Int. J. Math. And Appl., 9(1)(2021), 51–61
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Optimal Ordering and Screening Policy for Perishable Items with Imperfect Quality

Puspita Mahata[1], Sujit Kumar De[2,*] and Gour Chandra Mahata[3]

1 Department of Commerce, Srikrishna College, Bagula, Nadia, West Bengal, India.

2 Department of Mathematics, Midnapore College, Midnapore, West Bengal, India.

3 Department of Mathematics, Sidho-Kanho-Birsha University, Purulia, West Bengal, India.

Abstract: In reality, perishable products also contain defective ones before storage due to uncertainties during production process. In order to reduce the loss caused by the defective products, an inspection process is implemented by retailers. However, the screening is not always perfect due to the limited technical level. In view of the above, a retailer's perishable inventory model with defective items is established in this article. Two types of inspection errors, non-100% screening and sales returns are all considered in the proposed model. The objective of the model is to minimize the retailer's expected cost per unit time and to find the optimal retail price, the screening rate and the order quantity. Numerical example and sensitivity analysis are provided to illustrate the presented model and provide some managerial insights.

Keywords: Inventory strategy, Deteriorating items, Defective products, Screening.

1. Introduction

Some items could not always preserve their physical characteristics during their staying in the warehouse, such as fresh fruits, vegetables, drugs, volatile liquids and fashion goods. The quantity or the value of these products may decline over time, which is called the deterioration phenomena. The direct impacts caused by the deterioration are the cost loss and resource waste. In China, nearly ninety percent of fruits and vegetables without pre-cooling are transported to the downstream of the supply chain. The damage rate of goods is as high as 25%∼30% in the storage, which led to an annual economic loss of $1 billion (data source: "Report on the Investment Strategy of China's Cold Chain Logistics Market From 2017 to 2022") Therefore, in order to reduce the risk of cargo damage and maintain the cost advantage of the supply chain, inventory models considering the deteriorating items have been increasingly concerned by many scholars in these years.

Yang & Wee [1] discussed an integrated production-inventory system including one producer and multi-retailers, and established a mathematical model for a single perishable product in which the deteriorating rate is assumed to be a constant. Wang [2] introduced a linear time-varying deteriorating rate into a three-echelon inventory model to examine how the integrated decision was affected by the different deteriorating rate. Sarkar [3] proposed a function of time and the shelf life to depict the deteriorating rate of products in an EOQ model. Subsequently the uniform distribution, triangular distribution and beta distribution were mployed to describe the deterioration function respectively by Sarkar [4], Chaudhary & Sharma [5].

* *E-mail: skdemamo2008.com@gmail.com*

Mishra [6] all assumed that the deteriorating rate follows a two-parameter *Weibull* distribution in the study of perishable products EOQ. Similar studies on the deterioration function and the deteriorating inventory model could be found in the review documents Bakker [7] and Janssen [8].

Deteriorating inventory researches on the different types of the deteriorating rate showed by multiple perishable products have been quite rich. Some scholars also studied the inventory models with different decision mode, different demand, etc. (Tat [9]; Dash [10]; Kumar and Singh, [11]; Pandey [12]). However, some assumptions of these deteriorating inventory models are idealistic and unrealistic in nature. One of such assumptions is that the quality of the deteriorating items is always perfect. In reality, there are various uncertainties in the production process, which would cause the fluctuation in the quality of goods. For instance, some imperfect female fashions, such as those with the inaccurate logo, with the extra threads and with other minor problems, are produced due to workers' carelessness or machine stoppage.

Considering the obvious time effect of female fashions, these defective goods will often be discounted by retailers.

With this realistic consideration in mind, several scholars studied the deteriorating production/ inventory systems with imperfect items over the past decade. Guchhait [13] formulated different EPQ models with different demand rate for a breakable product whose quality is not uniform during the whole production process. The optimal reliability indicator of the production process and the optimal production strategy are obtained for the maximization of the profit. Lee & Kim [14] nalyzed the joint optimization problem of the production and distribution in a two-echelon supply chain. A single perishable product with the constant deteriorating rate and the constant defect rate is used as the research object. Jaggi [15] established a two-warehouse inventory model for the perishable items in which the percentage of defective items subjects to a known probability distribution. Alamri [16] introduced a learning curve to describe the relationship between the defect rate of the products and the replenishment batch, and presented a general deteriorating inventory model with the object of minimizing the total cost. Although the defect and deterioration factors are considered at the same time in a few papers, they all have overlooked the fact that the deterioration phenomena also occurs during the screening period. In other words, all the inventory models on the deteriorating and defective items mentioned above fail to account the deteriorated quantity of products in the inspection process, which would affect the length of the screening time and the accuracy of the inventory control.

Furthermore, another unrealistic assumption in above mentioned studies is that the screening process of defective products is perfect and the 100% screening is conducted. Contrary to these hypotheses, inspectors may erroneously divide the quality grade of goods during the actual screening process (Pickard [17]). The human errors could classify a good quality product as defective (Type I) or classify a defective product as non-defective (Type II). The imperfect products arising from the second type of error will be sold to customers, which directly lead to the return of goods and the decline of customer satisfaction. Meanwhile, owing to the deterioration of products during the screening period, the real amount of inspection is not the whole ordering quantity. The screening operation does not need to be conducted for some goods which had been deteriorated. Hsu & Hsu [18] developed one buyer's EOQ model with defective items and two types of inspection errors (Type I and Type II). The situations that the shortages are allowed and are not allowed are analyzed respectively. The results of the sensitivity analysis showed that the defective probability and the probability of Type I and Type II not only have a reverse impact on the optimal replenishment policy but also have a negative impact on buyer's profit. Yoo [19], Khan [20] (2011), Hsu & Hsu [21] and Jauhari [22] also made similar assumption on the screening error in different EPQ models. However, these studies are based on the normal temperature products, and they don't involve the deteriorating inventory system. Chen [23] synthetically analyzed the rework, pricing and replenishment strategy for multiple perishable products with imperfect quality. A two-layer mathematical model assuming constant deteriorating rate and constant defect rate was established to maximize the profit. Nevertheless, only a single screening error (Type II) was researched, and it failed to research the disposal of defective stock and the non-100% inspection activity in the presented model.

In view of the shortcomings of previous studies, this paper takes into account a retailer's inventory model with price dependent demand. The products not only deteriorate over time but also have a certain proportion of quality defects. The deteriorating rate and the defect rate of products subject to a two-parameter *Weibull* distribution and a known random distribution respectively. In order to reduce the impact of defective items on the goodwill, a non-100% screening is conducted by the retailer at a certain rate. The first and second types of errors occur during the screening process, and the probability of the errors also follows a known probability distribution. As pointed out earlier, defective products not screened which are sold to customers will be returned to the retailer in full price and then be discounted together with defective products screened at the end of the next screening period. The aim of this article is to determine the optimal retail price, the screening rate and the replenishment strategy so as to minimize the expected cost per unit time of the retailer.

The remainder of the paper is designed as follows. In Section 2, the inventory model for the perishable products with imperfect quality is established. Meanwhile, the notations and assumptions involved in the model are also presented in this section. The result of a numerical example is shown in Section 3 so as to demonstrate the feasibility of the proposed model. Subsequently, Section 4 introduces the sensitivity analysis on three parameters to research the impacts of the defect rate and screening errors on the inventory system. Finally, conclusions and the future research direction are made in Section 5.

2. Formulation of the Inventory Model

2.1. Environment setting and notations

The joint replenishment and screening policy of a single retailer is discussed in this paper. A single product with the deteriorating rate of a two-parameter *Weibull* distribution is ordered. There are some defects in the production process, which leads to the existence of the imperfect quality in the ordered products. The defect rate follows a known probability distribution. An inspection is conducted by the retailer as soon as the goods are transported to the warehouse. However, products that have already deteriorated will not be screened. The inspection cost depends on the length of the screening time. Two types of errors (Type I and Type II) exist in the screening process, and the probabilities of the errors are random. The imperfect products customers bought will be returned to the retailer. Then the inventory of the retailer is divided into three parts, which are used to store the good quality products, the screened defective products and the defective products returned by customers separately. The screening process and the demand proceed simultaneously, but shortages are not allowed. Considering the short screening process, the deterioration quantity of the defective products during the screening process is accounted preferentially.

Then the notations used in the proposed model are as follows:

Q the replenishment quantity per order.

x the screening rate.

$\theta(t)$ the deterioration rate, $\theta(t) = \alpha\beta t^{\beta-1}$, where α is the scale parameter and β is the shape parameter, $\alpha > 0$, $\beta > 1$.

D the demand rate.

Y the defect rate that subjects to a probability distribution with the desired value $E(Y) = y$.

M_1 the probability of Type I error (classifying a good product as defective), the desire value is $E(M_1) = m_1$.

M_2 the probability of Type II error (classifying a defective product as good), the desire value is $E(M_2) = m_2$.

c the purchasing cost per unit.

A the ordering cost per order.

h the holding cost per unit per unit time for good products (Unscreened products are regarded as good products).

h_s the holding cost per unit per unit time for defective products.

c_1 the cost of rejecting an non-defective product.

c_2 the cost of accepting a defective product.

ρ the screening cost coefficient.

Decision variables:

T the replenishment cycle time.

t_s the screening period.

2.2. The mathematical model

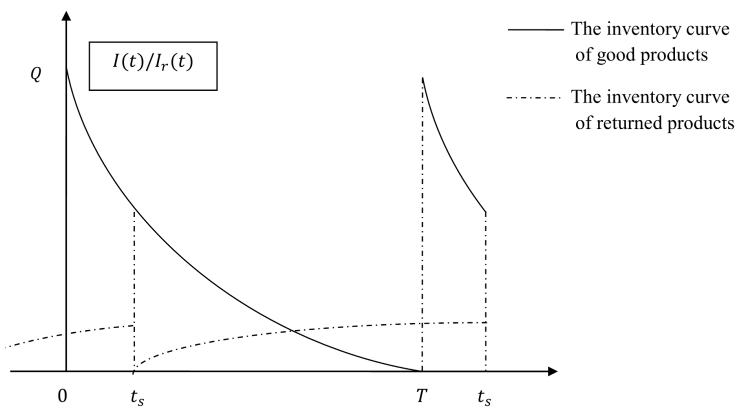

Figure 1. **The inventory curves of the good products and the returned**

Once the products arrive at the warehouse, the retailer begins to screen all the non-deteriorated products until $t = t_s$. The inventory curve of good products is shown by the full line $I(t)$ in Figure 1, and the inventory curve of defective products is shown in Figure 2. By definition, although the actual defect rate of products is y, the defect rate detected by the retailer is $y_r = (1-y)\, m_1 + y(1-m_2)$ due to the existence of two types of screening errors. In other words, there are still ym_2 defective products sold to customers. Hence, not only the screening of the defective products is completed but also the accumulation of the returned goods of the previous period is completed at the time of $t = t_s$. The broken line $I_r(t)$ in Figure 1 is the quantity of returned goods at time t. The accounting of three types of inventories are as follows:

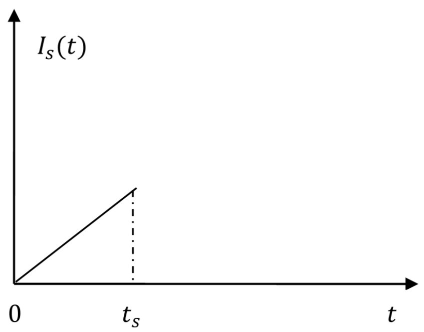

Figure 2. **The inventory curve of the defective products**

The inventory of the good products

As shown in Figure 1, the change of inventory level is driven by the deterioration, the consumption and the screening process simultaneously during the time $[0, t_s]$, which can be described by the following differential equation:

$$\frac{dI(t)}{dt} = -\theta(t) I(t) - D - x.y_r, \quad 0 \le t \le t_s \tag{1}$$

Using the boundary condition $I(0) = Q$, the inventory level of the retailer with time t in $[0, t_s]$ could be expressed as:

$$I(t) = e^{-\alpha t^\beta} \left[Q + (D + x.y_r) \int_t^0 e^{\alpha u^\beta} du \right], \quad 0 \le t \le t_s$$

Since it is quite difficult to obtain the analytical solution for the definite integral $\int_t^0 e^{\alpha u^\beta} du$, the Taylor series expansion is used to approximate the arithmetic solution drawing on the experience of Chen (2017). Let

$$A(t) = \int_t^0 \left[1 + \alpha u^\beta + \frac{(\alpha u^\beta)^2}{2!} \right] du = -t - \frac{\alpha}{\beta+1} t^{\beta+1} - \frac{\alpha^2}{2(2\beta+1)} t^{2\beta+1},$$

then we get

$$I(t) = e^{-\alpha t^\beta} \left[Q + (D + x y_r) A(t) \right], \quad 0 \le t \le t_s.$$

During the period of $[t_s, T]$, the inventory of the retailer is only affected by the demand rate and the deteriorating rate at the same time, the variation of inventory with time t is formulated by the following differential equation:

$$\frac{dI(t)}{dt} = -\theta(t) I(t) - D \tag{2}$$

Equation (2) satisfies the boundary condition: $I(T) = 0$, then we get the inventory level as follows:

$$I(t) = e^{-\alpha t^\beta} D \left[A(t) - A(T) \right], \quad t_s \le t \le T.$$

In summary, the inventory level of good items at time t is

$$I(t) = \begin{cases} e^{-\alpha t^\beta} \left[Q + (D + x y_r) A(t) \right], & 0 \le t \le t_s \quad (3) \\ e^{-\alpha t^\beta} D \left[A(t) - A(T) \right], & t_s \le t \le T \quad (4) \end{cases}$$

Equation (3) and Equation (4) intersect at $t = t_s$, thus the order quantity per order can be obtained:

$$Q = -DA(T) - x y_r A(t_s).$$

Then the total inventory of good products during the time $[0, T]$ is:

$$\begin{aligned} I &= \int_0^{t_s} I(t) dt + \int_{t_s}^T I(t) dt \\ &= Q \int_0^{t_s} e^{-\alpha t^\beta} dt + (D + x y_r) \int_0^{t_s} e^{-\alpha t^\beta} A(t) dt + D \int_{t_s}^T e^{-\alpha t^\beta} \left[A(t) - A(T) \right] dt \\ &= D \int_0^T e^{-\alpha t^\beta} A(t) dt + x y_r \int_0^{t_s} e^{-\alpha t^\beta} A(t) dt - DA(T) \int_0^T e^{-\alpha t^\beta} dt - x y_r A(t_s) \int_0^{t_s} e^{-\alpha t^\beta} dt \end{aligned}$$

The inventory of the defective products screened

When $t = t_s$, the screening process is over. According to the environment settings mentioned earlier, the accounting of deterioration quantity for defective products is mainly from the unscreened defective products. Thus, the inventory level $I_s(t)$ of the defective items at time t can be derived as follows:

$$\frac{dI_s(t)}{dt} = xy_r.$$

It is easy to get

$$I_s\left(t\right) = xy_r t, \quad 0 \leq t \leq t_s \tag{5}$$

It is worth noting that the screening process of the retailer is not 100%, only non-deteriorated products are inspected during the time $[0, t_s]$. Hence, the equation on the screening quantity is:

$$xt_s = Q - \int_0^{t_s} \theta\left(t\right) I(t)dt.$$

Then through the equation substitution, the optimal screening rate of the retailer is obtained

$$x = \frac{-DA\left(T\right)e^{-\alpha t_s^{\beta}} - \alpha\beta B(t_s)D}{t_s + y_r e^{-\alpha t_s^{\beta}} A\left(t_s\right) + \alpha\beta y_r B(t_s)}$$

where

$$B\left(t_s\right) = \int_0^{t_s} t^{\beta-1} e^{-\alpha t^{\beta}} A(t)dt.$$

The total inventory of the defective products in $[0, T]$ is

$$I_s = \int_0^{t_s} I_s\left(t\right)dt = \frac{1}{2}xy_r t_s^2.$$

The inventory of the defective products returned

The unscreened defective products of the previous period are returned by customers to the retailer, which would be handled together with the screened defective items at the end of the inspection. Thus, during the period of $[0, T]$, the inventory of the returned products $I_r(t)$ is driven by the combined effects of the deteriorating rate and the return rate, which can be depicted by the differential equation:

$$\frac{dI_r(t)}{dt} = m_2 yD - \theta\left(t\right) I_r(t) \tag{6}$$

Eq. (6) satisfies the boundary condition: $I_r\left(0\right) = 0$, and then the inventory of the returned products at time t is solved as:

$$I_r\left(t\right) = -m_2 yDe^{-\alpha t^{\beta}} A\left(t\right), \quad 0 \leq t \leq T \tag{7}$$

The total inventory of the returned products in [0,T] is:

$$I_r = \int_0^{T} I_r\left(t\right)dt = -m_2 yD \int_0^{T} e^{-\alpha t^{\beta}} A(t)dt.$$

Finally, in a replenishment cycle, the holding cost, the deterioration cost, the purchasing cost, the screening cost and the cost of the screening errors are involved in the total cost of the retailer. The costs are as follows:

The holding cost (HC): $HC = hI + h_s\left(I_r + I_s\right)$

The purchasing cost (PC): $PC = cQ$

The deterioration cost (DC): $DC = c\left(Q - DT - xy_r t_s\right) + c\left[m_2 yDT - I_r(T)\right]$

The Type I cost: $MC_1 = c_1 x(1-y)t_s m_1$

The Type II cost: $MC_2 = c_2 xyt_s m_2$

The screening cost: $IC = \frac{1}{2}\rho\left(\frac{T}{t_s}\right)^2$

Since the defect rate Y and the probabilities of Type I error M_1 and Type II error M_2 are random variables with known probability distribution, the expected cost per unit time of the retailer is:

$$E\left(\pi\right) = \frac{1}{T}\left[A + E\left(HC\right) + E\left(PC\right) + E\left(DC\right) + E\left(MC_1\right) + E\left(MC_2\right) + E(IC)\right].$$

Then the optimal decision variables of the proposed model can be obtained by the following objective function and the constraint conditions.

$$\min \quad E\left(\pi\left(t_s, T\right)\right)$$

such that

$$\begin{cases} I\left(t\right) \geq 0 \\ x(1-y_r) \geq D \\ t_s \leq \frac{Q}{x} \\ 0 \leq \frac{t_s}{T} \leq 1 \end{cases}$$

In order to get the optimal replenishment and screening policy, the expected cost per unit time of the retailer is employed as the decision making basis. The first constraint restricts that there is no shortage in the inventory of the retailer. Meanwhile, the second constraint indicates that the screened good products per unit time are greater than the unit demand when the screening process and the demand proceed simultaneously. The third inequality denotes that the actual screening quantity is less than the replenishment quantity. And the last inequality is the constraint of the parameter value.

3. Numerical Example

In this section, a numerical example is utilized to illustrate the applicability of the model. The data of this example are showed in Table 1.

Notation	Value
The demand rate, D	137 units/day
The ordering cost, A	\$50/order
The holding cost of non-defective items, h	\$0.012/unit/day
The holding cost of defective items, h_s	\$0.006/unit/day
The purchasing cost, c	\$25/order
The cost of rejecting a non-defective items, c_1	\$50/unit
The cost of accepting a defective items, c_2	\$200/unit
The coefficient of the screening cost, ρ	500
The coefficient of the deterioration rate, α	0.01
The coefficient of the deterioration rate, β	1.05
The probability of Type I error, m_1	0.04
The probability of Type II error, m_2	0.04
The defective probability, y	0.04

Table 1. **Parameters in the numerical test**

The optimal solutions of the example are as follows: the optimal replenishment cycle $T^* = 4$, the optimal order quantity is $Q^* = 605$, the optimal screening time and the screening rate are $t_s^* = 3$ and $x^* = 198.14$, respectively. At this time, the minimum cost is \$4312.42.

4. Sensitivity Analysis

In real life, with the purpose of minimizing costs, retailers should take reasonable management measures to deal with changes in the market environment or changes in their internal factors. Thus, in order to analyze the effects of the fluctuations of some parameters and get some relevant management inspiration, a sensitivity analysis is conducted on the main parameters: the coefficient of the deterioration rate β, the defect rate y, the probability of Type I error m_1 and the probability of Type II error m_2. The results of sensitivity analysis are shown in Table 2, Table 3 and Table 4, respectively.

β	y	$[T^*, t_s^*]$	Q^*/units	x^*/units/day	HC/\$	PC/\$	MC_1/\$	MC_2/\$	$E(\pi)$/\$
1.05	0.03	[4,3]	600	196.18	14.43	305.70	1141.79	141.25	4274.41
	0.04	[4,3]	605	198.14	14.56	284.20	1141.28	190.21	312.42
	0.05	[4,3]	612	200.13	14.76	309.73	1140.76	240.16	4374.96
2.05	0.03	[3,2]	454	222.26	8.17	324.52	862.37	106.68	421.42
	0.04	[3,2]	459	224.48	8.27	339.03	861.98	143.66	4480.15
	0.05	[3,2]	463	226.73	8.34	326.31	861.59	181.39	4521.71
3.05	0.03	[2,1]	299	298.13	3.54	121.65	578.38	71.55	4650.06
	0.04	[2,1]	302	301.10	3.57	122.54	578.12	96.35	4700.29
	0.05	[2,1]	305	304.14	3.60	121.93	577.86	121.65	4750.02

Table 2. **The optimal solution at different defect rate y and coefficient of the deterioration rate β**

As can be seen from Table 2, with the rise of β, the order quantity Q, inventory cost HC, Type I cost MC_1, Type II cost MC_2, replenishment cycle time T and the screening period t_s all show a substantial decline trend. On the contrary, the total cost and the screening rate x show a clear upward trend. From the formula of the deterioration rate θ, it is easy to know that with the rise of β, the degree of sensitivity of θ at the time t also increases, which means the deterioration rate θ will grow faster. In order to avoid the increase of the number of deterioration products caused by the increase of θ, retailer will shorten the replenishment cycle time and by the same token, the screening time will be shortened. Although the decreasing amount of two parameters is same, the ratio of t_s to T has been declining with the rise of β. That is why the screening rate shows an increasing trend when the replenishment quantity decreases. The shortening of the replenishment cycle leads to a reduction in aggregate demand during one cycle, and thus retailer tends to reduce the order quantity Q to avoid over-supply and high deterioration loss. Meanwhile, many costs associated with Q, including the Type I cost MC_1 and the Type II cost MC_2, show a substantial decline. However, the total cost is rising with the increase of β. Therefore, retailers should pay attention to the commodities with high deteriorating rate and take some measures to maintain the deterioration rate within a certain range.

It is also illustrated in Table 2 that with the increase of defective probability y, the total cost, order quantity Q and the Type II cost MC_2 all show a clear upward trend while the replenishment cycle time T and the screening period t_s do not change. The increase in defective probability y will lead to the rise of defective products and the reduction of quality products. Thus, in order to meet the market demand, the retailer would increase the order quantity. And the number of defective products that are mistaken classified as perfect quality also increases caused by the increase of defective goods, which leads to a increase of the number of returned goods, this is the reason for the rise of Type II cost MC_2. Therefore, the defective probability is also a parameter which should be focused by retailers in cost control.

As can be seen from Table 3, with the rise in the probability of Type I error m_1, the total cost, order quantity Q, Type I cost MC_1 and the screening rate x all show an upward trend. The increase of the probability of Type I error m_1 causes the reduction of the quantity of available goods. The retailer has to increase the order quantity Q to prevent out of stock. At the same time, the Type I cost MC_1 shows a significant increase, which is also the reason for the increase in total costs, but the other parameters have only a small change.

m_1	m_2	$[T^*, t_s^*]$	Q^*/units	x^*/units/day	PC/\$	MC_1/\$	MC_2/\$	$E(\pi)$/\$
0.02	0.02	[4,3]	594	194.27	299.35	559.49	93.25	4077.70
	0.04	[4,3]	593	194.10	286.95	559.01	186.34	4091.50
	0.06	[4,3]	593	193.94	299.54	558.54	279.27	4117.76
0.04	0.02	[4,3]	606	198.31	296.07	1142.27	95.19	4298.14
	0.04	[4,3]	605	198.14	284.20	1141.28	190.21	4312.42
	0.06	[4,3]	605	197.97	297.30	1140.29	285.07	4339.17
0.06	0.02	[4,3]	619	202.53	304.90	1749.83	97.21	4534.08
	0.04	[4,3]	618	202.35	293.58	1748.28	194.25	4548.87
	0.06	[4,3]	618	202.17	307.23	1746.73	291.12	4576.11

Table 3. **The optimal solution at different defect rate y and coefficient of the deterioration rate β**

From Table 3, it is not difficult to find that the total cost and the Type II cost MC_2 increase with the rise in the probability of Type II error m_2. The increase of m_2 means the number of defective products that are mistaken classified as perfect quality will increase, and then the quantity of returned goods will also rise. That is why there is a significant increase in the Type II cost. In addition, An interesting phenomenon is that, unlike m_1, the change in m_2 does not cause a significant change in the order quantity Q. From the above analysis and the data in the table, it can be seen that the probability of Type I error m_1 has a greater effect on the total cost than the probability of Type II error m_2. Therefore, retailer should be more vigilant about the occurrence of Type I error in cost control.

Parameter / Indicator	β	y	m_1	m_2
Rate – 1 (%)	190	67	200	200
Rate – 2 (%)	9	2	11	0.9
Rate – 3	0.05	0.03	0.06	0.0045

Table 4. **The growth rate of parameters and total cost**

Through a comprehensive analysis of Table 2 and Table 3, some parameters' variation rates could be obtained as shown in Table 4. The Rate-1 and Rate-2 represent the rate of increase of the parameters and the rate of increase of the total cost, respectively. And the Rate-3 is the ratio between the Rate-2 and the Rate-1, which indicates the size of the cost affected. Obviously the change in m_1 and β have a significant impact on the total cost. Hence, retailers should pay more attention to the probability of Type I error m_1, and the deterioration rate in daily perishable inventory management. Some measures, such as increase investment in preservation technology and the screening technology, outsource the warehousing activates, or increase penalties for inspection errors, could be employed to control the value of these two parameters. On this basis, the number of occurrence of Type II error also requires attention by managers.

5. Conclusion

The characteristics of deterioration and imperfect quality for a single product are taken into account in the inventory system of one retailer. The deteriorating rate of the product is assumed to be a two-parameter *Weibull* function which is superior to the hypotheses of constant or linear function in previous researches. Meanwhile, considering the fact that product was deteriorating all the time, non-100% inspection is conducted by the retailer. This differentiates our research from others. In the inspection process, two types of screening errors (Type I and Type II) are also considered in the inventory system, whose probabilities are random variables with known probability distribution. Then a mathematical model of joint replenishment, pricing and screening policy is established based on the above preconditions. The optimal solutions are obtained with the objective of minimizing the expected cost per unit time.

In our study, a numerical example is utilized to demonstrate the feasibility of the proposed model. And through the sensitivity analysis, it can be found that although the defect rate and the screening errors are able to affect the inventory strategy and the cost of the retailer, the deteriorating rate and the probability of Type I error have a significant impact on the total cost. Managers can reduce the loss caused by the increase of the deteriorating rate and the probability of Type I error through applying the relevant technology. In addition, some mechanisms between the retailers and the manufactures could be presented to improve the quality of perishable products. Hence, the investment in the quality of goods and the related contracts should be considered into the future research.

References

[1] P. C. Yang and H. M. Wee, *An integrated multi-lot-size production inventory model for deteriorating item*, Computers & Operations Research, 30(5)(2003), 671-682.

[2] K. J. Wang, Y. S. Lin and C. P. Jonas, *Optimizing inventory policy for products with time-sensitive deteriorating rates in a multi-echelon supply chain*, International Journal of Production Economics, 130(1)(2011), 66-76.

[3] B. Sarkar, *An EOQ model with delay in payments and time varying deterioration rate*, Mathematical and Computer Modelling, 55(3)(2012), 367-377.

[4] B. Sarkar, *A production-inventory model with probabilistic deterioration in two-echelon supply chain management*, Applied Mathematical Modelling, 37(5)(2013), 3138-3151.

[5] R. R. Chaudhary and V. Sharma, *A model for Weibull deteriorate items with price dependent demand rate and inflation*, Indian Journal of Science and Technology, 8(10)(2015), 975-981.

[6] U. Mishra, *An EOQ model with time dependent Weibull deterioration, quadratic demand and partial backlogging*, International Journal of Applied and Computational Mathematics, 2(4)(2016), 545-563.

[7] M. Bakker, J. Riezebos and R. H. Teunter, *Review of inventory systems with deterioration since 2001[J]*, European Journal of Operational Research, 221(2)(2012), 275-284.

[8] L. Janssen, T. Claus and J. Sauer, *Literature review of deteriorating inventory models by key topics from 2012 to 2015*, International Journal of Production Economics, 182(2016), 86-112.

[9] R. Tat, A. A. Taleizadeh and M. Esmaeili, *Developing economic order quantity model for non-instantaneous deteriorating items in vendor-managed inventory (VMI) system*, International Journal of Systems Science, 46(7)(2015), 1257-1268.

[10] B. P. Dash, T. Singh and H. Pattnayak, *An inventory model for deteriorating items with exponential declining demand and time-varying holding cost*, American Journal of Operations Research, 4(1)(2014).

[11] S. Kumar and A. K. Singh, *Optimal time policy for deteriorating items of two-warehouse inventory system with time and stock dependent demand and partial backlogging*, Sādhanā, 41(5)(2016), 541-548.

[12] R. Pandey, S. Singh and B. Vaish, *An EOQ model with quantity incentive strategy for deteriorating items and partial backlogging*, Uncertain Supply Chain Management, 5(2)(2017), 135-142.

[13] P. Guchhait, M. K. Maiti and M. Maiti, *Production-inventory models for a damageable item with variable demands and inventory costs in an imperfect production process*, International Journal of Production Economics, 144(1)(2013), 180-188.

[14] S. Lee and D. Kim, *An optimal policy for a single-vendor single-buyer integrated production–distribution model with both deteriorating and defective items*, International Journal of Production Economics, 147(1)(2014), 161-170.

[15] C. K. Jaggi, S. Tiwari and A. A. Shafi, *Effect of deterioration on two-warehouse inventory model with imperfect quality*, Computers & Industrial Engineering, 88(C)(2015), 378-385.

[16] A. A. Alamri, I. Harris and A. A. Syntetos, *Efficient inventory control for imperfect quality items*, European Journal of Operational Research, 254(1)(2016), 92-104.

[17] A. S. Pickard, H. W. Lin and M. D. Leon, *Impact of inspection errors on the performance measures of a general repeat inspection plan*, International Journal of Production Research, 43(23)(2005), 4945-4967.

[18] J. T. Hsu and L. F. Hsu, *An EOQ model with imperfect quality items, inspection errors, shortage backordering, and sales returns*, International Journal of Industrial Engineering Computations, 143(1)(2013), 162-170.

[19] S. H. Yoo, D. S. Kim and M. S. Park, *Economic production quantity model with imperfect-quality items, two-way imperfect inspection and sales return*. International Journal of Production Economics, 121(1)(2009), 255-265.

[20] M. Khan, M. Y. Jaber and M. Bonney, *An economic order quantity (EOQ) for items with imperfect quality and inspection errors*, International Journal of Production Economics, 133(1)(2011), 113-118.

[21] J. T. Hsu and L. F. Hsu, *Two EPQ models with imperfect production processes, inspection errors, planned backorders, and sales returns*, Computers & Industrial Engineering, 64(64)(2013), 389-402.

[22] W. A. Jauhari, *Integrated vendor-buyer model with defective items, inspection error and stochastic demand*, International Journal of Mathematics in Operational Research, 8(3)(2016), 342-359.

[23] T. H. Chen, *Optimizing pricing, replenishment and rework decision for imperfect and deteriorating items in a manufacturer-retailer channel*, International Journal of Production Economics, 183(2017), 539-550.

[24] G. Fauza, Y. Amer and S. H. Lee, *An integrated single-vendor multi-buyer production-inventory policy for food products incorporating quality degradation*, International Journal of Production Economics, 182(2016), 409-417.

[25] J. Zhang, G. Liu and Q. Zhang, *Coordinating a supply chain for deteriorating items with a revenue sharing and cooperative investment contract*, Omega, 56(2015), 37-49.