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Approximation of the Series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4n-3}$

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Abstract: In this paper we give a rational approximation to an alternating series, by applying a correction function to the series. The introduction of correction function certainly improves the value of sum of the series and gives a better approximation to it.

Keywords: Correction function, error function, remainder term, rational approximation.

1. Introduction

Commenting on the Lilavati rule for finding the value of circumference of a circle from its diameter, the commentator Sankara refers to various infinite series for computing the circumference from the diameter. One such series attributed to illustrious mathematician Madhava of 14th century is $C = \frac{4d}{1} - \frac{4d}{3} + \frac{4d}{5} - \cdots \pm \frac{4d}{2n-1} \mp \frac{4d\left(\frac{2n}{2}\right)}{(2n)^2+1}$, where $+$ or $-$ indicates that n is odd or even and C is the circumference of a circle of diameter d.

2. The Alternating Series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4n-3}$

The alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4n-3}$ is a convergent series and converges to $\frac{[\pi + 2\ln(\sqrt{2}+1)]}{4\sqrt{2}}$

Definition 2.1. *The remainder term after n terms of the series is $R_n = (-1)^n G_n$ where G_n is the correction function after n terms.*

Definition 2.2. *If G_n denotes the correction function, then the error function is defined as $E_n = G_n + G_{n+1} - \frac{1}{4n+1}$.*

Theorem 2.3. *The correction function for the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4n-3}$ is $G_n = \frac{1}{8n-2}$.*

Proof. For a fixed n and for any $r \in R$ and for $G_n(r) = \frac{1}{8n+2-r}$ the error function $E_n(r)$ is minimum for $r = 4$. Thus the correction function for the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4n-3}$ is $G_n = \frac{1}{8n-2}$. The corresponding error function is $|E_n| = \frac{16}{256n^3 + 192^2 - 16n - 12}$. Hence the proof. □

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Remark 2.4. *Clearly* $G_n < \dfrac{1}{4n+1}$, *absolute value of* $(n+1)^{th}$ *term. Also* $\dfrac{1}{8n+2} < \dfrac{1}{8n-2} < \dfrac{1}{8n-6}$ *i.e.* $\dfrac{1}{2}\{|t_{n+1}|\} < G_n < \dfrac{1}{2}\{|t_n|\}$ *where tn denotes* n^{th} *term of the series.*

Theorem 2.5. *The error function* $E_n(r)$ *is continuous at* $r = 4$.

Proof. The proof is left to the reader. □

Theorem 2.6. *The error function* $E_n(r)$ *has a local minimum at the point* $r = 4$.

Proof. The proof is obtained from the proof of Theorem 2.3.
Now for $G_n(r) = \dfrac{1}{6n+2-r}$ where $r \in R$ and n is fixed, we have the error function $E_n(r)$ is minimum for $r = 3$. Thus the correction function for the series $\sum_{n=1}^{\infty}(-1)^{n-1}\dfrac{1}{3n-2}$ is $G_n = \dfrac{1}{6n-1}$. The corresponding error function is $|E_n| = \dfrac{9}{108n^3+108n^2+9n-5}$. Hence the proof. □

Remark 2.7. *Clearly* $G_n < \dfrac{1}{3n+1}$, *absolute value of* $(n+1)^{th}$ *term. Also* $\dfrac{1}{6n+2} < \dfrac{1}{6n-1} < \dfrac{1}{6n-4}$ *i.e.* $\dfrac{1}{2}\{|t_{n+1}|\} < G_n < \dfrac{1}{2}\{|t_n|\}$.

Proposition 2.8. *The error function* $|E_n(r)|$ *is continuous at* $r = 3$.

Theorem 2.9. *The error function* $|E_n(r)|$ *has a local minimum at the point* $r = 3$.

Proof. The proof is obtained from Theorem 2.3. □

References

[1] Konrad Knopp, *Theory and Application of Infinite series*, Courier Corporation, (1990).

[2] G. H. Hardy, *A Course of Pure Mathematics*, Cambridge University Press, (1908).

[3] K. Knopp, *Infinite sequences and series*, Dover, (1956).