrak{T}_i(v_1)$ and $\mathfrak{T}_i(v_2)$ is not equal for some $v_1, v_2 \in \mathfrak{N}$ corresponding to the parameter ϕ_1. Thus $deg[v_1(\phi_1)]$ and $deg[v_2(\phi_1)]$ are not equal. Hence $\tilde{H}$ is not a totally regular IFSHG, which is a contradiction. Let $\tilde{H}$ is totally regular IFSHG. Then $deg[v_1(\phi_1)] = deg[v_2(\phi_1)]$. That is $deg(v_1(\phi_1)) + \mathfrak{T}(v_1(\phi_1)) = deg(v_2(\phi_1)) + \mathfrak{T}(v_2(\phi_1))$ and $deg(v_1(\phi_1)) - deg(v_2(\phi_1)) = \mathfrak{T}(v_2(\phi_1)) - \mathfrak{T}(v_1(\phi_1))$. Since $deg(v_1(\phi_1)) - deg(v_2(\phi_1)) \neq 0$ and $\mathfrak{T}(v_2(\phi_1)) - \mathfrak{T}(v_1(\phi_1)) \neq 0$. Thus $deg[v_1(\phi_1)] \neq deg[v_2(\phi_1)]$, So $\tilde{H}$ is not a regular IFSHG. which is a contradiction to our assumption. Hence $(\mathfrak{T}_\mu, \mathfrak{T}_\nu)$ must be a constant function. ☐

Theorem 2.16. *Let $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ be an IFSHG. If $\tilde{H}$ is both regular and totally regular, then $(\mathfrak{T}_\mu, \mathfrak{T}_\nu)$ is a constant function.*

Proof. Let $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ be an IFSHG and it is both regular and totally regular. Let $deg_\mu[v_1(\phi_1)] = p_1$ and $deg_\nu[v_1(\phi_1)] = p_1^{'}$, for all $\phi_i \in \Phi$ and $v_i \in \mathfrak{T}$. Let $deg_\mu(v_i(\phi_i)) = p_n$ and $deg_\nu(v_i(\phi_i) = p_n^{'}$, for all $\phi_i \in \Phi$ and $v_i \in \mathfrak{T}$. Consider, $deg_\mu[v_i(\phi_i)] = p_1$, for all $v_i \in \mathfrak{T} \Leftrightarrow deg_\mu(v_i(\phi_i)) + \mathfrak{T}_\mu(v_i(\phi_i)) = p_1 \Leftrightarrow p_n + \mathfrak{T}_\mu(v_i(\phi_i)) = p_1 \Leftrightarrow \mathfrak{T}_\mu(v_i(\phi_i)) = p_1 - p_n$,for all $v_i \in \mathfrak{T}$ and $\phi_i \in \Phi$. Consider, $deg_\nu[v_i(\phi_i)] = p_1^{'}$, for all $v_i \in \mathfrak{T} \Leftrightarrow deg_\nu(v_i(\phi_i)) + \mathfrak{T}_\nu(v_i(\phi_i)) = p_1^{'} \Leftrightarrow p_n + \mathfrak{T}_\nu(v_i(\phi_i)) = p_1^{'} \Leftrightarrow \mathfrak{T}_\nu(v_i(\phi_i)) = p_1^{'} - p_n^{'}$, for all $v_i \in \mathfrak{T}$ and $\phi_i \in \Phi$. Hence $(\mathfrak{T}_\mu, \mathfrak{T}_\nu)$ is a constant function. □

Note: The converse of the theorem need not be true.

Definition 2.17. *If all the hyper-edges corresponding to their parameters have the same cardinality, then IFSHG is said to be* $(k_i, k_i^{'})$ *- uniform IFSHG.*

Example 2.18. *Consider an IFSHG* $\tilde{H} = (H^*, \mathfrak{N}, \mathfrak{S}, \Phi)$*, such that* $V = \{v_1, v_2, v_3, v_4, v_5\}$ *and* $E = \{v_1v_2v_3, v_3v_4v_5\}$*. Let* $\Phi = \{\phi_1\}$ *be a parameter set. Let* $(\mathfrak{N}, \Phi)$ *be an intuitionistic fuzzy soft set over* V *with its approximate function* $\mathfrak{N} : \Phi \to P(V)$. $\mathfrak{N}(\phi_1) = \{v_1\langle 0.5, 0.4\rangle, v_2\langle 0.6, 0.3\rangle, v_3\langle 0.7, 0.2\rangle, v_4\langle 0.4, 0.3\rangle, v_5\langle 0.3, 0.2\rangle\}$*. Let* $(\mathfrak{S}, \Phi)$ *be an intuitionistic fuzzy soft set over* E *with its approximate function* $\mathfrak{S} : \Phi \to P(E)$. $\mathfrak{S}(\phi_1) = \{v_1v_2v_3\langle 0.8, 0.2\rangle, v_3v_4v_5\langle 0.8, 0.2\rangle\}$*. The* $(0.8, 0.2)$ *- Uniform IFSHG is shown in Figure 2.*

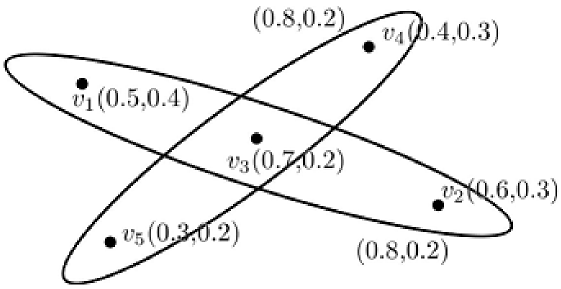

Figure 2. Uniform IFSHG

3. Conclusion

Hypergraphs are considered as the most efficient representation to handle the complicated practical problems in real life. An IFSS is an extension of fuzzy soft set, used to deal with uncertain information under complexity based on the parameters. So, by combining both IFSS and hypergraphs a notion of IFSHGs are given and also discussed about regular - IFSHGs and totally regular IFSHGs. Moreover the author intend to extent the research work in Edge-Regular and Edge-Irregular IFSHGs.

References

[1] K. T. Atanassov, *Intuitionistic fuzzy sets - Theory and Applications*, New York, Physica-verlag, Berlin, (1999).

[2] C. Berge, *Graphs and Hypergraphs*, North Holland, Newyork, (1976).

[3] T. K. Mathew and A. M. Shyla, *Some properties of intutionistic fuzzy soft graph*, International Journal of Mathematics and Applications, 7(2)(2019), 59-73.

[4] P. K. Maji, R. Biswas and A. R. Roy, *Soft set theory*, Computational Mathematics and Applications, 45(2003), 555-562.

[5] P. K. Maji, R. Biswas and A. R. Roy, *Intutionistic fuzzy soft sets*, The Journal of Fuzzy Mathematics, 12(3)(2004), 669-683.

[6] S. Mohinta and T. K. Samanta, *An introduction to fuzzy soft graph*, Mathematica Moravica, 19(2)(2015), 35-48.

[7] D. Molodstov, *Soft set theory-First results*, Computers and Mathematics with Applications, 37(1999), 19-31.

[8] A. Nagoor Gani and K. Radha, *On Regular Fuzzy Graphs*, Journal of Physical Sciences, 12(2008), 33-40.

[9] R. Parvathi and M. G. Karunambigai, *Intuitionistic fuzzy graphs*, Proceedings of 9^{th} Fuzzy days International Conference on Computational Intelligence, Advances in soft Computing: Computational Intelligence, Theory and Applications, Springer-Verlag, New York, 20(2006), 139-150.

[10] R. Parvathi, S. Thilagavathi and M. G. Karunambigai, *Intutionistic fuzzy hypergraphs*, Cybernatics and Information Technologies, 9(2)(2009), 46-53.

[11] I. Prandeepa and S. Vimala, *Regular and totally regular intuitionistic fuzzy hypergraphs*, International Journal of Mathematics and Applications, 4(2016), 137-142.

[12] S.Shahzadi and M. Akram, *Graphs in an intutionistic fuzzy soft environment*, DAxioms, 7(2018), 2-16.

[13] R. K. Thumbakara and B. George, *Soft graphs*, General Mathematics Notes, 21(2014), 75-86.

[14] S. Thilagavathi, *Intutionistic fuzzy soft hypergraphs*, International Journal of Engineering and Technology, 7(2018), 313-315.