International Journal of Mathematics And its Applications
Volume 4, Issue 2–A (2016), 45–49.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Soft g^*s-closed Mappings in Soft Topological Spaces

Research Article

M.Suraiya Begum[1]* and M.Sheik John[1]

1 Department of Mathematics, NGM College, Pollachi, Tamilnadu, India.

Abstract: Soft set theory was first proposed by Molodtsov [7] in 1999 as a general mathematical tool for dealing with not clearly defined objects. Modern topology depends strongly on the ideas of soft set theory.This prompted us to define soft g^*s-closed and soft g^*s-open mappings on soft topological spaces.

MSC: 54A40, 06D72.

Keywords: Soft Topological Spaces, Soft Closed, Soft Generalized Closed, Soft g^*s-closed set, Soft g^*s-closed mapping.

1. Introduction

The soft set theory is a rapidly processing field of mathematics. Molodtsov [7] shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, and so on. In 2010 Muhammad shabir, Munazza Naz [8] used soft sets to define a topology namely Soft topology. Soft generalized closed set was introduced by K.Kannan [4] in 2012. The investigation of generalized closed sets has led to several new and interesting concepts like new covering properties and new separation axioms. Some of these separation axioms have been found to be useful in computer science and digital topology. In this paper we defined soft g*s - closed mapping, soft g*s - open mapping and a detailed study of some of its properties in soft topological spaces.

2. Preliminaries

Throughout this paper, $(\tilde{X}, \tau, E)$ or $\tilde{X}$ denotes the soft topological spaces (STS in short). For a subset A_E of $\tilde{X}$, the closure, the interior and the complement of A_E are denoted by $cl(A_E)$, $int(A_E)$ and A_E^c respectively. We recall some basic definitions that are used in the sequel.

Definition 2.1 ([6])**.** *Let $\tilde{X}$ be an initial universal set and E be the set of parameters. Let $P(\tilde{X})$ denote the power set of $\tilde{X}$ and $A \subseteq E$. The pair F_E is called a soft set over $\tilde{X}$, where F is a mapping given by $F : A \to P(\tilde{X})$.*

Definition 2.2 ([6])**.** *A soft set F_E over $\tilde{X}$ is said to be*

(a). A null soft set, denoted by ϕ, if $\forall\ e \in E$, $F(e) = \phi$.

(b). An absolute soft set, denoted by $\tilde{X}$, if $\forall\ e \in E$, $F(e) = \tilde{X}$.

* *E-mail: suraiya0291@gmail.com*

The soft sets F_E over an universe $\tilde{X}$ in which all the parameters of the set E are same is a family of soft sets, denoted by $SS(\tilde{X})_E$.

Definition 2.3 ([8]). *Let τ be the collection of soft sets over $\tilde{X}$, then τ is said to be a soft topology on $\tilde{X}$ if*

(a). ϕ, $\tilde{X}$ are belongs to τ.

(b). The union of any number of soft sets in τ belongs to τ.

(c). The intersection of any two soft sets in τ belongs to τ.

In this case the triplet $(\tilde{X}, \tau, E)$ is called a soft topological space over $\tilde{X}$. and any member of τ is known as soft open set in $\tilde{X}$. The complement of a soft open set is called soft closed set over $\tilde{X}$.

Definition 2.4 ([4]). *Let $(\tilde{X}, \tau, E)$ be a soft topological space over $\tilde{X}$ and (F, E) be a soft set over $\tilde{X}$. Then*

(a). soft interior of a soft set F_E is defined as the union of all soft open sets contained in F_E. Thus $int(F_E)$ is the largest soft open set contained in F_E.

(b). soft closure of a soft set F_E is the intersection of all soft closed super sets F_E. Thus $cl(F_E)$ is the smallest soft closed set over $\tilde{X}$ which contains F_E.

We denote interior(resp. closure) of a soft set F_E as $int(F_E)$ (resp. $cl(F_E)$).

Definition 2.5 ([1]). *A subset F_E of a soft topological space $(\tilde{X}, \tau, E)$ is called*

(a). soft generalised closed (briefly g-closed) [1] if $cl(A_E) \tilde{\subseteq} U_E$ Whenever $A_E \tilde{\subseteq} U_E$ and U is soft open $\tilde{X}$.

(b). soft generalized-semi closed (soft gs-closed) if $scl(A_E) \tilde{\subseteq} U_E$ whenever $A_E \tilde{\subseteq} U_E$ and U_E is soft open in $\tilde{X}$.

(c). soft semi-generalised closed (briefly soft sg-closed) [9] $A_E \tilde{\subseteq} U_E$ and U_E is soft semiopen in $\tilde{X}$. Every soft semi closed set is soft sg-closed.

(d). soft generalised α-closed (briefly soft $g\alpha$-closed) [3] if $\alpha - cl(A_E) \tilde{\subseteq} U_E$ whenever $A_E \tilde{\subseteq} U_E$ and U is soft α-open in $\tilde{X}$.

Definition 2.6 ([2]). *Let $(\tilde{X}, \tau, E)$ be a soft topological space, F_E and G_E be soft closed sets in $\tilde{X}$ such that $F_E \tilde{\cap} G_E = \phi$. If there exist soft open sets A_E and B_E such that $F_E \tilde{\subseteq} A_E$, $G_E \tilde{\subseteq} B_E$ and $A_E \tilde{\cap} B_E = \phi$ then $\tilde{X}$ is called a soft normal space.*

Definition 2.7 ([9]). *Let $(\tilde{X}, \tau, E)$ be an STS and F_E be a subset of $\tilde{X}$. The set F_E is said to be soft g^*s - closed if $scl(F_E) \tilde{\subseteq} U_E$ whenever $F_E \tilde{\subseteq} U_E$ and U_E is soft gs- open set.*

Definition 2.8 ([9]). *Let A_E be a soft set in STS. Then soft g^*s- closure and soft g^*s-interior of A_E are ddefined as follows:*

(a). $g^*s \;\; cl(A_E) = \tilde{\bigcap}\{B_E : B_E$ is soft g^*s - closed set and $A_E \tilde{\subseteq} B_E\}$

(b). $g^*s \;\; int(A_E) = \tilde{\bigcup}\{C_E : C_E$ is soft g^*s - open set and $C_E \tilde{\subseteq} A_E\}$

Definition 2.9 ([1]). *A function $f : (\tilde{X}, \tau, E) \rightarrow (\tilde{Y}, \tau', E)$ is said to be soft continuous if $f^{-1}(G_E)$ is soft open in $(\tilde{X}, \tau, E)$ for each soft open set G_E of $(\tilde{Y}, \tau', E)$.*

3. Soft g^*s-closed and Soft g^*s-open Mappings

In this section we exhibit the concept of soft g^*s - closed and soft g^*s- open mappings and study some of its properties in soft topological spaces.

Definition 3.1. *A mapping $f : (\tilde{X}, \tau, E) \rightarrow (\tilde{Y}, \tau', E)$ is said to be soft g*s - closed mapping if for each soft closed set A_E of $\tilde{X}$, $f(A_E)$ is soft g^*s closed set in $\tilde{Y}$ and is called soft g*s open mapping if for each soft open set B_E of $\tilde{X}$, $f(B_E)$ is soft g^*s open in $\tilde{Y}$.*

Theorem 3.2. *Evert soft closed mapping is soft g^*s closed mapping.*

Proof. Let $f : (\tilde{X}, \tau, E) \rightarrow (\tilde{Y}, \tau', E)$ be a soft closed mapping. Let $A_E$be a soft closed set in $\tilde{X}$. Then $f(A_E)$ is soft closed in $\tilde{Y}$. Since every soft closed set is soft g^*s closed, $f(A_E)$ is soft g^*s closed. Hence f is soft g^*s closed mapping. ☐

Theorem 3.3. *Every soft g^*s closed mapping is soft g closed mapping.*

Proof. Let $f : (\tilde{X}, \tau, E) \rightarrow (\tilde{Y}, \tau', E)$ be a soft g^*s closed mapping and let V_E be a soft closed set in $\tilde{X}$. Then $f(V_E)$ is soft g^*s closed set in $\tilde{Y}$. Since every soft g^*s closed set is soft g closed, $f(V_E)$ is soft g closed. ☐

Theorem 3.4. *Every soft g^*s closed mapping is soft gs closed mapping.*

Proof. Let $f : (\tilde{X}, \tau, E) \rightarrow (\tilde{Y}, \tau', E)$ be a soft g^*s closed mapping and let V_E be a soft closed set in $\tilde{X}$. Then $f(V_E)$ is soft g^*s closed set in $\tilde{Y}$. Since every soft g^*s closed set is soft gs closed, $f(V_E)$ is soft gs closed. ☐

Theorem 3.5. *Every soft g^*s closed mapping is soft $g\alpha$ closed mapping.*

Proof. Let $f : (\tilde{X}, \tau, E) \rightarrow (\tilde{Y}, \tau', E)$ be a soft g^*s closed mapping and let V_E be a soft closed set in $\tilde{X}$. Then $f(V_E)$ is soft g^*s closed set in $\tilde{Y}$. Since every soft g^*s closed set is soft $g\alpha$ closed, $f(V_E)$ is soft $g\alpha$ closed. ☐

Theorem 3.6. *Every soft g^*s closed mapping is soft sg closed mapping.*

Proof. Let $f : (\tilde{X}, \tau, E) \rightarrow (\tilde{Y}, \tau', E)$ be a soft g^*s closed mapping and let V_E be a soft closed set in $\tilde{X}$. Then $f(V_E)$ is soft g^*s closed set in $\tilde{Y}$. Since every soft g^*s closed set is soft sg closed, $f(V_E)$ is soft sg closed. ☐

The converse of the above theorems may not be true as seen from the following counter examples.

Example 3.7. *Let $\tilde{X} = \{x_1, x_2, x_3\} = \tilde{Y}$ and $E = \{e_1, e_2, e_3\}$. Then $\tau = \{\phi, \tilde{X}, (F, E)\}$ is a soft topological space over $\tilde{X}$, $\tau' = \{\phi, \tilde{Y}, (G, E)\}$ is a soft topological space over $\tilde{Y}$. Now the soft sets over $\tilde{X}$ and $\tilde{Y}$are defined as follows: $F_1(e_1) = \{x_1\}$, $F_1(e_2) = \{x_1\}$, $F_2(e_1) = \{x_2\}$, $F_2(e_2) = \{x_2\}$, $F_3(e_1) = \{x_1, x_2\}$, $F_3(e_2) = \{x_1, x_2\}$. $G_1(e_1) = \{x_1\}$, $G_1(e_2) = \{x_1\}$, $G_2(e_1) = \{x_1, x_2\}$, $F_2(e_2) = \{x_1, x_2\}$, $G_3(e_1) = \{x_2, x_3\}$, $G_3(e_2) = \{x_2, x_3\}$. If the mapping $f : (\tilde{X}, \tau, E) \rightarrow (\tilde{Y}, \tau', E)$ is defined as $f(x_1) = x_1$, $f(x_2) = x_3$, $f(x_3) = x_2$. Considering the soft set $A_1(e_1) = \{x_3\}$, $A_1(e_2) = \{x_3\}$, it is clear that f is soft g^*s-closed mapping but not soft closed mapping.*

Example 3.8. *From the above Example, Considering the soft set $A_1(e_1) = \{x_1\}$, $A_1(e_2) = \{x_1\}$, it is clear that f is soft g-closed mapping but not soft g^*s closed mapping.*

Example 3.9. *Let $\tilde{X} = \{x_1, x_2, x_3\} = \tilde{Y}$ and $E = \{e_1, e_2, e_3\}$. Then $\tau = \{\phi, \tilde{X}, (F, E)\}$ is a soft topological space over $\tilde{X}$, $\tau' = \{\phi, \tilde{Y}, (G, E)\}$ is a soft topological space over $\tilde{Y}$. Now the soft sets over $\tilde{X}$ and $\tilde{Y}$are defined as follows: $F_1(e_1) = \{x_2\}$, $F_1(e_2) = \{x_1\}$, $F_2(e_1) = \{x_2\}$, $F_2(e_2) = \{x_2\}$, $F_3(e_1) = \{x_2, x_3\}$, $F_3(e_2) = \{x_2, x_3\}$. $G_1(e_1) = \{x_1\}$, $G_1(e_2) = \{x_1\}$, $G_2(e_1) = \{x_2, x_3\}$, $F_2(e_2) = \{x_2, x_3\}$, $G_3(e_1) = \{x_3\}$, $G_3(e_2) = \{x_3\}$. If the mapping $f : (\tilde{X}, \tau, E) \rightarrow$*

$(\tilde{Y}, \tau', E)$ is defined as $f(x_1) = x_2$, $f(x_2) = x_3$, $f(x_3) = x_2$. Considering the soft set $H_1(e_1) = \{x_1\}$, $H_1(e_2) = \{x_1\}$, and the soft set $B_1(e_1) = \{x_1, x_3\}$, $B_1(e_2) = \{x_1, x_3\}$, it is clear that f is soft $g\alpha$closed, gs closed and sg closed mapping but not soft g^*s-closed mapping .

Theorem 3.10. *If $f : (\tilde{X}, \tau, E) \to (\tilde{Y}, \tau', E)$ is a soft closed mapping and $g : (\tilde{Y}, \tau', E) \to (\tilde{Z}, \tau'', E)$ is soft g^*s closed mapping then $g \circ f : (\tilde{X}, \tau, E) \to (\tilde{Z}, \tau'', E)$ is a soft g^*s closed mapping.*

Proof. Let A_E be soft closed in $\tilde{X}$. Then $f(A_E)$ is soft closed in $\tilde{Y}$, since f is soft closed mapping. Now $g(f(A_E))$ is a soft g^*s closed set in $\tilde{Z}$, since g is soft g^*s closed mapping. Hence $g \circ f : (\tilde{X}, \tau, E) \to (\tilde{Z}, \tau'', E)$ is soft g^*s closed mapping. □

Example 3.11. *Let $\tilde{X} = \tilde{Z} = \{x_1, x_2, x_3\}$, $\tilde{Y} = \{x_1, x_2, x_3, x_4\}$ and $E = \{e_1, e_2\}$. Then $\tau = \{\phi, \tilde{X}, (F, E)\}$ is a soft topological space over $\tilde{X}$, $\tau' = \{\phi, \tilde{Y}, (G, E)\}$ is a soft topological space over $\tilde{Y}$ and $\tau'' = \{\phi, \tilde{Z}, (H_1, E), (H_2, E)\}$ is a soft topological space over $\tilde{Z}$. Here (F, E) is a soft set over $\tilde{X}$, (G, E) is a soft set over $\tilde{Y}$ and (H_1, E), (H_2, E) are soft sets over $\tilde{Z}$ defined as follows: $F(e_1) = \{x_1\}$, $F(e_2) = \{x_1\}$, $G(e_1) = \{x_1, x_3\}$, $G(e_2) = \{x_1, x_3\}$. $H_1(e_1) = \{x_3\}$, $H_1(e_2) = \{x_3\}$, $H_2(e_1) = \{x_1, x_2\}$, $H_2(e_2) = \{x_1, x_3\}$. If the mapping $I : (\tilde{X}, \tau, E) \to (\tilde{Y}, \tau', E)$ and $f : (\tilde{Y}, \tau', E) \to (\tilde{Z}, \tau'', E)$ defined as $f(x_1) = x_1$, $f(x_2) = f(x_4) = x_2$, $f(x_3) = x_3$. It is clear that each of I and f is soft g^*s-closed mapping but $f \circ I$ is not soft g^*s-closed, since $(f \circ I)^{-1}(H_1, E) = \{\{x_3\}, \{x_3\}\}$ is not a soft closed set over $\tilde{X}$.*

Theorem 3.12. *A mapping $f : (\tilde{X}, \tau, E) \to (\tilde{Y}, \tau', E)$ is soft g^*s closed mapping if and only if for each subset F_E of $\tilde{Y}$ and for each soft pen set G_E containing $f^{-1}(F_E)$ there is a soft g^*s open set H_E of $\tilde{Y}$ such that $F_E \tilde{\subseteq} H_E$ and $f^{-1}(H_E) \tilde{\subseteq} G_E$.*

Proof. Suppose f is a soft g^*s closed mapping and let F_E be a soft subset of $\tilde{Y}$ and G_E be soft open set of $\tilde{X}$ such that $f^{-1}(F_E) \tilde{\subseteq} G_E$. Then $H_E = \tilde{Y} - f(\tilde{X} - G_E)$ is a soft g^*s open set containing F_E such that $f^{-1}(H_E) \tilde{\subseteq} G_E$. Conversely, suppose F_E is soft closed in $\tilde{X}$. Then $f^{-1}(\tilde{Y} - f(F_E)) \tilde{\subseteq} \tilde{X} - F_E$, $\tilde{X} - F_E$ is soft open in $\tilde{X}$. By hypothesis, there is a soft g^*s open set V_E of $\tilde{Y}$ such that $\tilde{Y} - f(F_E) \tilde{\subseteq} V_E$ and $f^{-1}(V_E) \tilde{\subseteq} \tilde{X} - F_E \Rightarrow F_E \tilde{\subseteq} \tilde{X} - f^{-1}(V_E) \Rightarrow \tilde{Y} - V_E \tilde{\subseteq} f(F_E) \tilde{\subseteq} f(\tilde{X} - f^{-1}(V_E)) \tilde{\subseteq} \tilde{Y} - V_E \Rightarrow f(F_E) = \tilde{Y} - V_E$. Since$\tilde{Y} - V_E$ is a soft g^*s closed set in $\tilde{Y}$, $f(F_E)$ is soft g^*s closed set in $\tilde{Y}$. □

Theorem 3.13. *If a bijective mapping $f : (\tilde{X}, \tau, E) \to (\tilde{Y}, \tau', E)$ is soft g^*s closed and $H_E = f^{-1}(K_E)$ for some soft closed set K_E of $\tilde{Y}$, then $f : H_E \to (\tilde{Y}, \tau', E)$ is soft g^*s closed.*

Proof. Let F_E be a soft closed set in H_E. Then there exists a soft closed set G_E in $\tilde{X}$ such that $F_E = H_E \tilde{\cap} G_E \Rightarrow f(F_E) = f(H_E \tilde{\cap} G_E) = f(G_E) \tilde{\cap} K_E$. Since f is soft $g * s$ closed, $f(G_E)$ is soft g^*s closed in $\tilde{Y}$. Then $f(G_E) \tilde{\cap} K_E$ is soft g^*s closed $\Rightarrow f(F_E)$ is soft g^*s closed. □

Theorem 3.14. *If $f : (\tilde{X}, \tau, E) \to (\tilde{Y}, \tau', E)$ is soft continuous, soft g^*s closed map from a soft normal space $\tilde{X}$ onto a soft space $\tilde{Y}$, then $\tilde{Y}$ is soft normal.*

Proof. Let A_E, B_E be disjoint soft closed sets of $\tilde{Y}$. Then $f^{-1}(A_E)$, $f^{-1}(B_E)$ are disjoint soft closed sets of $\tilde{X}$. Since $\tilde{X}$ is soft normal, there are disjoint soft open set U_E, V_E of $\tilde{X}$ such that $f^{-1}(A_E) \tilde{\subseteq} U_E$ and $f^{-1}(B_E) \tilde{\subseteq} V_E$. By hypothesis, there exists soft g^*s closed sets C_E, D_E in $\tilde{Y}$ such that $A_E \tilde{\subseteq} C_E$ and $B_E \tilde{\subseteq} D_E$ and $f^{-1}(C_E) \tilde{\subseteq} U_E$, $f^{-1}(D_E) \tilde{\subseteq} V_E$. Since U_E and V_E are disjoint, $int(C_E)$ and $int(D_E)$ are also soft open sets. Since C_E is soft g^*s open, A_E is soft closed, $A_E \tilde{\subseteq} C_E \tilde{\subseteq} int(C_E)$ also $B_E \tilde{\subseteq} int(D_E)$. Hence $\tilde{Y}$ is soft normal. □

References

[1] I.Arockiarani and A.A.Lancy, *Generalized soft $g\beta$-closed sets and soft $gs\beta$-closed sets in soft topological spaces*, International journal of Mathematics Archive, 4(2013), 1-7.

[2] A.Aygunoglu and H.Aygun, *Some Notes on Soft Topological Spaces*, Neural Comput. and Applic., 21(1)(2012), 113-119.

[3] S.Hussain and B.Ahmad, *Some Properties of Soft Topological Spaces*, Comput.Math.Appl., 62(2011), 4058-4067.

[4] K.Kannan, *Soft Generalized Closed Sets in Soft Topological Spaces*, Journal of Theoretical and Applied Information Technology, 37(1)(2012), 17-21.

[5] J.Mahanta and P.K.Das, *On soft topological space via semiopen and semiclosed soft sets*, arXiv:1203.4133v1 [math.GN] 16 mar (2012).

[6] P.K.Maji, R.Biswas and A.R.Roy, *Soft set theory*, Computers and Mathematics with Applications, 45(2003), 555-562.

[7] D.Molodtsov, *Soft set Theory-First results*, Comput. Math. Appli., 37(1999), 19-31.

[8] M.Shabir and M.Naz, *On soft topological spaces*, Computers and Mathematics with Applications, 61(2011), 1786-1799.

[9] M.Suraiya Begum and M.Sheik John, *Soft g*s closed sets in soft topological spaces*, International Journal of Scientific Research and Education, 4(2016), 5466-5470.

[10] M.Suraiya Begum and M.Sheik John, *Soft g*s - continuous and soft g*s - irresolute functions in soft topological spaces*, International Journal of Mathematics Trends and Technology(accepted).

[11] S.Yuksel, Z.Guzel Ergul and N.Tozlu, *On Soft Compactness and Soft Separation Axioms*, Applied Mathematics and Information Sciences, (2013).