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A Bitopological $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - Homeomorphisms

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Abstract: In this paper, a new concept of homeomorphisms, say $(1,2)^*$ - $\overset{\bullet}{\alpha}$ homeomorphisms, $(1,2)^*$ - $\overset{\bullet}{\alpha}c$ - homeomorphisms, $(1,2)^*$ - αgg-homeomorphisms and $(1,2)^*$ - αggc - homeomorphisms are introduced and analyzed in bitopological spaces. Also $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - $T_{1/2}$ - space and $(1,2)^*$ - $T_{\overset{\bullet}{\alpha}}$ - space are introduced and studied.

Keywords: $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - homeomorphisms, $(1,2)^*$ - $\overset{\bullet}{\alpha}$c - homeomorphisms, $(1,2)^*$ - αgg - homeomorphisms, $(1,2)^*$ - αggc - homeomorphisms, $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - $T_{1/2}$ - space, $(1,2)^*$ - $T_{\overset{\bullet}{\alpha}}$ - space.

1. Introduction

H. Maki, P. Sundaram and K. Balachandran [2] , introduced the class of $(1,2)^*$ - g - homeomorphisms and $(1,2)^*$ - gc - homeomorphisms. O. Ravi, S. Pious Missier and T. Salai Parkunan [3] , introduced the class of $(1,2)^*$ - gs - homeomorphisms and $(1,2)^*$ - gsc - homeomorphisms. In this paper, a new class of mappings, say $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - homeomorphisms, $(1,2)^*$ - $\overset{\bullet}{\alpha}c$ - homeomorphisms, $(1,2)^*$ - αgg - homeomorphisms and $(1,2)^*$ - αggc - homeomorphisms are introduced in bitopological spaces. Also, the relationships among related $(1,2)^*$ - homeomorphism are investigated. Finally, a new spaces called $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - $T_{1/2}$ - space and $(1,2)^*$ - $T_{\overset{\bullet}{\alpha}}$ - space are introduced and studied.

2. Preliminaries

Throughout this paper, (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) denote bitopological spaces.

Definition 2.1. *A subset H of X is said to be* $\tau_{1,2}$ - *open [3] if* $H = A \cup B$ *where* $A \in \tau_1$ *and* $B \in \tau_2$.

The complement of $\tau_{1,2}$-open is called $\tau_{1,2}$-closed [3] .

Note 2.2 ([3]). $\tau_{1,2}$-*open sets of X need not necessarily form a topology.*

Definition 2.3. *Let H be a subset of X. Then*

(1). $\tau_{1,2}$ - *interior [3] of* H = *Union of all* $\tau_{1,2}$ - *closed sets of X contained in H.*

(2). $\tau_{1,2}$ - *closure [3] of* H = *Intersection of all* $\tau_{1,2}$ - *closed sets of X containing H.*

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Definition 2.4. *A subset H of X is said to be*

(1). $(1,2)^*$ - α - *closed set [1] . If* $\tau_{1,2}$ - $cl(\tau_{1,2}$ - $int(\tau_{1,2}$ - $cl(H))) \subseteq H$.

(2). $(1,2)^*$ - αg - *closed set [1] . If* $(1,2)^*$ - $\alpha cl(H) \subseteq U$ *whenever* $H \subseteq U$ *and* U *is* $\tau_{1,2}$*-open in* X.

(3). $(1,2)^*$ - αgg - *closed set [5] . If* $\tau_{1,2}$ - $cl(H) \subseteq U$ *whenever* $H \subseteq U$ *and* U *is* $(1,2)^*$ - αg - *open in* X.

The complements of the above mentioned closed sets are called their respective open sets.

Definition 2.5. *A subset H of X is said to be*

(1). $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *closed set [5] . If* $(1,2)^*$ - $\alpha cl(H) \subseteq U$ *whenever* $H \subseteq U$ *and* U *is* $(1,2)^*$ - αgg - *open in* X.

(2). $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *open set [5] . If* H^c *is* $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *closed set .*

Definition 2.6. *A map $f : X \to Y$ is called*

(1). $(1,2)^*$ - αgg - *continuous [5] . If inverse image of every* $\sigma_{1,2}$ - *closed set of* Y *is* $(1,2)^*$ - αgg - *closed set of* X.

(2). $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *continuous [5] . If the inverse image of every* $\sigma_{1,2}$ - *closed set (resp.* $\sigma_{1,2}$ - *open set) of* Y *is* $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *closed set (resp.* $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *open set) of* X.

(3). $(1,2)^*$ - αgg - *open map [5] . If the image of every* $\tau_{1,2}$ - *openset in* X *is* $(1,2)^*$ - αgg - *open set in* Y.

(4). $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *open map [5] . If the image of every* $\tau_{1,2}$ - *open set in* X *is* $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *open set in* Y.

(5). $(1,2)^*$ - αgg - *irresolute [5] . If the inverse image of every* $(1,2)^*$ - αgg - *closed set (resp.* $(1,2)^*$ - αgg - *open set) of* Y *is* $(1,2)^*$ - αgg - *closed set (resp.* $(1,2)^*$ - αgg - *open set) of* X.

(6). $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *irresolute [5] . If the inverse image of every* $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *closed set (resp.* $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *open set) of* Y *is* $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *closed set (resp.* $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *open set) of* X.

Definition 2.7. *A bitopological space X is said to be*

(1). $(1,2)^*$ - T_b - *space [3] if every* $(1,2)^*$ - gs - *closed set of* X *is* $\tau_{1,2}$ - *closed in* X.

(2). $(1,2)^*$ - *semi* - $T_{1/2}$ - *space [3] if every* $(1,2)^*$ - sg - *closed set of* X *is* $(1,2)^*$ - *semi - closed in* X.

(3). $(1,2)^*$ - T_c - *space [3] if every* $(1,2)^*$ - g^* - *closed set of* X *is* $\tau_{1,2}$ - *closed in* X.

Result 2.8 ([5]).

(a). Every $(1,2)^*$ - *continuous map is* $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *continuous.*

(b). Every $(1,2)^*$ - αgg - *continuous map is* $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *continuous.*

(c). Every $(1,2)^*$ - g^* - *continuous map is* $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *continuous.*

(d). Every $(1,2)^*$ - α - *continuous map is* $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *continuous.*

(e). Every $(1,2)^*$ - α^* - *continuous map is* $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *continuous.*

(f). Every $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *continuous map is* $(1,2)^*$ - αg - *continuous map.*

(g). Every $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - *continuous map is* $(1,2)^*$ - $\pi g\alpha$ - *continuous map.*

(h). Every $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- continuous map is* $(1,2)^*$ *-* $\pi g(\alpha g)^*$ *- continuous map.*

(i). Every $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- continuous map is* $(1,2)^*$ *- gs - continuous map.*

(j). Every $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- continuous map is* $(1,2)^*$ *- gsp - continuous map.*

(k). Every $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- continuous map is* $(1,2)^*$ *-* αgr *- continuous map.*

3. On $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - Homeomorphism

Definition 3.1. *A bijective map* $f : X \to Y$ *is called*

(1). $(1,2)^*$ *-* αgg *- homeomorphism if a map is both* $(1,2)^*$ *-* αgg *- continuous and* $(1,2)^*$ *-*αgg *- open map (i.e both f and* f^{-1} *are* αgg *- continuous).*

(2). $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- homeomorphism if a map is both* $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- continuous and* $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- open map (i.e both f and* f^{-1} *are* $\overset{\bullet}{\alpha}$ *- continuous).*

Proposition 3.2.

(a). Every $(1,2)^*$ *- homeomorphism is* $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- homeomorphism.*

(b). Every $(1,2)^*$ *-* αgg *- homeomorphism is* $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- homeomorphism.*

(c). Every $(1,2)^*$ *- g* - homeomorphism is* $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- homeomorphism.*

(d). Every $(1,2)^*$ *-* α *- homeomorphism is* $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- homeomorphism.*

(e). Every $(1,2)^*$ *-* $\alpha *$ *- homeomorphism is* $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- homeomorphism.*

(f). Every $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- homeomorphism is* $(1,2)^*$ *- gs - homeomorphism.*

(g). Every $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- homeomorphism is* $(1,2)^*$ *- gsp - homeomorphism.*

(h). Every $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- homeomorphism is* $(1,2)^*$ *-* αg *- homeomorphism.*

(i). Every $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- homeomorphism is* $(1,2)^*$ *-* αgr *- homeomorphism.*

(j). Every $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- homeomorphism is* $(1,2)^*$ *-* $\pi g\alpha$ *- homeomorphism.*

(k). Every $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- homeomorphism is* $(1,2)^*$ *-* $\pi g(\alpha g)^*$ *- homeomorphism.*

Proof. Straight forward. □

Remark 3.3. *The converse of the proposition 3.2 need not be true as seen from the following examples.*

Example 3.4. *Let* $X = Y = \{a,b,c\}$ *with* $\tau_1 = \{X, \phi, \{c\}\}$, $\tau_2 = \{X, \phi\}$, $\sigma_1 = \{Y, \Phi, \{b\}, \{c\}, \{b,c\}\}$ *and* $\sigma_2 = \{Y, \Phi\}$. *Then* $\tau_{1,2}$ *- open set* $= \{X, \phi, \{c\}\}$ *and* $\sigma_{1,2}$ *- open set* $= \{Y, \Phi, \{b\}, \{c\}, \{b,c\}\}$. *Define* $f : X \to Y$ *as bijective map. Clearly, f is* $(1,2)^*$ *-* $\overset{\bullet}{\alpha}$ *- homeomorphism but not* $(1,2)^*$ *- homeomorphim,* $(1,2)^*$ *-* αgg *- homeomorphism,* $(1,2)^*$ *-* g^* *- homeomorphism,* $(1,2)^*$ *-* α *- homeomorphism and* $(1,2)^*$ *-* α^* *- homeomorphism.*

Example 3.5. *Let* $X = Y = \{a,b,c,d\}$ *with* $\tau_1 = \{X, \phi, \{c,d\}\}$, $\tau_2 = \{X, \phi\}$, $\sigma_1 = \{Y, \Phi, \{a\}, \{b\}, \{a,b\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$ *and* $\sigma_2 = \{Y, \Phi\}$. *Then* $\tau_{1,2}$ *- open set* $= \{X, \phi, \{c,d\}\}$ *and* $\sigma_{1,2}$ *- open set* $= \{Y, \Phi, \{a\}, \{b\}, \{a,b\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$. *Define* $f : X \to Y$ *as bijective map. Clearly,* f *is* $(1,2)^*$ *-* αg *- homeomorphism but not* $(1,2)^*$ *-* $\dot{\alpha}$ *- homeomorphism.*

Example 3.6. *Let* $X = Y = \{a,b,c,d\}$ *with* $\tau_1 = \{X, \phi, \{c,d\}\}$, $\tau_2 = \{X, \phi\}$, $\sigma_1 = \{Y, \Phi, \{b\}, \{b,d\}, \{a,b,c\}\}$ *and* $\sigma_2 = \{Y, \Phi\}$. *Then* $\tau_{1,2}$ *- open set* $= \{X, \phi, \{c,d\}\}$ *and* $\sigma_{1,2}$ *- open set* $= \{Y, \Phi, \{b\}, \{b,d\}, \{a,b,c\}\}$. *Define* $f : X \to Y$ *as bijective map. Clearly,* f *is* $(1,2)^*$ *-* αgr *- homeomorphism,* $(1,2)^*$ *-* $\pi g\alpha$ *- homeomorphism and* $(1,2)^*$ *-* $\pi g(\alpha g)^*$ *- homeomorphism but not* $(1,2)^*$ *-* $\dot{\alpha}$ *- homeomorphism.*

Example 3.7. *Let* $X = Y = \{a,b,c\}$ *with* $\tau_1 = \{X, \phi, \{a\}, \{c\}, \{a,c\}\}$, $\tau_2 = \{X, \phi\}$, $\sigma_1 = \{Y, \Phi, \{a\}, \{b,c\}\}$ *and* $\sigma_2 = \{Y, \Phi\}$. *Then* $\tau_{1,2}$ *- open set* $= \{X, \phi, \{a\}, \{c\}, \{a,c\}\}$ *and* $\sigma_{1,2}$ *- open set* $= \{Y, \Phi, \{a\}, \{b,c\}\}$. *Define* $f : X \to Y$ *as bijective map. Clearly,* f *is* $(1,2)^*$ *- gs - homeomorphism and* $(1,2)^*$ *- gsp - homeomorphism but not* $(1,2)^*$ *-* $\dot{\alpha}$ *- homeomorphism.*

Remark 3.8. $(1,2)^*$ *-* $\dot{\alpha}$ *- homeomorphism is independent with* $(1,2)^*$ *- sg - homeomorphism and* $(1,2)^*$ *- b - homeomorphism.*

Example 3.9.

(1). Let $X = Y = \{a,b,c\}$ *with* $\tau_1 = \{X, \phi, \{a\}, \{c\}, \{a,c\}\}$, $\tau_2 = \{X, \phi\}$, $\sigma_1 = \{Y, \Phi, \{a\}, \{b,c\}\}$ *and* $\sigma_2 = \{Y, \Phi\}$. *Then* $\tau_{1,2}$ *- open set* $= \{X, \phi, \{a\}, \{c\}, \{a,c\}\}$ *and* $\sigma_{1,2}$ *- open set* $= \{Y, \Phi, \{a\}, \{b,c\}\}$. *Define* $f : X \to Y$ *as bijective map. Clearly,* f *is* $(1,2)^*$ *- sg - homeomorphism and* $(1,2)^*$ *- b - homeomorphism but not* $(1,2)^*$ *-* $\dot{\alpha}$ *- homeomorphism.*

(2). Let $X = Y = \{a,b,c\}$ *with* $\tau_1 = \{X, \phi, \{c\}, \{b,c\}\}$, $\tau_2 = \{X, \phi\}$, $\sigma_1 = \{Y, \Phi, \{b\}, \{a,c\}\}$ *and* $\sigma_2 = \{Y, \Phi\}$. *Then* $\tau_{1,2}$ *- open set* $= \{X, \phi, \{c\}, \{b,c\}\}$ *and* $\sigma_{1,2}$ *- open set* $= \{Y, \Phi, \{b\}, \{a,c\}\}$. *Define* $f : X \to Y$ *as bijective map. Clearly,* f *is* $(1,2)^*$ *-* $\dot{\alpha}$ *- homeomorphism but not* $(1,2)^*$ *- sg - homeomorphism and* $(1,2)^*$ *- b - homeomorphism.*

Remark 3.10. *The composition of two* $(1,2)^*$ *-* $\dot{\alpha}$ *- homeomorphism map need not be a* $(1,2)^*$ *-* $\dot{\alpha}$ *- homeomorphism map.*

Example 3.11. *Let* $X = Y = Z = \{a,b,c\}$ *with* $\tau_1 = \{X, \Phi, \{b,c\}\}$, $\tau_2 = \{X, \phi, \{c\}\}$, $\sigma_1 = \{Y, \Phi, \{b,c\}\}$, $\sigma_2 = \{Y, \Phi, \{b\}\}$, $\eta_1 = \{Z, \Phi, \{a,b\}\}$ *and* $\eta_2 = \{Z, \Phi, \{c\}\}$. *Then* $\tau_{1,2}$ *- open set* $= \{X, \Phi, \{c\}, \{b,c\}\}$, $\sigma_{1,2}$ *- open set* $= \{\{Y, \Phi, \{b\}, \{b,c\}\}$ *and* $\eta_{1,2}$ *- open set* $= \{Z, \Phi, \{c\}, \{a,b\}\}$. *Define* $f : X \to Y$ *and* $h : Y \to Z$ *be an identity maps. Let* f *and* h *be a* $(1,2)^*$ *-* $\dot{\alpha}$ *- homeomorphism maps. But* $(h \circ f)^{-1}(\{c\}) = f^{-1}(h^{-1}(\{c\})) = \{c\}$ *is not* $(1,2)^*$ *-* $\dot{\alpha}$ *- closed of X. Hence* $h \circ f$ *is not* $(1,2)^*$ *-* $\dot{\alpha}$ *- homeomorphism.*

Theorem 3.12. *For a bijective map* $f : X \to Y$, *the following statements are equivalent:*

(1). f^{-1} *is* $(1,2)^*$ *-* $\dot{\alpha}$ *- continuous,*

(2). f *is* $(1,2)^*$ *-* $\dot{\alpha}$ *- open map,*

(3). f *is* $(1,2)^*$ *-* $\dot{\alpha}$ *- closed map.*

Theorem 3.13. *If map* $f : X \to Y$ *is bijective* $(1,2)^*$ *-* $\dot{\alpha}$ *- continuous map, then following statements are equivalent:*

(1). f *is* $(1,2)^*$ *-* $\dot{\alpha}$ *- homeomorphism,*

(2). f *is* $(1,2)^*$ *-* $\dot{\alpha}$ *- open,*

(3). f *is* $(1,2)^*$ *-* $\dot{\alpha}$ *- closed.*

4. On $(1,2)^*$ - $\dot{\alpha}c$ - Homeomorphism and $(1,2)^*$ - αggc - Homeomorphism

Definition 4.1. *A bijective map $f : X \to Y$ is called*

(1). $(1,2)^*$ - αggc - *homeomorphism if both f and f^{-1} are* $(1,2)^*$ - αgg - *irresolute maps.*

(2). $(1,2)^*$ - $\dot{\alpha}c$ - *homeomorphism if both f and f^{-1} are* $(1,2)^*$ - $\dot{\alpha}$ - *irresolute maps.*

Proposition 4.2. *Every* $(1,2)^*$ - αggc - *homeomorphism is* $(1,2)^*$ - αgg - *homeomorphism.*

Remark 4.3. *The converse of the above proposition need not be true as seen from the following example: Let* $X = Y = \{a,b,c,d\}$ *with* $\tau_1 = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$, $\tau_2 = \{X, \phi\}$, $\sigma_1 = \{Y, \Phi, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}\}$ *and* $\sigma_2 = \{Y, \Phi\}$. *Then* $\tau_{1,2}$ - *open set* $= \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$ *and* $\sigma_{1,2}$ - *open set* $= \{Y, \Phi, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}\}$. *Define* $f : X \to Y$ *as bijective map. Clearly, f is* $(1,2)^*$ - αgg - *homeomorphism but not* $(1,2)^*$ - αggc - *homeomorphism.*

Proposition 4.4. *Every* $(1,2)^*$ - αggc - *homeomorphism is* $(1,2)^*$ - $\dot{\alpha}$ - *homeomorphism.*

Remark 4.5. *The converse of the above proposition need not be true as seen from the following example, Let* $X = Y = \{a,b,c\}$ *with* $\tau_1 = \{X, \phi, \{a\}\}$, $\tau_2 = \{X, \phi\}$, $\sigma_1 = \{Y, \Phi, \{b\}, \{a,c\}\}$ *and* $\sigma_2 = \{Y, \Phi\}$. *Then* $\tau_{1,2}$ - *open set* $= \{X, \phi, \{a\}\}$ *and* $\sigma_{1,2}$ - *open set* $= \{Y, \Phi, \{b\}, \{a,c\}\}$. *Define* $f : X \to Y$ *as bijective map. Clearly, f is* $(1,2)^*$ - $\dot{\alpha}$ - *homeomorphism but not* $(1,2)^*$ - αggc - *homeomorphism.*

Proposition 4.6. *Every* $(1,2)^*$ - $\dot{\alpha}c$ - *homeomorphism is* $(1,2)^*$ - $\dot{\alpha}$ - *homeomorphism.*

Remark 4.7. *The converse of the above proposition need not be true as seen from the following example, Let* $X = Y = \{a,b,c\}$ *with* $\tau_1 = \{X, \phi, \{a\}\}$, $\tau_2 = \{X, \phi\}$, $\sigma_1 = \{Y, \Phi, \{b\}, \{a,c\}\}$ *and* $\sigma_2 = \{Y, \Phi\}$. *Then* $\tau_{1,2}$ - *open set* $= \{X, \phi, \{a\}\}$ *and* $\sigma_{1,2}$ - *open set* $= \{Y, \Phi, \{b\}, \{a,c\}\}$. *Define* $f : X \to Y$ *as bijective map. Clearly, f is* $(1,2)^*$ - $\dot{\alpha}$ - *homeomorphism but not* $(1,2)^*$ - $\dot{\alpha}c$ - *homeomorphism.*

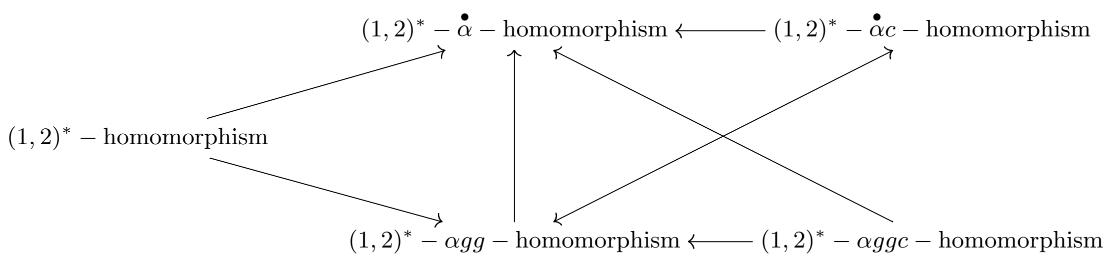
Remark 4.8. *Every* $(1,2)^*$ - αgg - *homeomorphism is independent with* $(1,2)^*$ - $\dot{\alpha}c$ - *homeomorphism.*

Example 4.9.

(1). Let $X = Y = \{a,b,c\}$ *with* $\tau_1 = \{X, \phi, \{a\}, \{b,c\}\}$, $\tau_2 = \{X, \phi\}$, $\sigma_1 = \{Y, \Phi, \{b\}, \{a,c\}\}$ *and* $\sigma_2 = \{Y, \Phi\}$. *Then* $\tau_{1,2}$ - *open set* $= \{X, \phi, \{a\}, \{b,c\}\}$ *and* $\sigma_{1,2}$ - *open set* $= \{Y, \Phi, \{b\}, \{a,c\}\}$. *Define* $f : X \to Y$ *as bijective map. Clearly, f is* $(1,2)^*$ - $\dot{\alpha}c$ - *homeomorphism but not* $(1,2)^*$ - αgg - *homeomorphism.*

(2). Let $X = Y = \{a,b,c\}$ *with* $\tau_1 = \{X, \phi, \{a\}, \{a,c\}\}$, $\tau_2 = \{X, \phi\}$, $\sigma_1 = \{Y, \Phi, \{a,c\}\}$ *and* $\sigma_2 = \{Y, \Phi\}$. *Then* $\tau_{1,2}$ - *open set* $= \{X, \phi, \{a\}, \{a,c\}\}$ *and* $\sigma_{1,2}$ - *open set* $= \{Y, \Phi, \{a,c\}\}$. *Define* $f : X \to Y$ *as bijective map. Clearly, f is* $(1,2)^*$ - αgg - *homeomorphism but not* $(1,2)^*$ - $\dot{\alpha}c$ - *homeomorphism.*

Diagram - I



Implications of the above diagram is not reversible.

Theorem 4.10. *For a bijective map $f : X \to Y$, the following statements are equivalent:*

(1). f^{-1} is $(1,2)^*$ - αgg - continuous,

(2). f is $(1,2)^*$ - αgg - open map,

(3). f is $(1,2)^*$ - αgg - closed map.

Theorem 4.11. *If map $f : X \to Y$ is bijective $(1,2)^*$ - αgg - continuous map, then following statements are equivalent:*

(1). f is $(1,2)^*$ - αgg - homeomorphism,

(2). f is $(1,2)^*$ - αgg - open,

(3). f is $(1,2)^*$ - αgg - closed.

5. Applications of $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - Closed set

Definition 5.1. *A bitopological space X is said to be*

(1). $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - space if every $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - closed set of X is $(1,2)^*$ - α - closed in X.

(2). $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - $T_{1/2}$ - space if every $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - closed set of X is $\tau_{1,2}$ - closed in X.

(3). $(1,2)^*$ - $T_{\overset{\bullet}{\alpha}}$ - space if every $(1,2)^*$ - gs - closed set of X is $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - closed in X.

Proposition 5.2. *Every $(1,2)^*$ - T_b - space is $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - $T_{1/2}$ - space.*

Remark 5.3. *Converse of the above proposition need not be true as seen from the following example, let $X = \{a, b, c\}$ with $\tau_1 = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ and $\tau_2 = \{X, \phi\}$. Then $\tau_{1,2}$ - open set $= \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. Clearly, X is $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - $T_{1/2}$ - space but not $(1,2)^*$ - T_b - space, since $\{b\}$ is $(1,2)^*$ - gs closed but not $\tau_{1,2}$ - closed set of X.*

Proposition 5.4. *Every $(1,2)^*$ - T_b - space is $(1,2)^*$ - $T_{\overset{\bullet}{\alpha}}$ - space.*

Remark 5.5. *Converse of the above proposition need not be true as seen from the following example, let $X = \{a, b, c\}$ with $\tau_1 = \{X, \phi, \{c\}\}$ and $\tau_2 = \{X, \phi\}$. Then $\tau_{1,2}$ - open set $= \{X, \phi, \{c\}\}$. Clearly, X is $(1,2)^*$ - $T_{\overset{\bullet}{\alpha}}$ - space but not $(1,2)^*$ - T_b - space, since $\{b, c\}$ is $(1,2)^*$ - gs closed but not $\tau_{1,2}$ - closed set of X.*

Proposition 5.6. *Every $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - $T_{1/2}$ - space is $(1,2)^*$ - T_c - space.*

Remark 5.7. *Converse of the above proposition need not be true as seen from the following example, let $X = \{a, b, c\}$ with $\tau_1 = \{X, \phi, \{c\}\}$ and $\tau_2 = \{X, \phi\}$. Then $\tau_{1,2}$ - open set $= \{X, \phi, \{c\}\}$. Clearly, X is $(1,2)^*$ - T_c - space but not $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - $T_{1/2}$ - space, since $\{a\}$ is $(1,2)^*$ - $\overset{\bullet}{\alpha}$ closed but not $\tau_{1,2}$ - closed set of X.*

Remark 5.8. *$(1,2)^*$ - $\overset{\bullet}{\alpha}$ - $T_{1/2}$ - space is independent with $(1,2)^*$ - $T_{\overset{\bullet}{\alpha}}$ - space from the following example.*

(1). Let $X = \{a, b, c\}$ with $\tau_1 = \{X, \phi, \{c\}\}$ and $\tau_2 = \{X, \phi\}$. Then $\tau_{1,2}$ - open set $= \{X, \phi, \{c\}\}$. Clearly, X is $(1,2)^*$ - $T_{\overset{\bullet}{\alpha}}$ - space but not $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - $T_{1/2}$ - space, since $\{a, c\}$ is $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - closed set but not $\tau_{1,2}$ - closed set of X.

(2). Let $X = \{a, b, c\}$ with $\tau_1 = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ and $\tau_2 = \{X, \phi\}$. Then $\tau_{1,2}$ - open set $= \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. Clearly, X is $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - $T_{1/2}$ - space but not $(1,2)^*$ - $T_{\overset{\bullet}{\alpha}}$ - space, since $\{c\}$ is $(1,2)^*$ - gs - closed set but not $(1,2)^*$ - $\overset{\bullet}{\alpha}$ - closed set of X.

Remark 5.9. $(1,2)^*$ - $T_{\dot{\alpha}}$ - *space is independent with* $(1,2)^*$ - *semi* - $T_{1/2}$ - *space from the following example.*

(1). Let $X = \{a,b,c\}$ *with* $\tau_1 = \{X, \phi, \{b\}, \{c\}, \{b,c\}\}$ *and* $\tau_2 = \{X, \phi\}$. *Then* $\tau_{1,2}$ - *open set* $= \{X, \phi, \{b\}, \{c\}, \{b,c\}\}$. *Clearly,* X *is* $(1,2)^*$ - *semi* - $T_{1/2}$ - *space but not* $(1,2)^*$ - $T_{\dot{\alpha}}$ - *space, since* $\{b\}$ *is* $(1,2)^*$ - *gs* - *closed set but not* $(1,2)^*$ - $\dot{\alpha}$ - *closed set of* X.

(2). Let $X = \{a,b,c\}$ *with* $\tau_1 = \{X, \phi, \{a\}, \{b,c\}\}$ *and* $\tau_2 = \{X, \phi\}$. *Then* $\tau_{1,2}$ - *open set* $= \{X, \phi, \{a\}, \{b,c\}\}$. *Clearly,* X *is* $(1,2)^*$ - $T_{\dot{\alpha}}$ - *space but not* $(1,2)^*$ - *semi* - $T_{1/2}$ - *space, since* $\{c\}$ *is* $(1,2)^*$ - *sg* - *closed set but not* $(1,2)^*$ - *s* - *closed set of* X.

Remark 5.10. $(1,2)^*$ - $T_{\dot{\alpha}}$ - *space is independent with* $(1,2)^*$ - $\dot{\alpha}$ - *space from the following example:*

(1). Let $X = \{a,b,c\}$ *with* $\tau_1 = \{X, \phi, \{c\}\}$ *and* $\tau_2 = \{X, \phi\}$. *Then* $\tau_{1,2}$ - *open set* $= \{X, \phi, \{c\}\}$. *Clearly,* X *is* $(1,2)^*$ - $T_{\dot{\alpha}}$ - *space but not* $(1,2)^*$ - $\dot{\alpha}$ - *space, since* $\{a,c\}$ *is* $(1,2)^*$ - $\dot{\alpha}$ - *closed set but not* $(1,2)^*$ - α - *closed set of* X.

(2). Let $X = \{a,b,c\}$ *with* $\tau_1 = \{X, \phi, \{b\}, \{c\}, \{b,c\}\}$ *and* $\tau_2 = \{X, \phi\}$. *Then* $\tau_{1,2}$ - *open set* $= \{X, \phi, \{b\}, \{c\}, \{b,c\}\}$. *Clearly,* X *is* $(1,2)^*$ - $\dot{\alpha}$ - *space but not* $(1,2)^*$ - $T_{\dot{\alpha}}$ - *space, since* $\{c\}$ *is* $(1,2)^*$ - *gs* - *closed set but not* $(1,2)^*$ - $\dot{\alpha}$ - *closed set of* X.

Theorem 5.11. *A map* $f : X \to Y$ *and* $h : Y \to Z$ *are two maps and if* f *is* $(1,2)^*$ - α - *irresolute and* h *is* $(1,2)^*$ - $\dot{\alpha}$ - *continuous and* Y *is* $(1,2)^*$ - $\dot{\alpha}$ - *space, then* $h \circ f : X \to Z$ *is* $(1,2)^*$ - α - *continuous.*

Theorem 5.12. *A map* $f : X \to Y$ *and* $h : Y \to Z$ *are two* $(1,2)^*$ - $\dot{\alpha}$ - *closed maps and* Y *is* $(1,2)^*$ - $\dot{\alpha}$ - $T_{1/2}$ - *space, then* $h \circ f : X \to Z$ *is* $(1,2)^*$ - $\dot{\alpha}$ - *closed map.*

Theorem 5.13.

(1). A map $f : X \to Y$ *is* $(1,2)^*$ - $\dot{\alpha}$ - *continuous and* X *is* $(1,2)^*$ - $\dot{\alpha}$ - $T_{1/2}$ - *space, then* f *is* $(1,2)^*$ - *continuous.*

(2). A map $f : X \to Y$ *is* $(1,2)^*$ - $\dot{\alpha}$ - *continuous and* X *is* $(1,2)^*$ - $\dot{\alpha}$ - *space, then* f *is* $(1,2)^*$ - α - *continuous.*

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