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Intuitive Approach for Nonlinear Transportation Problem

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Abstract: An intuitive approach has been developed for finding an optimal solution of nonlinear transportation problem. Method is presented in the form of an algorithm and illustrated through a numerical example. Approach is simple, easy to understand and apply.

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Keywords: Nonlinear transportation problem, intuitive approach, active cell, allocation, optimal solution.

1. Introduction

Classical Transportation problem (CTP) is to search a most economical transporting schedule of supplying a homogeneous commodity from a number of sources to a number of destinations satisfying supplying capacities and demand requirements respectively. A novel approximation method for finding initial basic feasible solution for CTP is given by Karagul, and Sahin, [2]. Sharma [7], have developed a modified zero suffix method for finding an optimal solution of CTP. This method provides optimal solution directly without initiating through initial basic feasible solution and improving the solution again and again to achieve optimal solution. If all the constraints of CTP are not of '=' type but some are of '$\leq$' type and some are of '$\geq$' type then transportation problem (TP) is said to be transportation problem with mixed constraints (TP-MC). Pandian and Natarajan [4] have developed a method for solving transportation problem with mixed constraints.

Some researchers had tried to further reduce the optimal transporting cost of TP, if possible, by allowing sources and destinations to act as transhipment point. Meaning that transportation is allowed from source to source, destination to destination, destination to source; respective from source to destination. In TP-MC if all the sources and destinations are allowed to act as transhipment point then the problem is said to be transhipment problem with mixed constraints (TsP-MC). Kumari and Kumar [3] have developed a method for solving TsP-MC. In TP-MC if time of transporting is given in place of per unit transporting cost and the objective is to minimize the maximum time then the problem is said to be time minimizing transportation problem with mixed constraints. Agarwal and Sharma, [5] have developed a shootout method for solving Time minimizing transportation problem. Sharma [6] explored that per unit transporting cost from a source to a destination in TP is not always constant, possibly reduces if supplied heavy amount of commodity. This idea leads the formulation of quadratic transportation problem in which cells of transportation table contains quadratic function of the amount supplied instead of per unit transporting cost. A maximin zero suffix method has been developed for solving it by

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Sharma [6]. Other non-linear transportation problem (NTP) was developed by Acharya [1] considering depreciation and profit on per unit transported goods also. The objective function in this situation becomes the sum of a linear function and a fractional function resulting a nonlinear objective function. They have developed a method to find more for less solution but not for its optimal solution.

In the proposed paper an attempt has been made to find an optimal solution of NTP. Basic idea to obtain optimal solution of NTP is to search firstly a basic feasible solution and then improve it again and again for getting an optimal solution. It takes too much time to improve worst basic feasible solution until an optimal solution is achieved The search of optimal solution becomes easier if a good basic feasible solution near to optimal is available. But unfortunately, there does not exist a method for getting a solution near to optimal. Adopting this idea, we have developed an intuitive method for getting an optimal solution directly without using any basic feasible solution. Rest of paper is organized as follows: Section 2 contains a mathematical model of NTP followed by tabular form. An algorithm followed by intuitive approach, illustrated with numerical example, is given in Section 3. At last paper ends with conclusion and references.

2. Mathematical Representation

Consider the following notations to represent the NTP of m sources O_i, $i = 1, 2, \dots, m$ and n destinations D_j, $j = 1, 2, 3, \dots, n$.

x_{ij} = amount of goods transporting from O_i to D_j.

c_{ij} = per unit transporting cost of goods from O_i to D_j.

d_{ij} = depreciation in per unit of transported goods from O_i to D_j.

p_{ij} = profit in per unit of transported goods from O_i to D_j.

a_i = supplying capacity of O_i.

b_j = demand at D_j.

These terminologies can be visualized in NTP table 1. The mathematical representation of NTP is to find x_{ij} which

	D_1	$\dots$	D_j	$\dots$	D_n	Supply
O_1	c_{11} d_{11} (x_{11}) p_{11}	$\dots$	c_{1j} d_{1j} (x_{1j}) p_{1j}	$\dots$	c_{1n} d_{1n} (x_{1n}) p_{1n}	a_1
$\vdots$		$\dots$		$\dots$		
O_i	c_{i1} d_{i1} (x_{i1}) p_{i1}	$\dots$	c_{ij} d_{ij} (x_{ij}) p_{ij}	$\dots$	c_{in} d_{in} (x_{in}) p_{in}	a_i
$\vdots$		$\dots$		$\dots$		
O_m	c_{m1} d_{m1} (x_{m1}) p_{m1}	$\dots$	c_{mj} d_{mj} (x_{mj}) p_{mj}	$\dots$	c_{mn} d_{mn} (x_{mn}) p_{mn}	a_m
Demand	b_1		b_j		b_n	

Table 1. **NTP Table**

$$Minimize\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \frac{\left(\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}x_{ij}\right)^2}{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}x_{ij}}$$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i; \quad i = 1, 2, \dots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j; \quad j = 1, 2, \ldots, n$$

$$x_{ij} \geq 0$$

3. Intuitive Approach

We are approaching towards allocating the cells one by one satisfying rim requirements until all the demands and supplies exhausted. The crux of the paper is to search a cell (active cell) to allocate so as to approach towards optimal solution. Observing the nature of the objective function of the problem, it is clear that it is an algebraic combination of three functions linear, quadratic and linear. Their gradients are $[c_{ij}]$, $[2\,(d_{ij})]$ and $[p_{ij}]$ at $[x_{ij}]$ which plays an important role to choose an active cell, smaller the first two gradient and larger the third one corresponding to a cell may be an active cell in the sense of minimizing the objective function. As the second gradient dominate all of the other two so the preference would be given firstly to a cell corresponding to which it is minimum. There may be possible that a tie of cell may occur corresponding to this minimum gradient. If so then tie is broken corresponding to a cell with maximum of the third gradient as it is of profit function existing in the denominator of objective function. Further tied cells, if exists, is broken by minimum of the first gradient. These Intuitive idea lead to the foundation stone of selecting an active cell. Accordingly, step 1 of the algorithm is developed for searching an active cell.

Once an active cell is observed it is allocated with minimum of supplying capacity and demand limit of corresponding source and destination respectively and the remaining cells in its row/column whose supply/demand are exhausted, are blocked. Again, an active cell among the unblocked cell is searched and allocated in the same manner. Selection and allocation of active cell continues until all the demands and supplies are exhausted, resulting an optimal solution is achieved. These ideas are compacted in the form of an algorithm to make easy to apply in solving the problem.

Algorithm

Step 1: Find active cell(u, v) in transportation table, where $C_{uv} = \min\limits_{q,t}\left[c_{qt} : p_{qt} = \left\{\max\limits_{r,s}\left(p_{rs} : d_{rs} = \min\limits_{i,j} d_{ij}\right)\right\}\right]$.

Step 2: Allocate the amount $\min\{a_u,\ b_v\}$ in active cell and update the rim requirements.

Step 3: Block the cells of its row/column whose rim requirements are exhausted.

Step 4: Check if all supply and demand exhausted. If yes, go to Step 5, otherwise go to Step 1 for unblocked cells.

Step 5: Find the Current flow with optimal transporting cost.

Example 3.1. *Consider a NTP with 3 sources and 4 destinations to illustrate the algorithm, which is taken from [1].*

	D_1	D_2	D_3	D_4	a_i
O_1	15 11 ( ) 12	7 5 ( ) 6	16 13 ( ) 10	8 5 ( ) 7	10
O_2	6 2 ( ) 8	25 20 ( ) 13	18 14 ( ) 11	21 16 ( ) 12	9
O_3	30 23 ( ) 16	9 6 ( ) 9	10 8 ( ) 6	14 9 ( ) 12	10
b_j	6	8	12	3	

Iteration 1

Step 1: $i = 1, 2, 3$; $j = 1, 2, 3, 4$; $(r, s) = (2, 1)$; $(q, t) = (2, 1)$; $(u, v) = (2, 1)$ corresponds to Active cell (2,1) i.e. $(O_2,\ D_1)$.

Step 2: Maximum possible allotment in active cell is $x_{21} = 6$.

Step 3: Blocked cells are shown by grey colour.

	D_1	D_2	D_3	D_4	a_i
O_1	15 11 ( ) 12	7 5 ( ) 6	16 13 ( ) 10	8 5 ( ) 7	10
O_2	6 2 (6) 8	25 20 ( ) 13	18 14 ( ) 11	21 16 ( ) 12	~~9~~ 3
O_3	30 23 ( ) 16	9 6 ( ) 9	10 8 ( ) 6	14 9 ( ) 12	10
b_j	~~6~~	8	12	3	

Step 4: All supply/demand are not exhausted.

Iteration 2

Step 1: $i = 1, 2, 3$; $j = 2, 3, 4$; $(r, s) = \{(1, 2),\ (1, 4)$; $(q, t) = (1, 4)$; $(u, v) = (1, 4)$ corresponds to active cell (1,4) i.e. $(O_1,\ D_4)$.

Step 2: Maximum possible allotment in active cell is $x_{14} = 3$.

Step 3 Blocked cells are shown by grey colour.

	D_1	D_2	D_3	D_4	a_i
O_1	15 11 ( ) 12	7 5 ( ) 6	16 13 ( ) 10	8 5 (3) 7	~~10~~ 7
O_2	6 2 (6) 8	25 20 ( ) 13	18 14 ( ) 11	21 16 ( ) 12	~~9~~ 3
O_3	30 23 ( ) 16	9 6 ( ) 9	10 8 ( ) 6	14 9 ( ) 12	10
b_j	~~6~~	8	12	~~3~~	

Step 4: All supply/demand are not exhausted.

Iteration 3

Using Step 1 to Step 4 allotment of current active cell and blocked cell are shown in table.

	D_1	D_2	D_3	D_4	a_i
O_1	15 11 ( ) 12	7 5 (7) 6	16 13 ( ) 10	8 5 (3) 7	~~10~~ ~~7~~
O_2	6 2 (6) 8	25 20 ( ) 13	18 14 ( ) 11	21 16 ( ) 12	~~9~~ 3
O_3	30 23 ( ) 16	9 6 ( ) 9	10 8 ( ) 6	14 9 ( ) 12	10
b_j	~~6~~	~~8~~ 1	12	~~3~~	

Iteration 4

Using Step 1 to Step 4 allotment of current active cell and blocked cell are shown in table.

	D_1	D_2	D_3	D_4	a_i
O_1	15 11 ( ) 12	7 5 (7) 6	16 13 ( ) 10	8 5 (3) 7	~~10~~ ~~7~~
O_2	6 2 (6) 8	25 20 ( ) 13	18 14 ( ) 11	21 16 ( ) 12	~~9~~ 3
O_3	30 23 ( ) 16	9 6 (1) 9	10 8 ( ) 6	14 9 ( ) 12	~~10~~ 9
b_j	~~6~~	~~8~~ ~~1~~	12	~~3~~	

Iteration 5

Using Step 1 to Step 4 allotment of current active cell and blocked cell are shown in table.

	D_1	D_2	D_3	D_4	a_i
O_1	15 11 ( ) 12	7 5 (7) 6	16 13 ( ) 10	8 5 (3) 7	~~10~~ ~~7~~
O_2	6 2 (6) 8	25 20 ( ) 13	18 14 ( ) 11	21 16 ( ) 12	~~9~~ 3
O_3	30 23 ( ) 16	9 6 (1) 9	10 8 (9) 6	14 9 ( ) 12	~~10~~ ~~9~~
b_j	~~6~~	~~8~~ ~~1~~	~~12~~ 3	~~3~~	

Iteration 6

Using Step 1 to Step 3 allotment of current active cell and blocked cell are shown in table.

	D_1	D_2	D_3	D_4	a_i
O_1	15 11 ( ) 12	7 5 (7) 6	16 13 ( ) 10	8 5 (3) 7	~~10~~ ~~7~~
O_2	6 2 (6) 8	25 20 ( ) 13	18 14 (3) 11	21 16 ( ) 12	~~9~~ ~~3~~
O_3	30 23 ( ) 16	9 6 (1) 9	10 8 (9) 6	14 9 ( ) 12	~~10~~ ~~9~~
b_j	~~6~~	~~8~~ ~~1~~	~~12~~ ~~3~~	~~3~~	

Step 4: All supply and demand are exhausted.

Step 5: obtained optimal solution is

$$\begin{aligned} Min\ Z &= [(7\times 7)+(8\times 3)+(6\times 6)+(18\times 3)+(9\times 1)+(10\times 9)] \\ &+\frac{[(5\times 7)+(5\times 3)+(2\times 6)+(14\times 3)+(6\times 1)+(8\times 9)]^2}{[(6\times 7)+(7\times 3)+(8\times 6)+(11\times 3)+(9\times 1)+(6\times 9)]} \\ Min\ Z &= 262+\frac{182^2}{207} \\ Min\ Z &= 422.02 \end{aligned}$$

Note that this optimal solution is the same as that of [1].

4. Conclusion

A method has been developed for finding an optimal solution of nonlinear transportation problem in the form of an algorithm. In this method there is no need of searching initial basic feasible solution and improving it again and again to get an optimal solution. Method is very simple easy to understand and apply. This approach may be extended to solve a non linear transportation problem with mixed constraints.

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