Int. J. Math. And Appl., 9(1)(2021), 67–72
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Cordial Labeling for Eight Sprocket Graph

J. C. Kanani[1,*] and V. J. Kaneria[2]

1 Government Polytechnic, Rajkot, Gujarat, India.

2 Department of Mathematics, Saurashtra University, Rajkot, Gujarat, India.

Abstract: We obtained a new graph which is called Eight Sprocket graph. We proved that the Eight Sprocket graph is cordial. We have investigated some Eight Sprocket graph related families of connected cordial graphs. We proved that path union of Eight Sprocket graph, cycle of Eight Sprocket graph and star of Eight Sprocket graph are cordial.

MSC: 05C78.

Keywords: Cordial labeling, Eight Sprocket graph, Path union of graphs, Cycle of graphs, Star of a graph.

1. Introduction

The concept of cordial labeling was introduced by Cahit [2] in 1987 and for numbering in graph was defined by S. W. Golomb [4]. It is found from Gallian [3] that many researchers have studied cordialness of several graphs. Large numbers of papers are found with variety of applications in coding theory, radar communication, cryptography etc. A depth details about applications of graph labeling is found in Bloom and Golomb [1] We have used all notations and terminology from Harary [5]. First of all let us recall some basic definitions, which are used in this paper.

Definition 1.1. *A function $f: V \to \{0,1\}$ is called binary vertex labeling of a graph G and $f(v)$ is called label of the vertex v of G under f. For an edge $e=(uv)$, the induced function $f^*: E \to \{0,1\}$ defined as $f^*(e) = |f(u)-f(v)|$. Let $vf(0)$, $vf(1)$ be number of vertices of G having labels 0 and 1 respectively under f and let $ef(0)$, $ef(1)$ be number of edges of G having labels 0 and 1 respectively under f^*. A binary vertex labeling f of a graph it is called cordial labeling if $\mid vf(0) - vf(1) \mid \leq 1$ and $\mid ef(0) - ef(1) \mid \leq 1$. A graph which admits cordial labeling is called cordial graph.*

Definition 1.2. *Let G be a graph and G_1,G_2,.....,G_n, $n \geq 2$ be n copies of graph G. Let $v \in V(G)$. Then the graph obtained by joining vertex v of $G^{(i)}$ with the same vertex of $G^{(i+1)}$ by an edge, $\forall i = 1,2,...,n-1$ is called a path union of n copies of a graph G. Also if the same vertex v of $G^{(n)}$ join by an edge with v of $G^{(1)}$, then such graph is known as cycle graph of n copies of G. These are denoted by $P(n.G)$ and $C(n.G)$ respectively. Obviously $P(n.K_1) = P_n$ and $C(n.K_1) = C_n$.*

Definition 1.3. *Let G be a graph on n vertices. The graph obtained by replacing each vertex of the star $K_{1,n}$ by a copy of G is called a star of G and is denoted by G^**

* E-mail: kananijagrutic@gmail.com

Definition 1.4. *Eight Sprocket graph is an union of eight copies of* C_{4n}*. If* $V_{i,j}(\forall\ i = 1, 2, ..., 8\ ,\ \forall\ j = 1, 2, ..., 4n)$ *be vertices of* i^{th} *copy of* C_{4n} *then we shall combine* $V_{1,4n}$ *and* $V_{2,1}$*,* $V_{2,4n}$ *and* $V_{3,1}$*,* $V_{3,4n}$ *and* $V_{4,1}$*,* $V_{4,4n}$ *and* $V_{5,1}$*,* $V_{5,4n}$ *and* $V_{6,1}$*,* $V_{6,4n}$ *and* $V_{7,1}$*,* $V_{7,4n}$ *and* $V_{8,1}$*,* $V_{8,4n}$ *and* $V_{1,1}$*, by a single vertex. Where* $n \in N - 1$*. So, graph seems like a Sprocket shape, and here number of Sprockets are eight, therefore, we gave name Eight Sprocket. It is denoted by* Sc_n *of n size, where* $n \in N - 1$*,* $\mid V(Sc_n) \mid= 16n - 8$*,* $\mid E(Sc_n) \mid= 16n$*. In this paper we introduced cordialness of Eight Sprocket graph, path union of Eight Sprocket graph, cycle of Eight Sprocket graph and star of Eight Sprocket graph. For detail survey of graph labeling, we refer Gallian [3].*

2. Main Section

Theorem 2.1. *An Eight Sprocket graph* Sc_n *is a cordial graph, where* $n \in N - \{1\}$*.*

Proof. Let $G = Sc_n$ be any Eight Sprocket graph of size n, where $n \in N - \{1\}$. We mention each vertices of Sc_n like $V_{i,j}$ $(\forall\ i = 1, 2, ..., 8,\ \ \forall\ j = 1, 2, ..., 4n)$. We see the numbers of vertices in G is $\mid V(Sc_n) \mid= p = 16n-8$ and $\mid E(Sc_n) \mid= q = 16n$. We define labeling function $f : V(G) \to \{0, 1\}$ as follows

$$f(v_{1,j}) = \begin{cases} 0, & \text{if } j = 1, 2, 5, 6, 9, 10, ..., 4n-3, 4n-2 \\ 1, & \text{if } j = 3, 4, 7, 8, 11, 12, ..., 4n-1, 4n. \end{cases}$$

$$f(v_{2,j}) = \begin{cases} 0, & \text{if } j = 1, 2, 5, 6, 9, 10, ..., 4n-3, 4n-2 \\ 1, & \text{if } j = 3, 4, 7, 8, 11, 12, ..., 4n-1, 4n. \end{cases}$$

$$f(v_{3,j}) = \begin{cases} 0, & \text{if } j = 2, 3, 6, 7, 10, 11, ..., 4n-2, 4n-1 \\ 1, & \text{if } j = 1, 4, 5, 8, 9, 12, ..., 4n-3, 4n. \end{cases}$$

$$f(v_{4,j}) = \begin{cases} 0, & \text{if } j = 3, 4, 7, 8, 11, 12, ..., 4n-1, 4n \\ 1, & \text{if } j = 1, 2, 5, 6, 9, 10, ..., 4n-3, 4n-2. \end{cases}$$

$$f(v_{5,j}) = \begin{cases} 0, & \text{if } j = 1, 4, 5, 8, 9, 12, ..., 4n-3, 4n \\ 1, & \text{if } j = 2, 3, 6, 7, 10, 11, ..., 4n-2, 4n-1. \end{cases}$$

$$f(v_{6,j}) = \begin{cases} 0, & \text{if } j = 1, 2, 5, 6, 9, 10, ..., 4n-3, 4n-2 \\ 1, & \text{if } j = 3, 4, 7, 8, 11, 12, ..., 4n-1, 4n. \end{cases}$$

$$f(v_{7,j}) = \begin{cases} 0, & \text{if } j = 2, 3, 6, 7, 10, 11, ..., 4n-2, 4n-1 \\ 1, & \text{if } j = 1, 4, 5, 8, 9, 12, ..., 4n-3, 4n. \end{cases}$$

$$f(v_{8,j}) = \begin{cases} 0, & \text{if } j = 3, 4, 7, 8, 11, 12, ..., 4n-1, 4n \\ 1, & \text{if } j = 1, 2, 5, 6, 9, 10, ..., 4n-3, 4n-2. \end{cases}$$

□

Above labeling pattern give rise a cordial labeling to the graph G. So G is a cordial graph.

Illustration 2.2. *Eight Sprocket graph* Sc_n *is shown consisting* $n = 8$ *Sprockets with cordial labeling.*

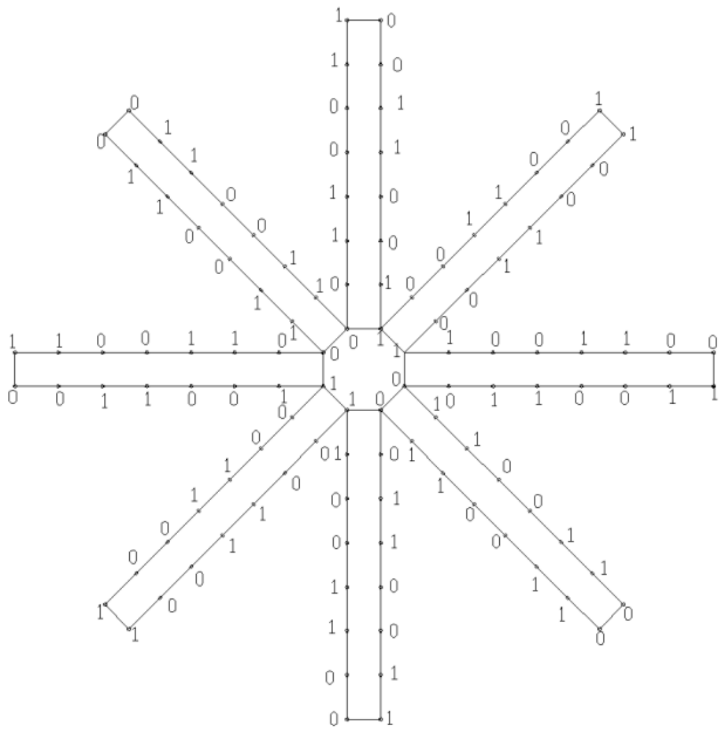

Figure 1. **Cordial labeling of Eight Sprocket graph with $p = 120$ and $q = 128$.**

Theorem 2.3. *Path union of finite copies of the Eight Sprocket graph Sc_n is a cordial graph, where, $n \in N - \{1\}$.*

Proof. Let $G = P(r.Sc_n)$ be a path union of r copies for the Eight Sprocket graph Sc_n, where $n \in N - \{1\}$. Let f be the cordial labeling of Sc_n as we mentioned in Theorem 2.1. In graph G, we see that the vertices $\mid V(G) \mid = P = r(16(n) - 8)$ and the edges $\mid E(G) \mid = Q = (r-1) + r16(n)$. Let $u_{k,i,j}$ ($\forall \ i = 1, 2, ..., 8, \forall \ j = 1, 2, ..., 4n$) be the vertices of k^{th} copy of Sc_n ($\forall \ k = 1, 2, ..., r$) where the vertices of k^{th} copy of Sc_n is $p = 16(n) - 8$ and edges of k^{th} copy of Sc_n is $q = 16n$. Join vertices $u_{k,1,2n+1}$ with $u_{k+1,1,2n+1}$ for $k = 1, 2, dots, r-1$ by an edge to form the path union of r copies of Eight Sprocket graph. To define labeling function $g : V(G) \rightarrow \{0, 1\}$ as follows

$$g(u_{1,i,j}) = \begin{cases} f(u_{,i,j}) \end{cases}$$

$$g(u_{2,i,j}) = \begin{cases} g(u_{1,i,j}) + 1, & \text{if } j = 1, 2, 5, 6, 9, 10, ..., 4n-3, 4n-2 \\ g(u_{1,i,j}) - 1, & \text{if } j = 3, 4, 7, 8, , ..., 4n-1, 4n \end{cases}$$

$$g(u_{3,i,j}) = \begin{cases} g(u_{2,i,j}) \end{cases}$$

$$g(u_{k,i,j}) = \begin{cases} g(u_{k-3,i,j}), & \text{if } k = 4, 5, 6, 7, 8, 9, ..., 3n+1, 3n+2, 3n+3. \end{cases}$$

□

Above labeling pattern give rise a cordial labeling to given graph G. So path union of finite copies of the Eight Sprocket graph is cordial graph.

Illustration 2.4. *Path union of 3 copies of Sc_3 and its cordial labeling shown in figure 2.*

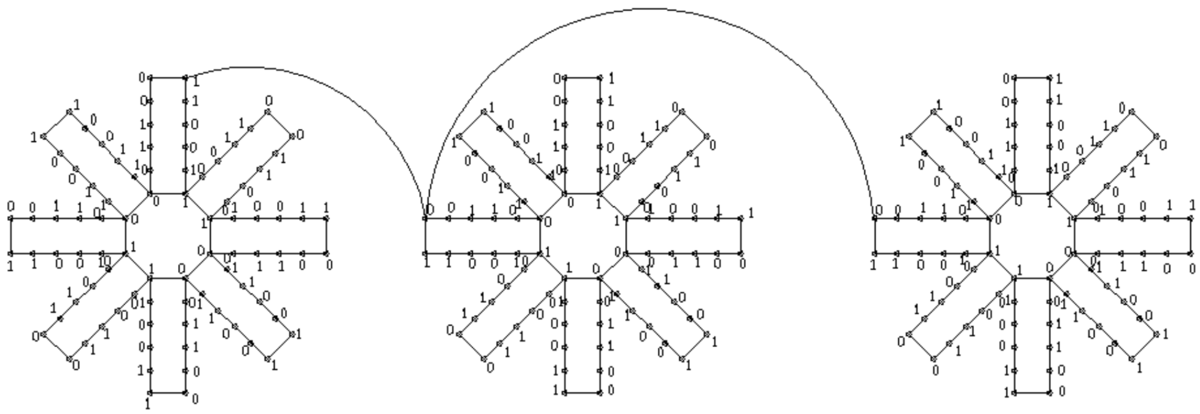

Figure 2. **A Path union of 3 copies of Sc_3 and its cordial labeling.**

Theorem 2.5. *Cycle of r copies of Eight Sprocket graph $C(r.Sc_n)$ is a cordial graph, where $n \in N-\{1\}$ and $r \equiv 0,3$ (mod 4).*

Proof. Let $G = C(r.Sc_n)$ be a cycle of Eight Sprocket graph Sc_n, where $n \in N - \{1\}$. Let f be the cordial labeling for Sc_n as we mentioned in Theorem 2.1. In graph G, we see that the vertices $\mid V(G) \mid= P = r(16(n) - 8)$ and the edges $\mid E(G) \mid= Q = r(16(n)+1)$. Let $u_{k,i,j}$ $(\forall\ i = 1, 2, ..., 8, \forall\ j = 1, 2, ..., 4n)$ be the vertices of k^{th} copy of Sc_n $(\forall\ k = 1, 2, ..., r)$, where the vertices of k^{th} copy of Sc_n is $p = 16(n) - 8$ and edges of k^{th} copy of Sc_n is $q = 16n$. Join vertices $u_{k,1,2n+1}$ with $u_{k+1,1,2n+1}$ for $k = 1, 2, \ldots, r - 1$ and $u_{r,1,2n+1}$ with $u_{1,1,2n+1}$ by an edge to form $C(r.Sc_n)$. We define labeling function $g : V(G) \to \{0, 1\}$ as follows

$$g(u_{1,i,j}) = \begin{cases} f(u_{,i,j}) \end{cases}$$

$$g(u_{2,i,j}) = \begin{cases} g(u_{1,i,j}) + 1, & \text{if } j = 1, 2, 5, 6, 9, 10, ..., 4n - 3, 4n - 2 \\ g(u_{1,i,j}) - 1, & \text{if } j = 3, 4, 7, 8, , ..., 4n - 1, 4n. \end{cases}$$

$$g(u_{3,i,j}) = \begin{cases} g(u_{2,i,j}) \end{cases}$$

$$g(u_{4,i,j}) = \begin{cases} g(u_{1,i,j}) \end{cases}$$

$$g(u_{k,i,j}) = \begin{cases} g(u_{k-3,i,j}), & \text{if } k = 5, 6, 7, 9, 10, ..., 4n + 1, 4n + 2, 4n + 3. \end{cases}$$

$$g(u_{k,i,j}) = \begin{cases} g(u_{k-4,i,j}), & \text{if } k = 8, 12, 16, 20, , ..., 4n + 4. \end{cases}$$

□

Above labeling pattern give rise a cordial labeling to cycle of r copies for Eight Sprocket graph.

Illustration 2.6. *Cycle of r copies for Eight Sprocket graph is cordial graph.*

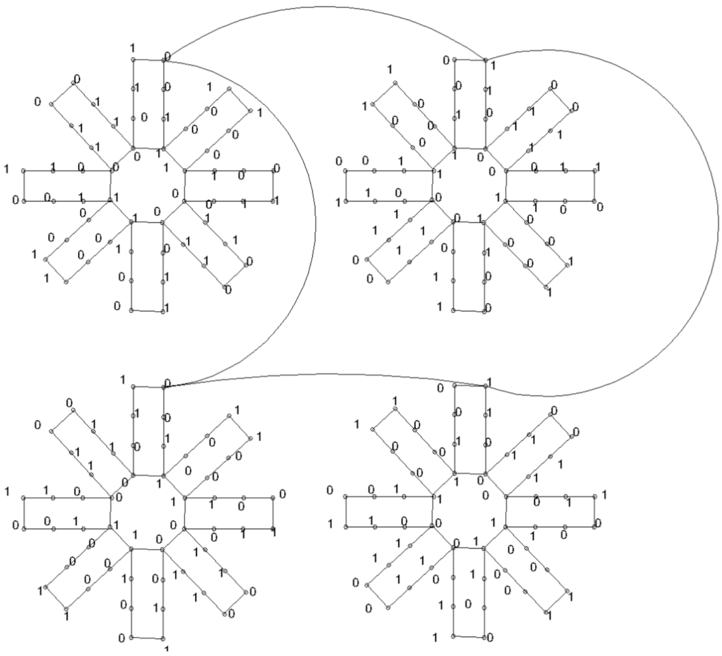

Figure 3. **Cycle of 4 copies of Eight Sprocket graph Sc_4 is cordial labeling.**

Theorem 2.7. *Star of Eight Sprocket graph (Sc_n)* is cordial, where* $n \in N - \{1\}$.

Proof. Let $G = (Sc_n)^*$ be a star of Eight Sprocket graph Sc_n, where $n \in N - \{1\}$, let f be the cordial labeling for Sc_n as we mention in Theorem 2.1. In graph G, we see that the vertices $| V(G) |= P = p(p+1)$ and the edges $| E(G) |= Q = (p+1)q + p$, where $p = 16(n) - 8$ and $q = 16(n)$. Let $u_{k,i,j}$ ($\forall\ i = 1, 2, ..., 8, \forall\ j = 1, 2, ..., 4n$) be the vertices of k^{th} copy of Sc_n ($\forall\ k = 1, 2, ...p$) where the vertices of k^{th} copy of Sc_n is $p = 16(n) - 8$ and edges of k^{th} copy of Sc_n is $q = 16(n)$. We mention that central copy of $(Sc_n)^*$ is $(Sc_n)^0$ and other copies of $(Sc_n)^*$ is $(Sc_n)^{(k)}$ ($\forall\ k = 1, 2, ..., p$). We define labeling function $g : V(G) \rightarrow \{0, 1\}$ as follows

$$g(u_{0,i,j}) = \begin{cases} f(u_{,i,j}) \end{cases}$$

$$g(u_{1,i,j}) = \begin{cases} g(u_{0,i,j}) + 1, & \text{if } j = 1, 2, 5, 6, , ..., 4n-3, 4n-2 \\ g(u_{0,i,j}) - 1, & \text{if } j = 3, 4, 7, 8, , ..., 4n-1, 4n. \end{cases}$$

$$g(u_{2,i,j}) = \begin{cases} g(u_{0,i,j}) \end{cases}$$

$$g(u_{3,i,j}) = \begin{cases} g(u_{1,i,j}) \end{cases}$$

$$g(u_{k,i,j}) = \begin{cases} g(u_{k-2,i,j}), & \text{if } k = 2, 3, 4, 5, , ..., 4n-2, 4n-1, 4n. \end{cases}$$

□

We see that the difference of vertices for the centre copy $(SC_n)^0$ of G and its other copies $(SC_n)^k$, $(1 \leq k \leq p)$ is g.
Using this sequence we can produce required edge label by joining corresponding vertices of $(Sc_n)^0$ with its other copy $(Sc_n)^k$ $(1 \leq k \leq p)$ in G. Thus G admits cordial labeling.

3. Concluding Remark

Present work contributes some new results. We discussed cordialness of Eight Sprocket graphs, path union of Eight Sprocket graph, cycle of Eight Sprocket graph and star of Eight Sprocket graph. The labeling pattern is demonstrated by means of illustrations which provide better understanding to derived results.

Acknowledgement

We are thankful to Prof. D. J. Marsonia for helping us in drawing 2D graphs with help of Autocad.

References

[1] G. S. Bloom and S. W. Golomb, *Application of numbered undirected graphs*, IEEE, 65(1977), 562-570.

[2] I. Cahit, *Cordial graphs: A weaker version of graceful and harmonious graphs*, Ars Combin, 23(1987), 201-207.

[3] J. A. Gallian, *A Dynamic survey of graph labeling*, The Electronics Journal of Combinatorics, 22(2019).

[4] S. W. Golomb, *How to number a graph, Graph Theory and Computing* , (R. C. Read. Ed.) Academic Press, (1972), 23–37.

[5] F. Harary, *Graph theory* , Narosa Publishing House, (2001).

[6] V. J. Kaneria, H. P. Chudasama and P. P. Andharia, *Absolute mean graceful labeling in path union of various graphs*, Mathematical Journal of Interdisciplinary Sciences, 7(2018), 51-56.