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Multiplicative Reduced Zagreb Indices of Some Networks

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Abstract: A graph index is a real number that is derived from molecular graphs of chemical compounds. In this paper, we introduce the multiplicative total reduced index, multiplicative reduced inverse degree, multiplicative reduced zeroth order index, general multiplicative reduced first Zagreb index of a graph and compute exact formulas for certain networks for chemical importance such as silicate networks, chain silicate networks, hexagonal networks, oxide networks and honeycomb networks.

MSC: 05C05, 05C07, 05C35.

Keywords: Multiplicative reduced first Zagreb index, multiplicative reduced F-index, multiplicative reduced zeroth order index, network.

1. Introduction

We consider only finite, simple, connected graph G with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. For other undefined notations, we refer [1]. In Chemical Graph Theory, the methods of graph index computation can help to find out chemical, biological information of drugs. So that chemical graph theory has an important effect on the development of chemical sciences. Numerous graph indices [2] considered in Theoretical Chemistry and have found same applications, especially in QSPR/QSAR study see [3, 4].

In [5], Kulli introduced the multiplicative reduced first Zagreb index and multiplicative reduced modified first Zagreb index of a graph, defined as

$$RM_1II(G) = \prod_{u\in V(G)} (d_G(u)-1)^2$$

$$^{m}RM_1II(G) = \prod_{u\in V(G)} \frac{1}{(d_G(u)-1)^2}.$$

Also in the same paper [5], Kulli introduced the multiplicative reduced F-index of a graph and it is defined as

$$RFII(G) = \prod_{u\in V(G)} (d_G(u)-1)^3.$$

We now introduce the following multiplicative reduced indices:

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The multiplicative total reduced index of a graph G is defined as

$$TRII(G) = \prod_{u \in V(G)} (d_G(u) - 1).$$

The multiplicative reduced inverse degree of a graph G is defined as

$$RIDII(G) = \prod_{u \in V(G)} \frac{1}{(d_G(u) - 1)}.$$

The multiplicative reduced zeroth order index of a graph G is defined as

$$RZII(G) = \prod_{u \in V(G)} \frac{1}{\sqrt{d_G(u) - 1}}.$$

We continue this generalization and introduce the general multiplicative first Zagreb index of a graph G, defined it as

$$RM_1^a II(G) = \prod_{u \in V(G)} (d_G(u) - 1)^a, \tag{1}$$

where a is a real number. Recently some reduced indices were studied, for example, in [6–20].
In this paper, the general multiplicative first Zagreb index for certain networks are computed. Also we compute some other multiplicative reduced indices for certain networks.

2. Result for Silicate Networks

Silicates are very interesting and most complicated minerals. These are obtained by fusing metal oxides or metal carbonates with sand. A silicate network of dimension n is denoted by SL_n, where n is the number of hexagons between the center and boundary of SL_n. A 2-dimensional silicate network is depicted in Figure 1.

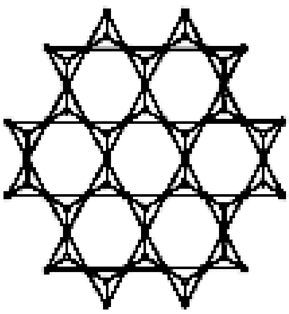

Figure 1. **A 2-dimensional silicate network**

Let G be the graph of a silicate network SL_n with $15n^2 + 3n$ vertices and $36n^2$ edges. From Figure 1, it is easy to see that the vertices of SL_n are either degree 3 or 6. In G, there are two types of vertices as given in Table 1.

$d_G(u) \setminus u \in V(G)$	3	6
Number of vertices	$6n^2 + 6n$	$9n^2 - 3n$

Table 1. **Vertex partition of SL_n**

Theorem 2.1. *The general multiplicative first Zagreb index of a silicate network SL_n is*

$$RM_1^a II\left(SL_n\right) = 2^{a\left(6n^2+6n\right)} \times 5^{a\left(9n^2-3n\right)}. \tag{2}$$

Proof. Let G be the graph of SL_n. From Equation (1) and by using Table 1, we deduce

$$\begin{aligned} RM_1^a II\left(SL_n\right) &= \prod_{u \in V(G)} \left(d_G\left(u\right)-1\right)^a \\ &= (3-1)^{a\left(6n^2+6n\right)} \times (6-1)^{a\left(9n^2-3n\right)} \\ &= 2^{a\left(6n^2+6n\right)} \times 5^{a\left(9n^2-3n\right)}. \end{aligned}$$

□

By using Definitions and from Theorem 2.1, we establish the following results.

Corollary 2.2. *Let SL_n be an 2-dimensional silicate network. Then*

(1). $RM_1 II\left(SL_n\right) = 2^{12n^2+12n} \times 5^{18n^2-6n}$.

(2). ${}^m RM_1 II\left(SL_n\right) = \left(\frac{1}{2}\right)^{12n^2+12n} \times \left(\frac{1}{5}\right)^{18n^2-6n}$.

(3). $RFII\left(SL_n\right) = 2^{18n^2+18n} \times 5^{27n^2-9n}$.

(4). $TRII\left(SL_n\right) = 2^{6n^2+6n} \times 5^{9n^2-3n}$.

(5). $RIDII\left(SL_n\right) = \left(\frac{1}{2}\right)^{6n^2+6n} \times \left(\frac{1}{5}\right)^{9n^2-3n}$.

(6). $RZII\left(SL_n\right) = \left(\frac{1}{2}\right)^{3n^2+3n} \times \left(\frac{1}{\sqrt{5}}\right)^{9n^2-3n}$.

Proof. Put $a = 2, -2, 3, 1, -1, -\frac{1}{2}$ in Equation (2), we get the desired results. □

3. Results for Chain Silicate Networks

We now consider a family of chain silicate networks. This network is obtained by arranging n tetrahedral linearly and is denoted by CS_n. A chain silicate network is shown in Figure 2.

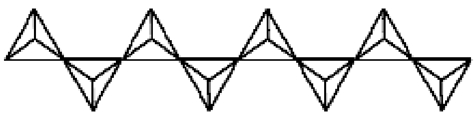

Figure 2. **A chain silicate network**

Let G be the graph of a chain silicate network CS_n with $3n+1$ vertices and $6n$ edges. The vertices of CS_n are either of degree 3 or 6. In G, there are two types of vertices as given in Table 2.

$d_G(u) \setminus u \in V(G)$	3	6
Number of vertices	$2n+2$	$n-1$

Table 2. **Vertex partition of CS_n**

Theorem 3.1. *The general multiplicative first Zagreb index of a chain silicate network CS_n is*

$$RM_1^a II\left(CS_n\right) = 2^{a(2n+2)} \times 5^{a(n-1)}. \tag{3}$$

Proof. Let G be the graph of CS_n. From Equation (1) and by using Table 2, we derive

$$\begin{aligned} RM_1^a II\left(CS_n\right) &= \prod_{u \in V(G)} \left(d_G\left(u\right) - 1\right)^a \\ &= (3-1)^{a(2n+2)} \times (6-1)^{a(n-3)} \\ &= 2^{a(2n+2)} \times 5^{a(n-1)}. \end{aligned}$$

□

By using definitions and from Equation (3), we obtain the following results.

Corollary 3.2. *Let CS_n be an 2-dimensional chain silicate network. Then*

(1). $RM_1 II\left(CS_n\right) = 2^{4n+4} \times 5^{2n-2}$.

(2). ${}^m RM_1 II\left(CS_n\right) = \left(\frac{1}{2}\right)^{4n+4} \times \left(\frac{1}{5}\right)^{2n-2}$.

(3). $RFII\left(CS_n\right) = 2^{6n+6} \times 5^{3n-3}$.

(4). $TRII\left(CS_n\right) = 2^{2n+2} \times 5^{n-1}$.

(5). $RIDII\left(CS_n\right) = \left(\frac{1}{2}\right)^{2n+2} \times \left(\frac{1}{5}\right)^{n-1}$.

(6). $RZII\left(CS_n\right) = \left(\frac{1}{2}\right)^{n+1} \times \left(\frac{1}{\sqrt{5}}\right)^{n-1}$.

Proof. Put $a = 2, -2, 3, 1, -1, -\frac{1}{2}$ in Equation (3), we obtain the desired results. □

4. Results for Hexagonal Networks

It is known that there exist three regular plane tilings with composition of same kind of regular polygons such as triangular, hexagonal and square. Triangular tiling is used in the construction of hexagonal networks. This network is denoted by HX_n, where n is the number of vertices in each side of hexagon. A 6-dimensional hexagon network is shown in Figure 3.

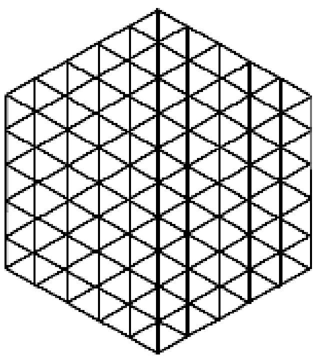

Figure 3. **Hexagonal network of dimension six**

Let G be the graph of a hexagonal network HX_n. By calculation, G has $3n^2-3n+1$ vertices and $9n^2-15n+6$ edges. From Figure 3, it is easy to see that the vertices of HX_n are either of degree 3, 4, or 6. In HX_n, there are three types of vertices as given in Table 3.

$d_G(u) \setminus u \in V(G)$	3	4	6
Number of vertices	6	$6n-12$	$3n^2-9n+7$

Table 3. **Vertex partition of** HX_n

Theorem 4.1. *The general multiplicative first Zagreb index of a hexagonal network* HX_n *is*

$$RM_1^a II(HX_n) = 2^{6a} \times 3^{a(6n-12)} \times 5^{a\left(3n^2-9n+7\right)}. \tag{4}$$

Proof. Let G be the graph of HX_n. From Equation (1) and by using Table 3, we obtain

$$\begin{aligned} RM_1^a II(HX_n) &= \prod_{u \in V(G)} (d_G(u)-1)^a \\ &= (3-1)^{6a} \times (4-1)^{a(6n-12)} \times (6-1)^{a\left(3n^2-9n+7\right)} \\ &= 2^{6a} \times 3^{a(6n-12)} \times 5^{a\left(3n^2-9n+7\right)}. \end{aligned}$$

□

By using definitions and Table 3, we establish the following results.

Corollary 4.2. *Let* HX_n*, be a hexagonal network. Then*

(1). $RM_1 II(HX_n) = 2^{12} \times 3^{12n-24} \times 5^{6n^2-18n+14}$.

(2). ${}^m RM_1 II(HX_n) = \left(\frac{1}{2}\right)^{12} \times \left(\frac{1}{3}\right)^{12n-24} \times \left(\frac{1}{5}\right)^{6n^2-18n+14}$.

(3). $RFII(HX_n) = 2^{18} \times 3^{18n-36} \times 5^{9n^2-27n+21}$.

(4). $TRII(HX_n) = 2^{6} \times 3^{6n-12} \times 5^{3n^2-9n+7}$.

(5). $RIDII(HX_n) = \left(\frac{1}{2}\right)^{6} \times \left(\frac{1}{3}\right)^{6n-12} \times \left(\frac{1}{5}\right)^{3n^2-9n+7}$.

(6). $RZII(HX_n) = \left(\frac{1}{2}\right)^{3} \times \left(\frac{1}{3}\right)^{3n-6} \times \left(\frac{1}{\sqrt{5}}\right)^{3n^2-9n+7}$.

Proof. Put $a = 2, -2, 3, 1, -1, -\frac{1}{2}$ in Equation (4), we obtain the desired results. □

5. Results for Oxide Networks

Oxide networks are of vital importance in the study of silicate networks. An n-dimensional oxide network is denoted by OX_n. A 5-dimensional oxide network is shown in Figure 4.

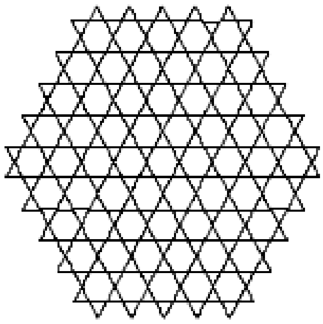

Figure 4. **A 5-dimensional oxide network**

Let G be the graph of an oxide network OX_n. By calculation, G has $9n^2+3n$ vertices and $18n^2$ edges. From Figure 4, it is easy to see that the vertices of OX_n are either of degree 2 or 4. In OX_n, there are two types of vertices as given in Table 4.

$d_G(u) \setminus u \in V(G)$	2	4
Number of vertices	$6n$	$9n^2-3n$

Table 4. **Vertex partition** OX_n

Theorem 5.1. *The general multiplicative first Zagreb index of an oxide network* OX_n *is*

$$RM_1^a II\left(OX_n\right) = 3^{a\left(9n^2-6n\right)}. \tag{5}$$

Proof. Let G be the graph of OX_n. From Equation (1) and by using Table 4, we deduce

$$\begin{aligned} RM_1^a II\left(OX_n\right) &= \prod_{u \in V(G)} \left(d_G\left(u\right)-1\right)^a \\ &= (2-1)^{a6n} \times (4-1)^{a\left(9n^2-3n\right)} = 3^{a\left(9n^2-3n\right)}. \end{aligned}$$

□

By using definitions and Table 4, we obtain the following results.

Corollary 5.2. *Let* OX_n *be an oxide network. Then*

(1). $RM_1 II\left(OX_n\right) = 3^{18n^2-6n}$.

(2). ${}^{m}RM_1 II\left(OX_n\right) = \left(\frac{1}{3}\right)^{18n^2-6n}$.

(3). $RFII\left(OX_n\right) = 3^{27n^2-9n}$.

(4). $TRII\left(OX_n\right) = 3^{9n^2-3n}$.

(5). $RIDII\left(OX_n\right) = \left(\frac{1}{3}\right)^{9n^2-3n}$.

(6). $RZII\left(OX_n\right) = \left(\frac{1}{\sqrt{3}}\right)^{9n^2-3n}$.

Proof. Put $a = 2, -2, 3, 1, -1, -\frac{1}{2}$ in Equation (5), we get the desired results. □

6. Results for Honeycomb Networks

A honeycomb network of dimension n is denoted by HC_n, where n is the number of hexagons between central and boundary hexagon. These networks are very useful in Computer Graphics and Chemistry. The number of vertices in HC_n is $6n^2$ and the number of edges in HC_n is $9n^2 - 3n$. A honeycomb network of dimension 4 is shown in Figure 5.

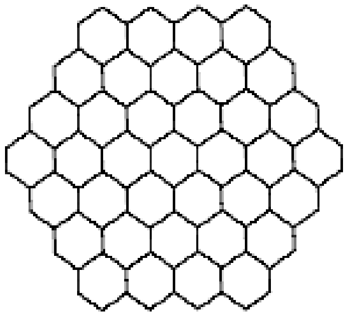

Figure 5. **Honeycomb network of dimension 4**

From Figure 5, it easy to see that the vertices of HC_n are either of degree 2 or 3. Let G be the graph of HC_n. In HC_n, there two types of vertices as given in Table 5.

$d_G(u) \setminus u \in V(G)$	2	3
Number of vertices	$6n$	$6n^2 - 6n$

Table 5. **Vertex partition of HC_n**

Theorem 6.1. *The general multiplicative first Zagreb index of a honeycomb network HC_n is*

$$RM_1^a II\,(HC_n) = 2^{a\left(6n^2-6n\right)}. \tag{6}$$

Proof. Let G be the graph HC_n. From Equation (1) and by using Table 5, we deduce

$$\begin{aligned} RM_1^a II\,(HC_n) &= \prod_{u \in V(G)} (d_G(u) - 1)^a \\ &= (2-1)^{a6n} \times (3-1)^{a\left(6n^2-6n\right)} = 2^{a\left(6n^2-6n\right)}. \end{aligned}$$

□

By using definitions and Table 5, we establish the following results.

Corollary 6.2. *Let HC_n be a honeycomb network. Then*

(1). $RM_1II\,(HC_n) = 2^{12n^2-12n}$.

(2). ${}^mRM_1II\,(HC_n) = \left(\frac{1}{2}\right)^{12n^2-12n}$.

(3). $RFII\,(HC_n) = 2^{18n^2-18n}$.

(4). $TRII\,(HC_n) = 2^{6n^2-6n}$.

(5). $RIDII\,(HC_n) = \left(\frac{1}{2}\right)^{6n^2-6n}$.

(6). $RZII\,(HC_n) = \left(\frac{1}{2}\right)^{3n^2-3n}$.

Proof. Put $a = 2, -2, 3, 1, -1, -\frac{1}{2}$ in Equation (6), we get the desired results. □

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