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k-Isolate Domination Number of Splitting Graph of Simple Graphs

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Abstract: A dominating set S of a graph G is said to be a k-isolate dominating set if $\langle S \rangle$ has at least k-isolated vertices [4]. In this paper the k-isolate dominating set of splitting graph of some graphs such as path, cycle and complete graphs are found.

MSC: 05C69.

Keywords: Dominating set, Isolate dominating set, k-isolate dominating set, k-isolate domination number, splitting graph, path, cycle and complete graph.

1. Introduction

In a graph $G = (V, E)$, the degree of a vertex v in V is the number of edges incident with v and is denoted by $\deg(v)$. A dominating set S is such that the sub graph $\langle S \rangle$ induced by S has at least one isolated vertex is called an isolate dominating set. The concept of isolate domination number is first developed by I.Sahul Hamid and S. Balamurugan [1]: A dominating set S of a graph G is said to be a k-isolate dominating set if $\langle S \rangle$ has at least k-isolated vertices [4]. The k-isolate dominating set S is said to be a minimal k -isolate dominating set if proper subset of S is not an isolate dominating set. Splitting graph was first studied by Sampathkumar and Walikar [2]. Also it was developed by Patil and Thangamari [3]. For each vertex v of a graph G, take a new vertex v' and v' is adjacent with all the vertices of G which are adjacent to v. The graph $S(G)$ thus obtained is called splitting graph of G. In this paper we discussed about the k-isolate domination number of splitting graph of path, cycle and complete graph.

2. Preliminary Results

Theorem 2.1 ([4]). *For the path P_n we have*

$$\gamma_{ki}(P_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil, & k \leq \left\lceil \frac{n}{3} \right\rceil \\ k, & \left\lceil \frac{n}{3} \right\rceil < k \leq \left\lceil \frac{n}{2} \right\rceil \\ \text{Does not exists}, & k > \left\lceil \frac{n}{2} \right\rceil \end{cases}$$

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Remark 2.2 ([5]). *Let C_n be a cycle with n vertices $(n \geq 3)$, then*

$$\gamma_{ki}(C_n) = \begin{cases} \lceil \frac{n}{3} \rceil, & k \leq \lceil \frac{n}{3} \rceil \\ k, & \lceil \frac{n}{3} \rceil < k \leq \lfloor \frac{n}{2} \rfloor \\ \text{Does not exists}, & k > \lfloor \frac{n}{2} \rfloor \end{cases}$$

3. Main Result

Theorem 3.1. *For the splitting graph of path $S[P_n]$ we have*

$$\gamma_{ki}[S(P_n)] = \begin{cases} 2\lceil \frac{n}{3} \rceil, & \text{for } k \leq 2\lceil \frac{n}{3} \rceil \\ k, & \text{for } 2\lceil \frac{n}{3} \rceil < k \leq 2\lceil \frac{n}{2} \rceil \\ \text{Does not exists}, & \text{for } k > 2\lceil \frac{n}{2} \rceil \end{cases}$$

Proof. Let P_n be the path on n vertices namely $\{v_1, v_2, \ldots, v_n\}$ and $S[P_n]$ be a splitting graph of path containing $2n$ vertices namely $\{v_1, v_2, \ldots, v_n, v_1{}', v_2{}', \ldots, v_n{}'\}$.

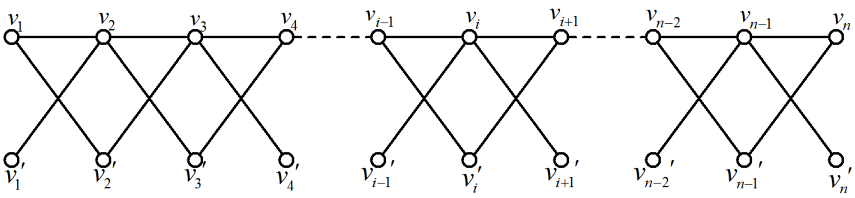

Figure 1. **Splitting graph of path**

Case (i): $k \leq 2\lceil \frac{n}{3} \rceil$.

We can split $S[P_n]$ into two paths namely P_1 and P_2 with each $'n'$ vertices. That is P_1 is a path with $v_1, v_2{}', v_3, v_4{}', \ldots, v_{n-1}{}', v_n$ vertices and P_2 is a path with $v_1{}', v_2, v_3{}', \ldots, v_{n-1}{}', v_n$ vertices and no one vertices are common to both the path. Hence by Case (i) of Theorem 2.1, we have,

$$\begin{aligned} \gamma_{ki}[S(P_n)] &= \gamma_{ki}[S(P_1)] + \gamma_{ki}[S(P_2)] \\ &= \left\lceil \frac{n}{3} \right\rceil + \left\lceil \frac{n}{3} \right\rceil \\ &= 2\left\lceil \frac{n}{3} \right\rceil \end{aligned}$$

Hence $\gamma_{ki}[S(P_n)] = 2\lceil \frac{n}{3} \rceil$, when $k \leq 2\lceil \frac{n}{3} \rceil$.

Case (ii): $2\lceil \frac{n}{3} \rceil < k \leq 2\lceil \frac{n}{2} \rceil$.

From Case (i), when $k = 2\lceil \frac{n}{3} \rceil$, we have $\gamma_{ki}[S(P_n)] = 2\lceil \frac{n}{3} \rceil$ with k vertices. Again by applying Case (ii) of Theorem 2.1, to $S[P_n]$, we have $\gamma_{ki}[S(P_n)] = k$ if $2\lceil \frac{n}{3} \rceil < k \leq 2\lceil \frac{n}{2} \rceil$.

Case (iii): $k > 2\lceil \frac{n}{2} \rceil$.

Clearly for the graph $S[P_n]$ which contains the pathP_1 and P_2, we have only $\frac{n}{2}$ vertices for the dominating set for each path. Hence when k exceeds more than $2\lceil \frac{n}{2} \rceil$ vertices, $\gamma_{ki}[S(P_n)]$ does not exists. □

Theorem 3.2. *Let $S(C_n)$ be the splitting graph of cycle C_n $(n \geq 3)$, then*

$$\gamma_{ki}[S(C_n)] = \begin{cases} 2\left\lceil \frac{n}{3} \right\rceil, & \text{for } k \leq 2\left\lfloor \frac{n}{3} \right\rfloor \\ k, & \text{for } 2\left\lfloor \frac{n}{3} \right\rfloor < k \leq n \\ \text{Does not exists, for } k > n \end{cases}$$

Proof. Let C_n be the cycle of n vertices with n edges whose degree of each vertex be two. Let the vertex of C_n be $\{v_1, v_2, \ldots, v_n\}$. And $S(C_n)$ be the splitting graph of C_n. Let the vertices of $S(C_n)$ be adjacent to $\{v_1, v_2, \ldots, v_n, v_1', v_2', \ldots, v_{n-1}', v_n'\}$ where v_i is not adjacent with v_i' and v_i is adjacent with $v_{i+1}', v_{i+1}, v_{i-1}, v_{i-1}'$where $i = 2, 3, 4, \ldots, n-1$ and in particular v_1 is adjacent with v_2, v_2', v_n, v_n'. Therefore, in $S(C_n), d(v_i) = 4, 1 \leq i \leq n$ and $d(v_i') = 2$, $1 \leq i \leq n$ where v_i' is adjacent with v_{i+1}, v_{i-1} and in particular v_1' is adjacent with v_2 and v_n.

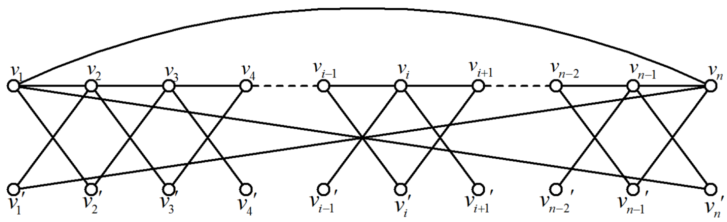

Figure 2. **Splitting graph of cycle**

Case (i): $k \leq 2\left\lceil \frac{n}{3} \right\rceil$.

In this case, the vertices of $S(C_n)$ can be rearranged as two cycles namely $v_1, v_2', v_3, v_4', \ldots, v_{n-1}', v_n, v_1$ and $v_1', v_2, v_3', \ldots, v_{n-1}', v_n, v_1'$ along with the edge v_1v_n containing the same $2n$ vertices. Consider the set of vertices $\{v_2', v_5, v_8', \ldots, v_{n-4}', v_{n-1}\}$ which belongs to the cycle $C_n(1)$ and $\{v_2, v_5', v_8, \ldots, v_{n-4}, v_{n-1}'\}$ which belongs to the cycle $C_n(2)$ (where the cycle $C_n(1)$ and $C_n(2)$ connected with the edge v_1v_n) in which the above two sets does not adjacent with each other. The above two sets together forms a minimal $k-$isolate dominating set. Hence by Case (i) of Remark 2.2, we have when $k \leq 2\left\lceil \frac{n}{3} \right\rceil$

$$\begin{aligned} \gamma_{ki}[S(C_n)] &= \gamma_{ki}[S(C_n(1))] + \gamma_{ki}[S(C_n(2))] \\ &= \left\lceil \frac{n}{3} \right\rceil + \left\lceil \frac{n}{3} \right\rceil \\ &= 2\left\lceil \frac{n}{3} \right\rceil \end{aligned}$$

Hence $\gamma_{ki}[S(C_n)] = 2\left\lceil \frac{n}{3} \right\rceil$, when $k \leq 2\left\lceil \frac{n}{3} \right\rceil$.

Case (ii): $2\left\lceil \frac{n}{3} \right\rceil < k \leq n$.

By Case (i), We have $k-$isolate dominating set as $\{v_2, v_2', v_5, v_5', v_8, v_8', \ldots, v_{n-4}, v_{n-4}', v_{n-1}, v_{n-1}'\}$ with k vertices when $k = 2\left\lceil \frac{n}{3} \right\rceil$. To obtain the $(k+1)-$isolate dominating set, v_2 is replaced with v_1' and v_3'. Hence we get the minimal $(k+1)-$isolate dominating set as $\{v_1', v_2', v_3', v_5, v_5', v_8, v_8', \ldots, v_{n-4}, v_{n-4}', v_{n-1}, v_{n-1}'\}$ containing $k+1$ vertices. Again to obtain the $(k+2)-$isolate dominating set v_5 is replaced with v_4' and v_6'. Continuing this process by replacing the vertex v_i by v_{i-1}' and v_{i+1}', we get the maximal $k-$isolate dominating set $\{v_1', v_2', v_3', \ldots, v_n'\}$. Hence $\gamma_{ki}[S(C_n)] = k$ when $2\left\lceil \frac{n}{3} \right\rceil < k \leq n$.

Case (iii): $k > n$.

Let $k = n$. By Case (i), $\{v_1', v_2', v_3', \ldots, v_n'\}$ is the minimal $k-$isolate dominating set. Since in $S(C_n)$ we have n vertices which are non-adjacent with themselves, there is no isolate dominating set with $(n+1)$ vertices. Hence when $k > n$, $\gamma_{ki}[S(C_n)]$ does not exists. ☐

Theorem 3.3. *If $S(K_n)$ is the splitting graph of the complete graph K_n,then*

$$\gamma_{ki}[S\,(K_n)] = \begin{cases} 2, for\ 1 \leq k \leq 2 \\ n, & for\ 2 < k \leq n \\ Does\ not\ exists, for\ k > n \end{cases}$$

Proof. Let K_n be the complete graph with n vertices. Let $S(K_n)$ be the splitting graph of the complete graph K_n with $2n$ vertices namely $v_1, v_2, v_3, \ldots, v_n, v_1{}', v_2{}', v_3{}', \ldots, v_n{}'$.

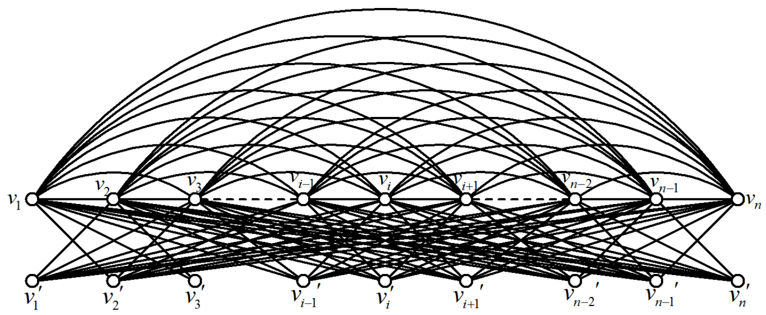

Figure 3. **Splitting graph of complete**

Case (i): $1 \leq k \leq 2$.

Obviously $\{v_i, v_i{}'\}(1 \leq i \leq n)$ is the minimal $2-$isolate dominating set. Hence $\gamma_{ki}[S(K_n)] = 2$, when $k \leq 2$.

Case (ii): $2 < k \leq n$.

Let $k = 2$. By Case (i), $\{v_i, v_i{}'\}(1 \leq i \leq n)$ is the minimal $k-$isolate dominating set. Since all the v_i $(1 \leq i \leq n)$ vertices are adjacent with each other and also with all $v_i{}'$ $(1 \leq i \leq n)$ vertices except their corresponding duplicate vertex, omit the particular v_i $(1 \leq i \leq n)$ vertex from the minimal $k-$isolate dominating set and add all the $v_i{}'$ $(1 \leq i \leq n)$ vertices (i.e.). $\{v_1{}', v_2{}', v_3{}', \ldots, v_n{}'\}$. Hence $\gamma_{ki}[S(K_n)] = n$, when $2 < k \leq n$.

Case (iii): $k > n$.

Obviously, $\gamma_{ki}[S(K_n)]$ does not exists when $k > n$. □

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