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On t-Perfect Codes in Corona Product of Graphs

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Abstract: A perfect code in a graph is a subset of a vertex set with the property that each vertex is adjacent to exactly one vertex in the subset. The corona product of two graphs G and H is the graph $G \circ H$ is obtained by taking one copy of G, called the centre graph and $|V(G)|$ copies of H, called the outer graph and by joining each vertex of the i^{th} copy of H to the i^{th} vertex of G, where $1 \leq i \leq |V(G)|$. The aim of this paper is to discuss the sufficient condition for the existence of t-perfect codes in corona product of two graphs.

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Keywords: Perfect code, radius of graph, corona product.

1. Introduction

In classical setting, a q-ary code is a subset of F_q^n, where q and n are positive integers and F_q^n is the set of codewords of length n over a set of size q. A q- ary code $C \subseteq F_q^n$ is called a perfect t-code, if every words in F_q^n is at distance no more than t to exactly one codeword of C, where the distance (Hamming distance) between two words is the number in which they differ. From the beginning of coding theory in the late 1940s perfect codes have been an important and interesting subject in information theory. Perfect codes in graphs have also attracted considerable attention, especially perfect codes in Cayley graphs of finite groups. In general, a product of two graphs G and H is a new graph whose vertex set is $V(G) \times V(H)$ and any of two vertices in the product, the adjacency of those two vertices are determined by the adjacency of the vertices of graph G and$H$. There are many graph operations helpful for computing invariants of big graphs in terms of invariants of their factors. Harary and Frucht [9] introduced a new product on two graphs G and H called corona product denoted by $G \circ H$. The corona product $G \circ H$ of two graphs G and H is obtained by taking one copy of G and $|V(G)|$ copies of H; and by joining each vertex of i^{th} copy of H to the i^{th} vertex of G, where $1 \leq i \leq |V(G)|$. The aim of this paper is to discuss the existence of t-perfect codes in corona product of graph. The paper is organized as follows: In section 2, we recall some basic concepts of graphs and codes. Section 3 will describe corona product of graph with example. In section 4, we will discuss some sufficient conditions for the existence of t-perfect codes in corona product of graphs.

2. Pre-requisites

In this section, we recall some basic concepts of graphs (finite, simple and connected) and codes (linear) which are essential for further discussions [3, 4, 7].

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2.1. Graphs

- A graph $G = (V, E)$ is a triple (V, E, φ) consisting of a non-empty set $V(G)$ of vertices, a set $E(G)$ of edges and an incidence function φ that associates with each edge of G a pair of vertices (not necessarily distinct) of G.
- Let $u, v \in V(G)$ are said to be adjacent, if there is an edge e, so that $\varphi(e) = (u, v)$ or $\phi(e) = (v, u)$ and the degree of a vertex u, denoted by $\deg(u)$, is the number of edges incident with it.
- The eccentricity $e(u)$ of a vertex u in a graph G is defined to be the largest distance between u and other vertices of G.
- For the graph $G = (V, E)$, the maximum eccentricity among the vertices is called diameter and is denoted by $diam(G)$. The minimum eccentricity among the vertices is called radius, denoted by $rad(G)$. The vertex v is a central vertex, if $e(v) = rad(G)$.

2.2. Codes and Perfect Codes

- In classical setting, if F_q is a field and $n \in N$, then the set of all n-tuples, F_q^n represents a vector space. A code C is any non-empty subset of the vector space F_q^n. The code C is called linear, if it is an F_q- linear subspace of F_q^n.
- The Hamming distance between vectors u and v in F_q^n is given by the number of non-zero entries in their difference. The Hamming weight of a vector v in F_q^n is given by the number of non-zero entries in v.
- A code C of length n and odd distance $d = 2t + 1$ is called a perfect code, if C attains the Hamming bound and also a perfect t-code, if every words in F_q^n is at a Hamming distance.
- Let $G = (V, E)$ be a graph (finite, simple and connected) and $t \geq 1$ an integer. A subset C of $V(G)$ is called a perfect t-code or t-perfect code in$G = (V, E)$, if every vertex of G is at a distance no more than t to exactly one vertex of C.
- The subset C of $V(G)$ of a graph $G = (V, E)$ is called a code and Suppose $S_t(G) = \{u/u \in V(G); d(u, v) \leq t\}$ for a positive integer t we call C a t-perfect code iff $\bigcup_{c \in C} S_t(c) = V(G)$ and $S_t(c_i) \bigcap S_t(c_j) = \phi$ for each c_i, c_j with $i \neq j$.

3. Corona Product of a Graph

Harary and Frucht [9] introduced a new product of on two graphs G and H called corona product denoted by $G \circ H$. Let and $H = (V', E')$ be the two graphs. The corona product of G and H is the graph $G \circ H$ is obtained by taking one copy of $G = (V, E)$, called the centre graph and $|V(G)|$ copies of H, called the outer graph and by joining each vertex of the i$^{\text{th}}$ copy of H to the i$^{\text{th}}$ vertex of G, where $1 \leq i \leq |V(G)|$. In general, the corona product $G \circ H$ are neither commutative nor associative.

The following Figures 1 & 2 depict the corona product $G \circ H$ of two graphs G and H.

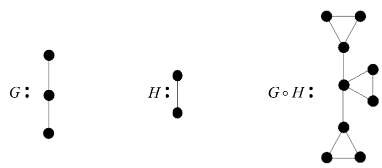

Figure 1.

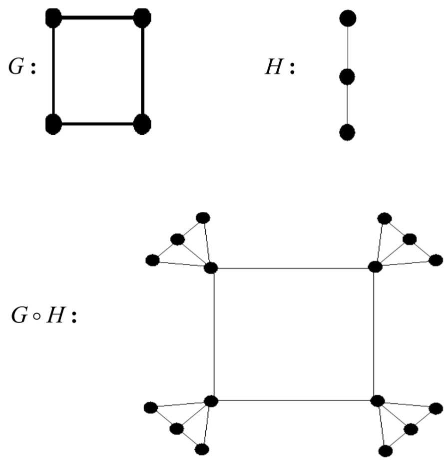

Figure 2.

4. Main Results

In this section, we discuss the sufficient condition for the existence of t-perfect codes in corona product of two graphs. Here G' denote the copy of G and H_i denotes i$^{\text{th}}$ copy of H corresponding to $x_i \in V(G')$ in $G \circ H$.

Proposition 4.1. *If C^* is a t-perfect code in $G \circ H$ such that $C^* \bigcap V(G') \neq \phi$ then $|C^*| = 1$.*

Proof. Let C^* is a t-perfect code in and $u \in V(G') \bigcap C^*$. Let us assume that $|C^*| > 1$. Then there exist $x_i \in V(G')$ such that $d(u, x_i) = t$ and so $d(u, w) = t + 1$, for each $w \in V(H_i)$ which is contradictory to the hypothesis. Hence, $|C^*| = 1$. □

Corollary 4.2. *If C^* is a t-perfect code in $G \circ H$ then all elements of C^* are either in G' or in $\bigcup_{i=1}^{|V(G)|} V(H_i)$.*

Proposition 4.3. *If C^* is a t-perfect code in $G \circ H$ such that $C^* \bigcap V(G') = \phi$, then $|C^* \bigcap V(H_i)| \leq 1$ for $i = 1, 2, ..., |V(G)|$.*

Proof. Let $u, v \in V(H_i)$, then $d(u, x_i) = d(v, x_i) = 1$, which is not possible. □

Proposition 4.4. *If $G \circ H$ has a t-perfect code C^* with $t > 1$ and $C^* \notin V(G')$ then $|C^*| = 1$.*

Proof. Suppose C^* is a t-perfect code in $G \circ H$ and $v \in C^* \bigcap V(H_i)$. Let us assume that $|C^*| > 1$ and $v \neq u \in C^*$. By Corollary 4.2. $u \notin V(G')$ and by Proposition 4.3. $u \in V(H_i)$ with $i \neq j$. Thus, there exist a vertex $x_i \in G'$ such that $d(u, x_i) = t$ and $d(w, u) = t + 1$ for each $w \in V(H_i)$ which is contradictory to the assumption that C^* is a t-perfect code. □

Proposition 4.5. *C^* is a 1-perfect code in $G \circ H$ whenever G is the trivial graph or* $\deg(H) = |V(H)| - 1$.

Proof. If G is the trivial graph and $u \in V(G')$, then $\{u\}$ is the 1-perfect code in $G \circ H$. Also, if u_i be a vertex of H_i corresponding to the vertex $u \in V(H)$ and if u is a vertex of degree $|V(H)| - 1$, then $\bigcup_{i=1}^{|V(G)|} \{u_i\}$ is the 1-perfect code in $G \circ H$. □

Proposition 4.6. *Let G and H be the two graphs and $t \geq 2$. Then $G \circ H$ is a t-perfect code if and only if, $rad(G) \leq t - 1$.*

Proof. Let $rad(G) \leq t - 1$ and x_i be the central vertex of G'. Then $\{x_i\}$ is the t-perfect code in $G \circ H$. Conversely, suppose that $G \circ H$ has t-perfect code C^* then there will be two possible cases.

Case (i): Let $C^* \bigcap G' = \phi$, then by Proposition 4.3, $|C^* \bigcap V(H_i)| \leq 1$ for $i = 1, 2, ..., |V(G)|$.

Let $rad\,(G) > t-1$ and let $u \in V\,(H_i)$ be an element of C^*. Since $rad\,(G) > t-1$, then there exist $(x_i x_j)$ is the edge in G' such that $d\,(u, x_i) = t$ and $d\,(u, x_j) = t+1$, which contradicts the assumption that C^* is a t-perfect code.

Case (ii): All elements of C^* are in G', then by Proposition 4.1, $|C^*| = 1$ and so $rad\,(G) \leq t-1$. □

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